Signal Reconstruction After Severe Spectral Excision

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Abstract

When cochannel interference covers less than 100% of the spectral band of a signal, that interference can be totally spectrally excised without total excision of the signal. But severe signal distortion can result. When the signal is cyclostationary, some degree of signal restoration can be achieved by exploiting the spectral redundancy exhibited by such signals. For example, perfect reconstruction is theoretically possible for a BPSK signal with 100% excess bandwidth after 75% spectral excision. In this paper, several signal and interference scenarios are considered and the degree of signal restoration, after spectral excision, that is attainable using an adaptive FIR frequency-shift filter, is determined by computer simulation.

1 Introduction

The novelty of frequency-shift (FRESH) filtering relative to conventional linear time-invariant (LTI) filtering is that spectral components in some frequency bands are used to estimate, enhance, or cancel those in other bands by exploiting the spectral redundancy exhibited by cyclostationary signals [1]. Hence, in addition to the complex scaling operation that LTI filters apply to individual spectral components of the received signal, FRESH filters also use frequency-shifting operations. A canonical structure for FRESH filters consists of a parallel bank of frequency-shift operations each followed by an LTI filter, the outputs of which are summed to yield an estimate of the desired signal. This structure provides a general representation for linear periodic (or polyperiodic) time-variant filters of which harmonics of the reciprocal(s) of the (incommensurate) period(s) correspond to the frequency shifts in the FRESH filter.

Consider a complex-valued received signal $r(t)$ consisting of a cyclostationary signal of interest $s(t)$ plus noise and interference. The general expression for linear-conjugate-linear (LCL) FRESH filtering of $r(t)$ to produce an estimate $\hat{s}(t)$ of $s(t)$ is given by

$$\hat{s}(t) = \sum_{\tau} h_1(t,\tau) r(\tau) + \sum_{\tau} h_2(t,\tau) r^*(\tau), \qquad (1)$$

where $h_1(t,\tau)$ and $h_2(t,\tau)$ are periodically or polyperiodically time-varying linear filters with Fourier-series representations

$$h_1(t,\tau) = \sum_{p=0}^{L-1} h_p(t-\tau)\exp(i2\pi\alpha_p\tau) \qquad (2)$$

and

$$h_2(t,\tau) = \sum_{q=0}^{M-1} g_q(t-\tau)\exp(i2\pi\beta_q\tau). \qquad (3)$$

The inclusion of the conjugate signal in (1) can be essential (depending on the signal and/or interference types) for optimum time-variant filtering if $r(t)$ and $s(t)$ are analytic-signal or complex-envelope representations of two real-valued signals [1]. The frequency-shift parameters α_p and β_q can be any of the cycle frequencies and conjugate cycle frequencies of the signal and interference and their linear combinations with integer coefficients. (For unconstrained optimum FRESH filtering, the maximal set of such frequencies that are useful must be included.) A cycle frequency η is one for which the frequency-shifted version $x(t)e^{i2\pi\eta t}$ is correlated with $x(t+\tau)$ for some values of τ. A conjugate cycle frequency ν is one for which the frequency-shifted version $x(t)e^{-i2\pi\nu t}$ is correlated with $x^*(t+\tau)$ for some values of τ. The values L and M correspond to the number of linear paths and the number of conjugate-linear paths in the filter, respectively, and can be infinite in the optimum unconstrained filter. Using (2) and (3) in (1), we obtain

$$\hat{s}(t) = \sum_{p=0}^{L-1} h_p(t) \otimes [r(t)\exp(i2\pi\alpha_p t)] + \sum_{q=0}^{M-1} g_q(t) \otimes [r^*(t)\exp(i2\pi\beta_q t)], \qquad (4)$$

where $\otimes$ denotes convolution; (4) is equivalent to a multivariate (dimension $L+M$) LTI filtering problem in which the input signals are frequency-shifted

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versions of $r(t)$ and $r^*(t)$. The version of (4) that minimizes the MSE between $\hat{s}(t)$ and $s(t)$ yields the optimum FRESH filter, which is also known as the cyclic Wiener filter for optimally chosen L, M, $\{\alpha_p\}$, and $\{\beta_q\}$ [1]. Therefore, the FRESH filtering problem can (but does not necessarily) have a much larger dimension than the optimum LTI filtering problem does due to the need to simultaneously solve $(L + M)$ LTI filtering equations. It is also more challenging due to the important and difficult task of optimally (or, at least, adequately) choosing the numbers and values of frequency shifts.

FRESH filtering can accommodate much more challenging communications problems than LTI filtering methods can, because in cochannel interference environments where two or more signals overlap in both time and frequency, there is very little that an LTI filter can do to separate the signals. On the other hand, since many communications signals exhibit cyclostationarity, FRESH filters can be used to mitigate severe cochannel interference, and some signals that overlap completely can, in principle, be perfectly separated. Computer simulations reveal that moderate to significant improvements in signal quality in terms of mean-squared error (MSE) and bit-error rate (BER) can be obtained with the use of a FRESH filter instead of the optimum LTI filter. Examples of cochannel interference include those found in cellular radio communications and other dense communications environments where strong interference from nearby antennas occurs.

2 The LS FRESH Filter

In practical applications where adaptive FRESH filters are to be implemented, FIR (finite-impulse response) filters are used following the frequency-shifting modulators. For an FIR FRESH filter with weight vector $\mathbf{w}$, (4) can be expressed as

$$\hat{s}(t) = \mathbf{w}^H \mathbf{y}(t), \tag{5}$$

where the elements of $\mathbf{w}$ correspond to the coefficients of each of the LTI filters $h_p(t)$ and $g_q(t)$, and $\mathbf{y}(t)$ is defined by

$$\begin{aligned}\mathbf{y}(t) \triangleq\ & [p_{\alpha_0}(t-t_1)\cdots p_{\alpha_0}(t-t_m)\cdots \\ & p_{\alpha_{L-1}}(t-t_1)\cdots p_{\alpha_{L-1}}(t-t_m) \\ & p_{-\beta_0}(t-t_1)^*\cdots p_{-\beta_0}(t-t_m)^*\cdots \\ & p_{-\beta_{M-1}}(t-t_1)^*\cdots p_{-\beta_{M-1}}(t-t_m)^*]^T,\end{aligned} \tag{6}$$

where $p_\mu(t) \triangleq r(t)e^{i2\pi\mu t}$ is a frequency-shifted version of $r(t)$, the t_k are appropriately chosen delays (e.g. $t_k = k - m/2$, corresponding to an effectively non-causal filter that extends equally into the past and the future), and m is the length of each of the LTI filters $h_p(t)$ and $g_q(t)$.

The least squares (LS) weight vector $\mathbf{w}$ that minimizes the average-squared error (over N time samples), $\langle|\hat{s}(t) - s(t)|^2\rangle_N$, is given by

$$\mathbf{w} = \hat{\mathbf{R}}_{\mathbf{yy}}^{-1}\hat{\mathbf{R}}_{\mathbf{y}s}, \tag{7}$$

where $\hat{\mathbf{R}}_{\mathbf{yy}} = \langle\mathbf{y}(t)\mathbf{y}^H(t)\rangle_N$ and $\hat{\mathbf{R}}_{\mathbf{y}s} = \langle\mathbf{y}(t)s^*(t)\rangle_N$. (The notation $\langle\cdot\rangle_N$ represents time averaging over N samples, the $\hat{}$ symbol represents estimates obtained from finite-time averaging, and superscript H denotes conjugate transposition.)

This LS FRESH filter is used in the next section to illustrate attainable performance in several applications. The theoretical capability of optimum FRESH filters is characterized in [1], and the performance that is attainable with practical FIR FRESH filters is illustrated through simulations in [2]–[4] for problems of separating cochannel signals, e.g., by suppressing cochannel interference while preserving the signal of interest. In this paper, we consider the problem where partial-band cochannel interference (i.e., interference that overlaps only part of the spectrum of the signal) has been removed by spectral excision (band stop filtering), and FRESH filtering is to be used to remove the resultant distortion from the signal of interest by reconstructing its spectrum by exploiting its spectral redundancy.

Two types of partial-band interference problems are considered, one where a contiguous band of up to 75% of the band of a BPSK signal had been removed, and the other in which a comb notch filter removes about 50% of the band of a CPFSK signal. In the next section, the results of LS adaptive FRESH filtering (reconstruction) using a training signal for these two scenarios are reported. Progress on blind adaptation of FRESH filters is reported in [3].

3 Simulations

In the first simulation reported here, LS FRESH filtering with training signals is employed with four frequency-shift parameters: $\alpha_{0,1} = 0, -f_b, \beta_{0,1} = 2f_c, 2f_c + f_b$, where f_b and f_c are the baud-rate and carrier frequency, respectively, of the BPSK signal of interest. Table 1 (Cases 1–8) shows simulation parameters and results for restoring a BPSK signal with 100% excess bandwidth (EBW) raised-cosine pulses given only spectral components in the band $[B_l, B_h]$ as specified in the table. Noise is added to the signal before excision. The in-band SNR (denoted SNR in table) is computed using the 3-dB signal bandwidth f_b. The percent bandwidth remaining after excision (denoted % in table) is computed using the absolute bandwidth, $2f_b$, of the original signal.

In these simulations, the use of $N = 2048$ time samples and $m = 10$ filter weights yields reasonably good results. No significant improvement with either $m = 20$ or $m = 40$ was found. Results of Case 1 are shown in Figure 1. Figure 2 shows results of restoring a BPSK signal with rectangular pulse (Case 9).

The presence of noise in spectral bands where signal components are weak has adverse effects on the performance of the LS FRESH filter. As seen in the rectangular-pulse case, the DC component is not recoverable because the spectrum contains nulls at non-zero integer multiples of f_b. Similarly, for the raised-cosine pulse, reconstruction using the middle half of either sideband (Cases 7–8) yields much better performance compared with the use of the edge-quarter

Case	m	SNR	$[B_l, B_h]$	%	MSE (dB)
1*	10	30 dB	$[\frac{f_b}{2}, f_b]$	25	$-0.1 \rightarrow -5.0$
2	20	50 dB	$[\frac{f_b}{2}, f_b]$	25	$-0.1 \rightarrow -8.8$
3	10	50 dB	$[\frac{f_b}{2}, f_b]$	25	$-0.1 \rightarrow -9.2$
4	5	50 dB	$[\frac{f_b}{2}, f_b]$	25	$-0.1 \rightarrow -8.7$
5	40	50 dB	$[\frac{f_b}{2}, f_b]$	25	$-0.1 \rightarrow -8.4$
6	10	50 dB	$[\frac{f_b}{4}, f_b]$	38	$-1.0 \rightarrow -7.2$
7	10	25 dB	$[0, \frac{f_b}{2}]$	25	$-2.7 \rightarrow -23.7$
8	10	10 dB	$[0, \frac{f_b}{2}]$	25	$-2.4 \rightarrow -10.0$
9*	10	30 dB	$[\frac{f_b}{2}, 3f_b]$	–	$-0.5 \rightarrow -6.7$

* Shown in Figs. 1 and 2.

Table 1: FRESH filtering parameters and results for restoring excised BPSK signal with 100%-EBW raised-cosine (Cases 1–8) and rectangular (Case 9) pulses using 4 frequency-shift paths and N = 2048.

for a given level of additive white noise.

Comparison of Cases 3 and 6 indicates that the presence of additional spectral components does not necessarily improve performance and may in fact degrade performance due to demand on the filter to achieve appropriate weighting of the redundant components, as demonstrated in Case 6. In Case 3, frequency-shifted versions of the excised signal comprise disjoint and contiguous spectral bands that cover the entire band of the non-excised signal. Therefore, only in-band weighting and simple recombination are required, and as a result, Case 3 yields better performance than Case 6 does for the same number of filter weights and frequency shifts. However, it is conceivable that the performance order of these two cases will reverse as SNR is decreased, because Case 6 contains some stronger spectral components. Although BERs were not computed in these simulations, it can be easily seen from the time records in Figures 1–2 that the BER is very substantially reduced by the LS FRESH reconstruction filters.

In the second simulation, we consider the reconstruction of a CPFSK signal from which TV interference is removed by a comb notch filter. The CPFSK signal $s(t)$ is generated with carrier frequency $f_c = 0$, baud-rate $f_b = f_s/16 = 19.2$kHz, where $f_s = 307.2$kHz is the sampling frequency, a tone-separation of 100kHz, and spectrally-raised cosine pulses with 100% EBW. Noise is added to this signal and a TV interference suppressor is applied to the resulting noisy signal. Therefore, the input to the FRESH filter, $y(t)$, is given by $y(t) = [s(t) + n(t)] \otimes h(t)$, where $h(t) = q(t) \otimes q(t)$ is the impulse response of the TV suppressor that exploits line-to-line redundancy with $q(t) = \delta(t-T) - \frac{1}{2}\delta(t) - \frac{1}{2}\delta(t-2T)$. The specified value for $1/T$ is 15.75kHz. The transfer function of the suppressor is $H(f) = \exp(i4\pi fT)[1 - \cos(2\pi fT)]^2$. This is a comb notch filter.

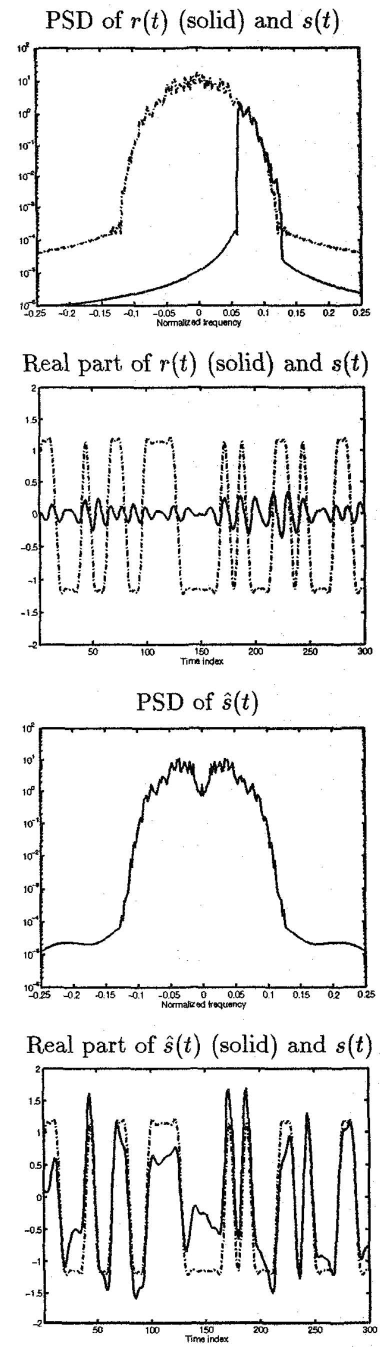

Figure 1: Restoration of an excised BPSK signal with 100%-EBW raised-cosine pulses, given only spectral components in $[f_b/2, f_b]$, with an adaptive 4-path LS FRESH filter, 10 weights per path, N = 2048, and 30dB in-band SNR. A 5dB MSE reduction is achieved – Case 1 in Table 1.

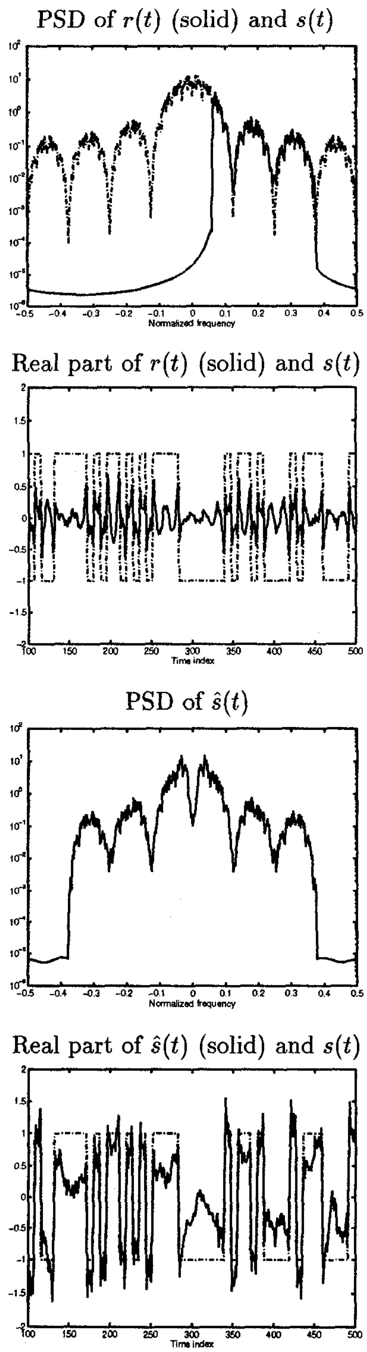

Figure 2: Restoration of an excised BPSK signal with rectangular pulses, given only spectral components in $[f_b/2, 3f_b]$, with an adaptive 4-path LS FRESH filter, 10 weights per path, N = 2048, and 30dB in-band SNR. A 6dB MSE reduction is achieved – Case 9 in Table 1.

The FRESH filter used in the simulations reported here consists of three frequency-shift paths corresponding to $\alpha_{0,1,2} = 0, \pm f_b$. In the two non-zero-shift paths, a complementary filter $G(f) = 1 - H(f)$ is inserted after the frequency-shift operation, prior to linear time-invariant filtering. This complementary filter helps alleviate the demand on the number of filter weights that would otherwise be required to reconstruct the signal.

In this simulation, $N = 4096$ and $m = 60$ is used. The in-band SNR is 30dB, and an MSE reduction of 9dB is obtained. It can be seen from the time records in Figure 4 that the BER is very substantially reduced.

References

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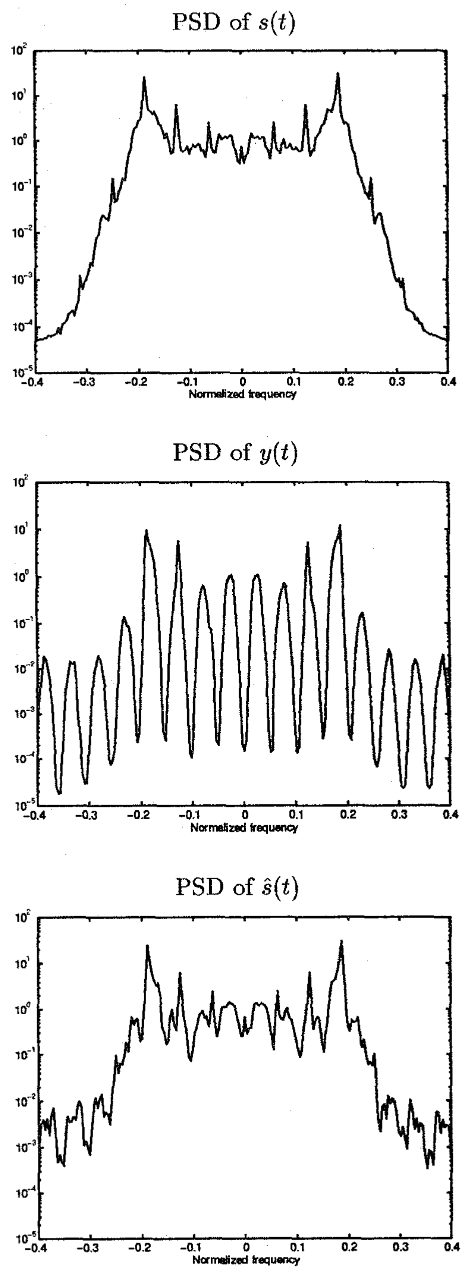

Figure 3: Restoration of a comb-notched CPFSK signal with spectrally-raised cosine pulse-shaping using an adaptive 3-path LS FRESH filter with 60 weights per path, N = 4096, and 30dB in-band SNR. A 9dB MSE reduction is achieved. (Power spectral densities shown.)

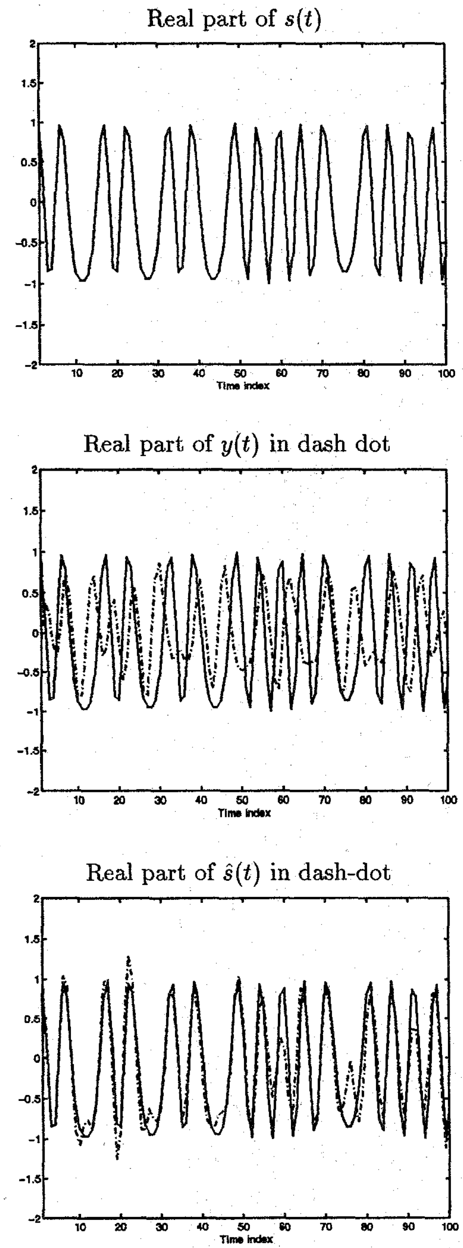

Figure 4: Restoration of a comb-notched CPFSK signal with spectrally-raised cosine pulse-shaping using an adaptive 3-path LS FRESH filter with 60 weights per path, N = 4096, and 30dB in-band SNR. A 9dB MSE reduction is achieved. (Time waveforms corresponding to Figure 3.)