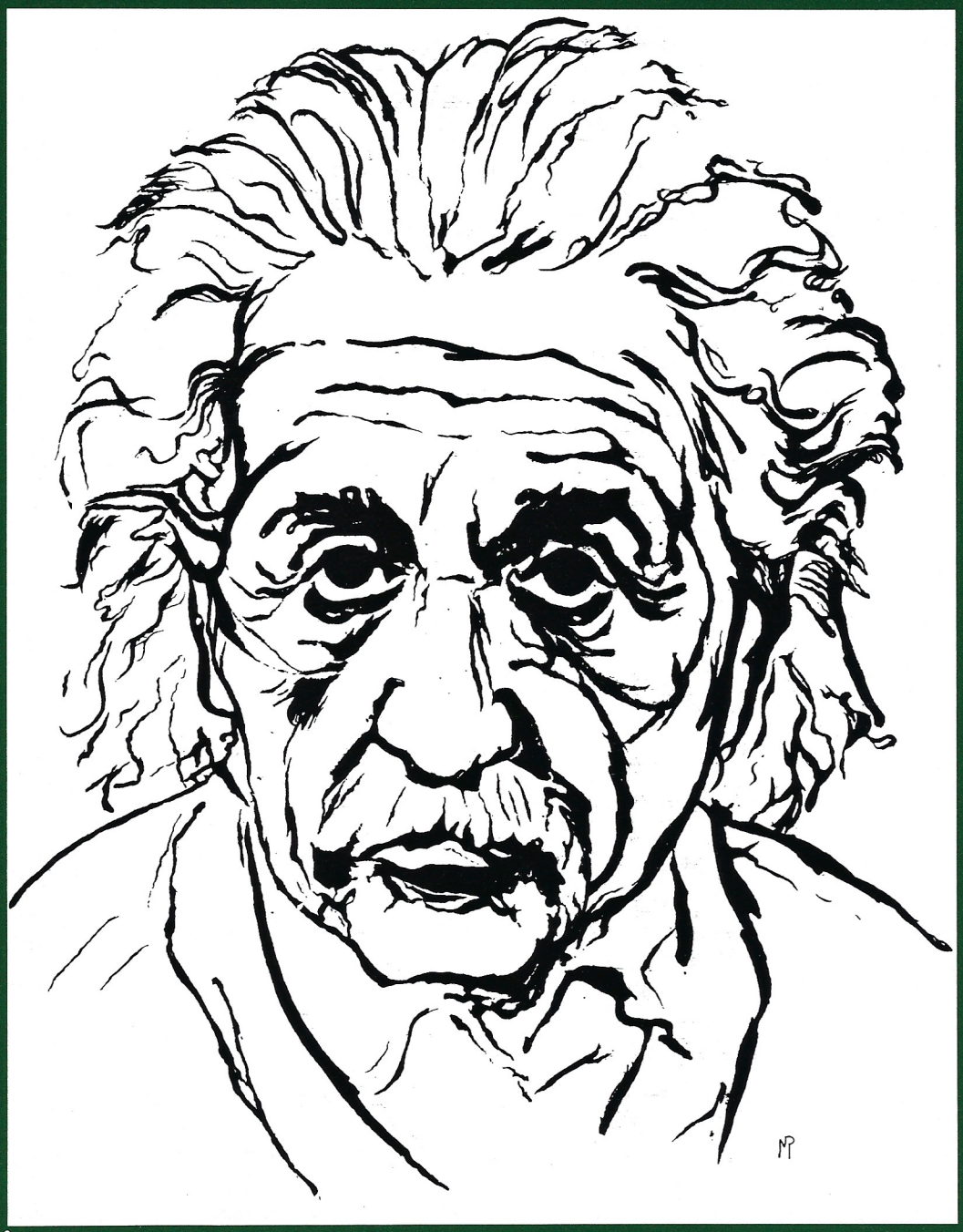
# IEEE ASSP MAGAZINE

A Publication of the IEEE Acoustics, Speech, and Signal Processing Society



OCTOBER 1987 VOLUME 4 NUMBER 4 (ISSN-0740-7467)

# Introduction to Einstein's Contribution to Time-series Analysis

*William A. Gardner*

THE HISTORY of ideas evolves endlessly, sometimes becoming more accurate with the passage of time, sometimes more distorted. This is so even for ideas originating as recently as 100 years ago. In spite of the documentation provided by published books and papers, personal letters, and recollections, historians are inevitably limited by the substantial loss of information with the passing of participants and by the unavoidable subjectivity of the historians' own interpretations of the available information. The recent discovery of a long forgotten early paper by Albert Einstein on the topic of time-series analysis [1] provides an interesting example of the evolutionary process of tracing the history of ideas. This very brief paper, entitled "Method for the determination of the statistical values of observations concerning quantities subject to irregular fluctuations," was originally published in the *Archives des Sciences Physiques et Naturelles* in 1914 following an oral presentation at a meeting of the Swiss Physical Society earlier that year. An English translation of this paper is reproduced in this issue of the *ASSP Magazine*. The paper discusses the autocorrelation function and its relationship to the spectral content of a time-series, the cross-correlation function as a measure of interdependence of two time-series, and two methods for measurement or computation of spectral content: the frequency-smoothed periodogram method and the Fourier-transformed autocorrelation method. Two experts in the field of time-series analysis, Professor A. M. Yaglom from the Institute of Atmospheric Physics, Academy of Sciences of the USSR, and Professor P. R. Masani from the Department of Mathematics, University of Pittsburgh, have written extensive commentaries on this paper and its place in the history of time-series analysis [2], [3]. The differences in the backgrounds and points of view of this physicist and this mathematician have led to some interesting differences in their interpretations of Einstein's accomplishments in his paper.

The author's address is: Signal and Image Processing Laboratory, Department of Electrical and Computer Engineering, University of California, Davis, CA 95616.

As an example of the differences in interpretation, Professor Yaglom argues that Einstein presents and proves the result that we currently refer to as the *Wiener-Khinchin relation* between the spectral density of average power of a time-series and its autocorrelation function (a result that Weiner and Khinchin each independently derived in the 1930s), whereas Professor Masani explains that the sketchy proof outlined by Einstein can be valid only for a restricted class of time-series that contains primarily sums of sine-waves and excludes essentially all time-series that exhibit random fluctuations. Professor Masani further argues that in order to obtain a valid proof of the relation for randomly fluctuating time-series, we must adopt Wiener's approach, which is based on the integrated spectral density [4]. However, it follows from the derivation given in [5] that Einstein's approach would have been valid for randomly fluctuating time-series if only he had incorporated a spectral smoothing operation in the expression from which he claims the desired relation follows. Although Einstein explicitly recognizes the need for spectral smoothing to reduce random fluctuations in practice, he does not include this crucial operation in his derivation of the relation. Without an appropriate treatment of the frequency smoothing operation (or an approximately equivalent time-averaging operation, which is explained in [5]) required for the limit spectral density function to exist, the Wiener-Khinchin relation is nothing more than the relatively simple periodogram-correlogram relation, which is just an application of the convolution theorem for Fourier series or Fourier transforms. This simpler result was well understood and used in practice long before Wiener's and Khinchin's derivations of their relation. In fact, the relationship between the spectral content of a time-series and the autocorrelation of that time-series was being used in meteorology as early as 1917 [6] and was apparently understood and used by Albert A. Michelson in his studies of the spectral content of lightwaves in the 1890s [7]. Michelson used a mechanical harmonic analyzer to compute the Fourier transform of an autocorrelation function obtained from an interferometer for the purpose of examining the fine structure of the spectral lines of light waves. Thus, besides seeing that we should be uncertain about



0740-7467/87/1000-0004$01.00©1987IEEE

Einstein's understanding of the subtleties of the Wiener-Khinchin relation, we also see that his ideas about the relationship between spectral content and the autocorrelation were not without precedent.

In connection with the Wiener-Khinchin relation, Professor Yaglom also argues that the proof proposed by Einstein is more physically lucid than those provided by Wiener and Khinchin, because Khinchin's derivation relies on the abstraction of probabilistic models and because Wiener's derivation is very complicated by virtue of the fact that he does not use probabilistic methods. This point of view provides another example of the evolutionary process of understanding and tracing the history of ideas in the field of time-series analysis. As illustrated by Professor Yaglom's comments, it has long been believed that without the abstraction of probabilistic concepts, we are forced to accept as the only other alternative Wiener's relatively complicated approach known as *generalized harmonic analysis* [4] in order to derive the Wiener-Khinchin relation. That this commonly held belief is false might well have been understood as early as the 1930s or 1940s, but this understanding, which might even have been possessed as far back as 1914 by Einstein, was not brought to the foreground where misconception could be dispelled until only recently [5].

Also related to the issue of the precedence of Einstein's ideas is the fact that Professor's Yaglom and Masani do not mention that the autocorrelation function was being used in economics as early as 1901 [8] and that the cross-correlation function was being used to study the interrelation of two time-series by investigators other than Einstein at least as early as 1914 [9].

In any case, the commentaries by Professors Yaglom and Masani are enlightening, and Einstein's paper certainly establishes that he was one of the earliest contributors of ideas of fundamental importance for time-series analysis. A valuable interpretation of the historical significance of this paper is presented in the excellent commentary by Professor Yaglom, the English translation of which is presented following the English translation of Einstein's paper. This commentary originally appeared in *Problemy Peredachi Informatsii* in 1985. For further reading in the history of spectral analysis, see [5], [10]–[13].

## REFERENCES


[1] A. Einstein, "Method for the determination of the statistical values of observations concerning quantities subject to irregular fluctuations," *Archives des Sciences et Naturelles,* vol. 37, 1914, pp. 254–256.

[2] A. M. Yaglom, "Einstein's 1914 paper on the theory of irregularly fluctuating series of observations," *Problemy Peredachi Informatsii,* vol. 21, 1985, pp. 101–107.

[3] P. R. Masani, "Einstein's contribution to Generalized Harmonic Analysis and his intellectual kinship with Norbert Wiener," *Jahrbuch Überblicke Mathematik,* 1986, pp. 191–209.

[4] N. Wiener, "Generalized harmonic analysis," *Acta Mathematika,* vol. 55, 1930, pp. 117–258.

[5] W. A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory,* Englewood Cliffs, N.J.: Prentice-Hall, 1987.

[6] H. H. Clayton, "Effect of short period variation of solar radiation on the earth's atmosphere," *Smithsonian Miscellaneous Collections,* vol. 68, no. 3, publ. 2446, 1917.

[7] A. A. Michelson and S. W. Stratton, "A new harmonic analyzer," *American Journal of Science,* vol. 5, 1898, pp. 1–13.

[8] R. H. Hooker, "Correlation of the marriage-rate with trade," *Journal of the Royal Statistical Society,* vol. 64, 1901, pp. 485–492.

[9] H. L. Moore, *Economic Cycles: Their Law and Cause,* New York: Macmillan, 1914.

[10] N. Wiener, "The historical background of harmonic analysis," *American Mathematical Society Semicentennial Publication,* vol. 2, 1938, pp. 56–68.

[11] H. T. Davis, *The Analysis of Economic Time Series,* Bloomington, Ind.: The Principia Press, 1941.

[12] D. R. Brillinger, "Some history of the data analysis of time-series in the United States," *History of Statistics in the United States,* ed. D. B. Owen, New York: Marcel Dekker, 1976, pp. 267–280.

[13] E. A. Robinson, "A historical perspective of spectrum estimation," *Proceedings of the IEEE,* vol. 70, 1982, pp. 885–907.