# Blind Adaptive Spatiotemporal Filtering for Wide-Band Cyclostationary Signals

Stephan V. Schell and William A. Gardner

*Abstract*—**An algorithm that blindly adapts spatiotemporal filters to extract from sensor array data one or more desired signals having known cyclostationarity properties is presented. It can perform well without knowledge of a training signal, spatiotemporal characteristics of the interference and noise, the directions of arrival of the desired signal(s), or any array calibration data.**

## I. Introduction

The need to spatially filter data received by a sensor array arises in many applications, including radar and sonar, communications, signals intelligence, geophysical and astrophysical exploration, and biomedicine. However, most existing methods for selecting the beamformer weights used in the spatial filter require prior knowledge of the signal or interference characteristics that may be difficult, costly, or simply impossible to obtain in some applications, especially for signals intelligence or where the receiver and/or transmitter(s) are moving. In particular, existing methods typically require known reference signals or spreading codes [1], [2], known

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spatial autocorrelation matrices of the desired signal and/or interference [3], known direction of arrival of the desired signal [4], or known array calibration data [5]. When the knowledge required by these methods is only approximate, the quality of the estimated signal can be degraded (e.g., via well-known signal cancellation effects [6]), although some methods [7] are more robust than others.

In contrast, the method in [8] adapts the array so as to maximize the degree of cyclostationarity (at a particular cycle frequency and time lag) of the output signal. This method, referred to as the spectral self-coherence restoral (SCORE) method, can accommodate multiple signals of interest, multiple interferers, and unknown noise, does not require complicated searches, and requires knowledge of only a cycle frequency (e.g., a baud rate or doubled carrier frequency) of the desired signal(s). Although knowledge of a cycle frequency may not be available in some applications, it can be estimated directly from the data, given a sufficiently large number of samples [9]. Thus, if the cycle frequency is known or can be estimated, SCORE can be a viable alternative to other methods of adaptive spatial filtering.

In this correspondence, the SCORE method is reinterpreted using the framework of canonical correlation analysis that is well known in the multivariate statistics community (e.g., see [10]) and is then generalized to accommodate wide-band received data that do not adhere to the narrow-band assumption. In Section II, some background information on cyclostationarity is presented, and canonical correlation analysis is explained briefly in Section III. The SCORE method for wide-band array data is presented in Section IV from the perspective of canonical correlation analysis. Results of computer simulations are presented in Section V to illustrate the performance of the new method and to compare it with that of the minimum-variance distortionless response beamformer.

## II. Cyclostationarity

In this section some relevant concepts from the theory of cyclostationary signals are presented. More detailed treatments can be found in [11], [12].

A zero-mean signal $s(t)$ exhibits second-order cyclostationarity if there exist one or more finite-amplitude additive sine waves having nonzero frequencies in one of its lag-product waveforms or, equivalently, if its cyclic autocorrelation function $R_{ss}^{\alpha}(\tau)$ is not identically zero for at least one nonzero value of the cycle frequency parameter $\alpha$, where $R_{ss}^{\alpha}(\tau)$ is defined as $R_{ss}^{\alpha}(\tau) \triangleq \langle s(t+\tau/2)s^*(t-\tau/2)e^{-j2\pi\alpha t}\rangle_{\infty}$, and $\langle\cdot\rangle_T$ denotes the time-average operation over a time interval of length $T$. Equivalently, a signal that exhibits second-order cyclostationarity at cycle frequency $\alpha$ contains spectral components separated in frequency by $\alpha$ that are correlated; that is, the signal exhibits spectral correlation. The degree to which a signal $s(t)$ exhibits correlation of spectral components centered at frequency $f$ and separated by $\alpha$ is specified by the spectral autocoherence magnitude

$$|C_{ss}^{\alpha}(f)| \triangleq \left|\frac{S_{ss}^{\alpha}(f)}{\sqrt{S_{ss}(f+\alpha/2)S_{ss}(f-\alpha/2)}}\right|$$

which is the magnitude of a correlation coefficient, where $S_{ss}^{\alpha}(f)$ is the cyclic spectrum (or spectral correlation) which is equal to the Fourier transform of the cyclic autocorrelation.

## III. Canonical Correlation Analysis

The task of determining weight vectors to extract $L$ desired signals from $\boldsymbol{x}(t)$ can be phrased in terms of the canonical correlation problem as follows. Given observations over a time interval $t \in [0, T]$ of two zero-mean signals $\boldsymbol{x}(t)$ and $\boldsymbol{r}(t)$ that both contain the desired signals but do not contain any other common components, find the two sets of $L$ weight vectors $\boldsymbol{W} = [\boldsymbol{w}_1 \cdots \boldsymbol{w}_L]$ and $\boldsymbol{C} = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_L]$ that extract the $L$ most significant signal components that are common to both $\boldsymbol{x}(t)$ and $\boldsymbol{r}(t)$, where the signals extracted from $\boldsymbol{x}(t)$ and $\boldsymbol{r}(t)$ are given by $\boldsymbol{y}(t) = \boldsymbol{W}^H\boldsymbol{x}(t)$, $\boldsymbol{u}(t) = \boldsymbol{C}^H\boldsymbol{r}(t)$, respectively. The standard solution to this problem (e.g., see [10]) is to find $\boldsymbol{w}_1$ and $\boldsymbol{c}_1$ that jointly maximize the cross-correlation coefficient between $y_1(t)$ and $u_1(t)$, and then to sequentially find $\boldsymbol{w}_l$ and $\boldsymbol{c}_l$ for $l = 2, \cdots, L$ that maximize the magnitude of the cross-correlation coefficient $\rho_{y_l u_l}$ between $y_l(t)$ and $u_l(t)$ such that $y_l(t)$ is uncorrelated with $y_1(t), \cdots, y_{l-1}(t)$ and $u_l(t)$ is uncorrelated with $u_1(t), \cdots, u_{l-1}(t)$. It is shown in [10] that the solutions are the eigenvectors satisfying $\hat{\boldsymbol{R}}_{xr}\hat{\boldsymbol{R}}_{rr}^{-1}\hat{\boldsymbol{R}}_{rx}\boldsymbol{w}_l = \lambda_l^{(w)}\hat{\boldsymbol{R}}_{xx}\boldsymbol{w}_l$ and $\hat{\boldsymbol{R}}_{rx}\hat{\boldsymbol{R}}_{xx}^{-1}\hat{\boldsymbol{R}}_{xr}\boldsymbol{c}_l = \lambda_l^{(c)}\hat{\boldsymbol{R}}_{rr}\boldsymbol{c}_l$, where $\lambda_l^{(w)}$ and $\lambda_l^{(c)}$ are the $l$th largest eigenvalues, respectively, the cross-correlation matrix $\hat{\boldsymbol{R}}_{xr}$ is defined as $\hat{\boldsymbol{R}}_{xr} \triangleq \langle \boldsymbol{x}(t)\boldsymbol{r}(t)^H\rangle_T$, and the autocorrelation matrices are defined in the obvious way. It can be shown [10] that $\lambda_l^{(w)} = \lambda_l^{(c)} = |\rho_{y_l u_l}|^2$ for all $l = 1, \cdots, L$, so the superscripted identifiers $(w)$ and $(c)$ are typically omitted from the symbols denoting the eigenvalues. It should be noted that choosing $\boldsymbol{r}(t) = \boldsymbol{x}(t-\tau)e^{j2\pi\alpha t}$ yields the narrow-band cross-SCORE method presented and derived from the alternative self-coherence restoral framework in [8].

## IV. Wide-Band SCORE

In wide-band environments the received data $\boldsymbol{x}(t)$ can be modeled in terms of finite-time Fourier transforms (FTFT):

$$\begin{aligned}\tilde{\boldsymbol{x}}(t,f) &\approx \sum_{l=1}^{L} \tilde{\boldsymbol{a}}(\theta_l, f)\tilde{s}_l(t,f) + \tilde{\boldsymbol{i}}(t,f)\\ &= \tilde{\boldsymbol{A}}(\Theta, f)\tilde{\boldsymbol{s}}(t,f) + \tilde{\boldsymbol{i}}(t,f)\end{aligned} \tag{1}$$

where

$$\tilde{\boldsymbol{x}}(t,f) \triangleq (1/\sqrt{\Delta})\int_{t-\Delta/2}^{t+\Delta/2} \boldsymbol{x}(t)e^{-j2\pi f t}\,dt$$

is the FTFT of the received data, $\tilde{\boldsymbol{a}}(\theta, f)$ is the transfer function of the array for a signal arriving from angle $\theta$, $\tilde{\boldsymbol{s}}(t,f)$ is the FTFT of the $L$ cyclostationary signals impinging on the array from angles $\theta_1, \cdots, \theta_L$, respectively, and having cycle frequency $\alpha$, and $\tilde{\boldsymbol{i}}(t, f)$ is the FTFT of all other signals and noise that do not have cycle frequency $\alpha$. The approximation in (1) holds well if the FTFT integration time $\Delta$ is greater than the duration of the impulse response of the array. Similarly, the FTFT of the $l$th extracted signal is given by $\tilde{y}_l(t,f) = \tilde{\boldsymbol{w}}_l(t,f)^H\tilde{\boldsymbol{x}}(t,f)$ for $l = 1, \cdots, L$.

In order to frame the problem of extracting the cyclostationary signals having cycle frequency $\alpha$ in terms of the canonical correlation analysis problem discussed in Section III, notice that the FTFT $\tilde{\boldsymbol{r}}(t,f)$ of the auxiliary signal defined by $\boldsymbol{r}(t) \triangleq \boldsymbol{x}(t)e^{j2\pi\alpha t}$ is given by

$$\begin{aligned}\tilde{\boldsymbol{r}}(t,f) &= \tilde{\boldsymbol{x}}(t,f-\alpha)\\ &= \tilde{\boldsymbol{A}}(\Theta, f-\alpha)\,\tilde{\boldsymbol{s}}(t,f-\alpha) + \tilde{\boldsymbol{i}}(t,f-\alpha).\end{aligned} \tag{2}$$

Since $\boldsymbol{s}(t)$ is cyclostationary with cycle frequency $\alpha$, $\tilde{\boldsymbol{s}}(t,f-\alpha)$ is correlated (shares a common component) with $\tilde{\boldsymbol{s}}(t,f)$. Consequently, a solution based on canonical correlation analysis can be found for each value of $f$:

$$\hat{\boldsymbol{S}}_{xr}(f)\hat{\boldsymbol{S}}_{rr}(f)^{-1}\hat{\boldsymbol{S}}_{rx}(f)\tilde{\boldsymbol{w}}_l(f) = \lambda_l^{(w)}(f)\hat{\boldsymbol{S}}_{xx}(f)\tilde{\boldsymbol{w}}_l(f) \tag{3}$$

$$\hat{\boldsymbol{S}}_{rx}(f)\hat{\boldsymbol{S}}_{xx}(f)^{-1}\hat{\boldsymbol{S}}_{xr}(f)\tilde{\boldsymbol{c}}_l(f) = \lambda_l^{(c)}(f)\hat{\boldsymbol{S}}_{rr}(f)\tilde{\boldsymbol{c}}_l(f) \tag{4}$$

where $\hat{\boldsymbol{S}}_{xr}(f)$ is the cross-correlation matrix of $\tilde{\boldsymbol{x}}(t,f)$ and $\tilde{\boldsymbol{r}}(t,f)$ and approaches the cross-spectrum matrix of $\boldsymbol{x}(t)$ and $\boldsymbol{r}(t)$ as $\Delta$ increases.

The role of cyclostationarity (or spectral coherence) in computing the weight vectors can be seen more clearly by noting that $\hat{S}_{xr}(f)$ is equal to the cyclic spectrum $\hat{S}_{xx}^{\alpha}(f - \alpha/2)$ and reinterpreting (3)–(4) accordingly. As in Section III, the eigenvalues $\lambda_l^{(w)}(f)$ and $\lambda_l^{(c)}(f)$ are identical for each $l$. For reasons explained in [8], the spatial filtering method based on the solution of (3)–(4) is called SCORE.

Although it is not immediately apparent from the preceding discussion, the support in the frequency domain of the weight vectors $\{w_l(f)\}$ computed according to (3) does not include the entire support of the desired signals. It can be shown that this problem is solved by defining an effective set of weight vectors, denoted by $\{\tilde{w}_l^e(f)\}$, as $\tilde{w}_l^e(f) = \tilde{w}_l(f)$ for $f_t < f \leq B_H$ and $\tilde{w}_l^e(f) = \tilde{c}_l(f + \alpha)$ for $B_L \leq f \leq f_t$, where the transition frequency $f_t$ can be set to any value in the interval $[B_L + \alpha, B_H - \alpha]$, and $\alpha$ is assumed to be positive and less than $(B_H - B_L)/2$. A similar construction can be made if $\alpha$ is negative. If $\alpha$ is greater than $(B_H - B_L)/2$ then the frequency range $[B_H - \alpha, B_L + \alpha]$ in the center of the receiver band $[B_L, B_H]$ is not covered by $\{\tilde{w}_l^e(f)\}$.

In practice, the received data may be decomposed into spectrally disjoint subbands rather than computing the FTFT for all possible $f$. Then (3) is solved once for each band, and the $L$ weight vectors resulting from a given band are then treated as being constant over this band. For example, in the simulations presented in Section V, eight bands are used.

For any particular frequency $f$, the capabilities of the wide-band SCORE method based on (3)–(4) are similar to those of the narrow-band SCORE algorithm presented in [8]. That is, if only one signal has the property that narrow-band components at frequencies $f$ and $f - \alpha$ are correlated, then $\tilde{w}_l^e(f)$ converges toward a weight vector that extracts a maximum-SNR estimate of the narrow-band component at frequency $f$ of that signal. If $L$ uncorrelated signals share this property (e.g., independent signals which have the same baud rate), then each of the $L$ weight vectors found for frequency $f$ extracts an estimate of the narrow-band component at frequency $f$ of a different signal, provided that the spectral autocoherence magnitude (defined in Section II) at frequency $f$ of each signal is different (e.g., see [8]).

It should be noted that the wide-band SCORE method presented here differs from the frequency-dependent SCORE method presented in [13]. In most environments of interest in which $\alpha < (B_H - B_L)/2$, the latter method requires that a complicated set of coupled matrix equations be solved, which hinders practical application.

Finally, although the wide-band SCORE method does not require knowledge of training signals, spreading codes, or the spatial characteristics of the signals or interference, it does require knowledge of a cycle frequency of the signal(s) of interest. In [9] it is shown (in the context of direction finding methods that exploit cyclostationarity) that the cycle frequency (normalized to the sampling rate) need be known or estimated only to within about $\pm 1/2N$, where $N$ is the number of data samples used to compute the weights of the spatial filter. Also, a simple method presented there requires only a search for peaks in the FFT of a lag-product waveform, and simulation results in [9] show that this method produced exact estimates for $N \geq 1024$ when the input SINR was 0 dB. Nonetheless, cycle frequency estimation and the degradation in performance due to cycle frequency error do merit further study.

## V. Simulation Results

Results from computer simulations of the wide-band SCORE algorithm processing wide-band data are presented here. The simulated environment models a low-SNR binary phase-shift-keyed direct-sequence spread-spectrum signal of interest hidden beneath five high-SNR narrow-band digital communication signals (also BPSK). The results are presented, followed by a brief discussion of their implications.

The array is linear and consists of four identical equispaced omnidirectional sensors; the sensor spacing is equal to one-half of the wavelength corresponding to the highest frequency in the receiver band. The lowest RF frequency is 0.75 times the highest, yielding a relative bandwidth (defined here as the ratio of the width to the center frequency of the receiver band) of about 28%. The noise consists of stationary complex white Gaussian noise that is uncorrelated from sensor to sensor. The signals not of interest (SNOI's) are identical BPSK signals having bit rate $1/10$ (normalized to the sampling rate) and independent and identically distributed (i.i.d.) random bit sequences and Nyquist-shaped pulses with 100% excess bandwidth. Five such SNOI's having carrier offsets (relative to the center frequency of the receiver band and normalized to the sampling rate) $-2/5$, $-1/5$, 0, $1/5$, and $2/5$ arrive from $-20°$, $10°$, $-10°$, $40°$, and $0°$, respectively. Each SNOI has in-band SNR (defined here as the ratio of the signal power to the power of the noise within the band occupied by the signal) equal to 15 dB. The signal of interest (SOI) is a BPSK signal having chip rate $1/2$ with chip sequence modeled as an i.i.d. random bit sequence, and rectangular chip pulses; it has zero carrier frequency offset and arrives from $25°$. The SNR of the SOI is $-5$ dB, for a total signal-to-interference-and-noise ratio (SINR) of $-20$ dB. The periodogram of the received data at one of the sensors is shown in Fig. 1, wherein the contribution of the SOI is obscured by the interference and noise.

One hundred independent trials are conducted in which the wide-band SCORE algorithm is applied using $\alpha = 1/2$ to extract the SOI. The wide-band data is decomposed into data from eight spectrally disjoint subbands using an FFT channelizer, such that each subband closely follows the narrow-band model. The average SINR (averaged over the 100 trials) of the extracted signal in each of the eight subbands is plotted in Fig. 2 for different numbers $N$ of data samples used to compute the weight vector. As can be seen from the figure, substantial improvement in SINR (from 5 to 15 dB higher than the input SINR, depending on the subband) is obtained when only 128 data samples are used to compute the weight vectors, and from 10 to 20 dB improvement when more data samples are used. Also, when greater than 2048 data samples are used, the output SINR of the SCORE processor is within a fraction of a decibel of the maximum-attainable SINR (which is computed using a perfectly known training signal).

For comparison, the minimum variance distortionless response (MVDR) beamformer (e.g., see [14]) is also simulated using knowledge of the exact direction of arrival (25°) of the signal of interest and perfectly known array calibration data. As shown in Fig. 3, the MVDR beamformer converges more quickly than SCORE. However, when the look direction is erroneously given as 35°, the SINR of the MVDR beamformer at convergence is about 6 dB lower than the SINR of the SCORE beamformer in the central subbands, as shown in Fig. 4.

These results indicate that the use of a sensor array for this cochannel interference problem could be exploited in the design of a spread spectrum communication system to accomplish either of two objectives without requiring a training signal or other potentially troublesome prior knowledge: 1) reduce the bit error rate in the despread SOI while keeping the spreading factor constant, or 2) allow a much smaller spreading factor to be used at the transmitter while keeping the bit error rate constant. For example, if a BER of $10^{-6}$ is desired (which can be attained with an uncoded BPSK sig-

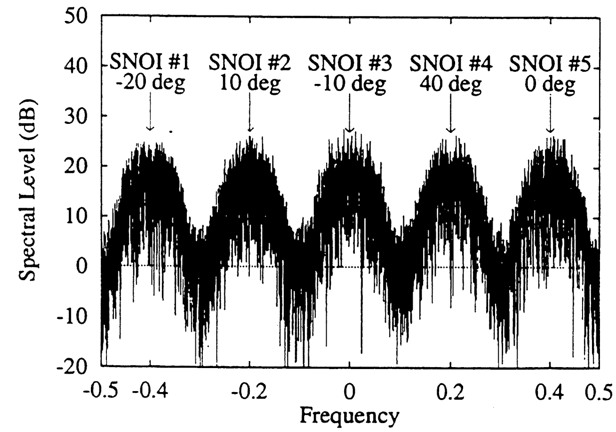


Fig. 1. Periodogram of the data received at one of the sensors. The five SNOI's are labeled with their DOA's. The SOI, which arrives from 25°, is not visible because its total SINR is −20 dB.

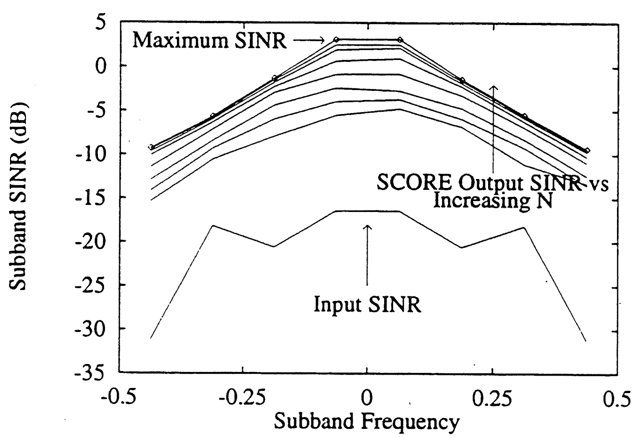


Fig. 2. Average SINR (averaged over 100 trials) in each of the eight subbands of SCORE processor output, plotted for different numbers $N$ of data samples: 128, 256, 512, $\cdots$, 8192. The input SINR at the first sensor and the maximum-attainable output SINR are shown at the extreme bottom and top of the graph, respectively.

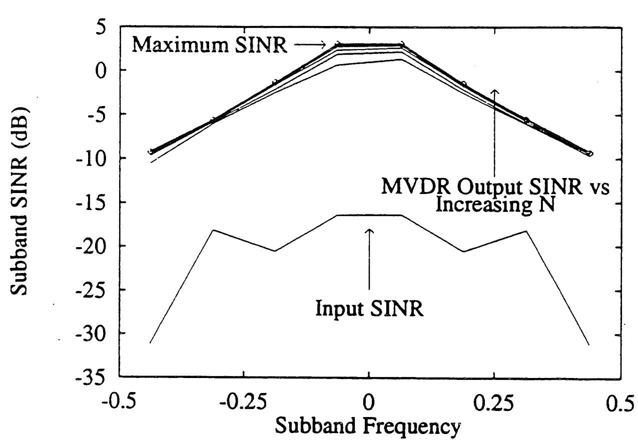


Fig. 3. Average SINR in each of the eight subbands of MVDR processor output using knowledge of the exact look direction and exact calibration data plotted for different numbers $N$ of data samples: 128, 256, 512, $\cdots$, 8192. The input SINR at the first sensor and the maximum-attainable output SINR are shown at the extreme bottom and top of the graph, respectively.

nal having about 10-dB SINR [15]), then a spreading factor (processing gain) of about 1000 would be needed without the use of the array to overcome the −20 dB SINR of the spread signal. In con-

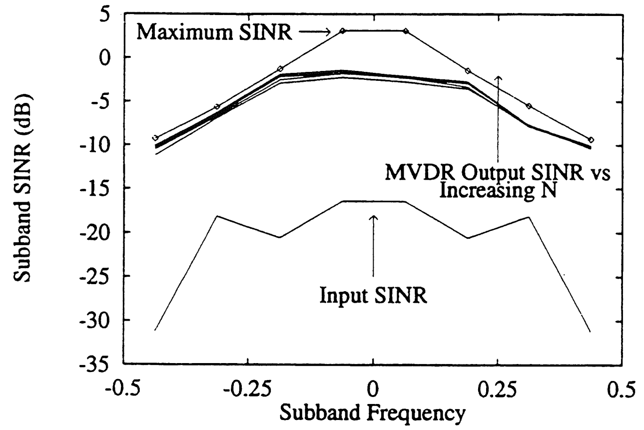


Fig. 4. Average SINR in each of the eight subbands of MVDR processor output using erroneous look direction, plotted for different numbers $N$ of data samples: 128, 256, 512, $\cdots$, 8192. The input SINR at the first sensor and the maximum-attainable output SINR are shown at the extreme bottom and top of the graph, respectively.

trast, the increase in output SINR of between 10 and 20 dB due to the SCORE processor implies that a much lower spreading factor between 100 and 10, respectively, would be needed.

The results also indicate that in signal interception applications, the SINR of the SOI at the output of the SCORE processor could be sufficiently high that a blind despreading technique might be applied successfully.

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