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maps are more interesting to a driver than the automobile's blueprints.

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AUTHOR'S COMMENT

Professor Hinich, of the University of Texas at Austin, has written a severely critical review [1] of the book [2], and of engineers and scientists in general with regard to their understanding of statistics. As author of this book, I am responding to this criticism not to try to establish whether or not this is a good book, but rather to counter Professor Hinich's bold claim that the philosophy put forth in the book, regarding the utility of a nonstochastic approach to time-series analysis, is misguided and that engineers and scientists who accept this philosophy as a viable conceptual tool do not understand statistics. In Professor Hinich's own words,

> "Both the author's Preface and the Forward by Bracewell reveal the confusion that reigns concerning the role of probability models in a statistical analysis of data...It is clear to me from reading this book that most engineers do not understand the need for careful thinking about statistical matters. It is time for the professional statisticians to rise to the challenge of teaching statistics to engineers and scientists."

Everyone agrees, I think, that stochastic processes have their place—they are indeed useful in some applications. The question before us is, "Must we accept stochastic processes as the only viable approach to dealing with time-series data that is erratic or unpredictable? Or is there a viable alternative that is more useful in some applications?"

The standard approach in statistics is well suited to experimental design, data analysis, and inference for populations: When a population actually exists in the real world, application of the standard conceptual framework of orthodox statistics can be appropriate. However, when no population associated with the available data exists or when no such population can exist (e.g., in astronomy, the concept of a population of universes is not usually considered viable—replicating the "experiment" of creating the universe is rather far-fetched), then the appropriateness of pretending that a population exists should be questioned.

Many statisticians accept and invoke the concept of a population in developing their theories and methods. When one has a single time-series of data to analyze and use as a basis for making inferences about the real-world situation that gave rise to the data, and when one knows that there can be no access to a population of such time-series (because it does not or cannot exist), then one would be remiss in not questioning the appropriateness of the orthodox conceptualization of the time-series as one member of a population, or ensemble, mathematically modeled as a stochastic process.

Many—but by no means all—real-world problems in communications engineering and signal processing involve time-series data for which no population exists, that is, data for which replication of the experiment is impossible or impractical. However, many of these time-series are known to arise from physical phenomena that can be considered to be unchanging in their basic nature for very long periods of time. In such cases, conceptually idealizing this time-invariance by extending the length of the data record without bound enables us to conceive of a model that is derivable from the data in the limit as the amount of data used for measuring the parameters of the model approaches infinity. This leads us to the concept of a fraction-of-time (FOT) probability model that is free from the abstract concept of a population. For example, the FOT probability that a time-series exceeds some specified level is defined to be the fraction of time that this event occurs over the life of the time-series.

Once we have accepted the idea of an infinitely long time-series with an FOT probability model, we can develop a theory of statistical inference and decision that is isomorphic to the theory for stationary stochastic processes.

In summary, I believe Professor Hinich's admitted confusion about the message delivered in [2] results not from any flaw in the philosophy put forth in [2], but rather from his unwillingness to accept this philosophy as a viable alternative to his philosophy—a philosophy to which he clings tightly. Moreover, in defense of engineers and scientists, I mention that the book [3] on stochastic processes, coupled with [2], illustrates that some nonstatisticians are capable of understanding, using, and teaching both the orthodox theory of stochastic processes (for those situations where it is appropriate) and the unorthodox theory of time-series based on FOT probability (for those other situations where it is the more appropriate of the two).

Not all statisticians share Professor Hinich's inflexible position on what he calls "an odd misdirection of intellectual effort." Professor A. M. Yaglom of the Academy of Sciences of Russia, author of well known books on time-series analysis and stochastic processes, states the following in his review [4] of [2] and [3]:

> "It is important, however, that until Gardner's second book was published there was no attempt to present the modern spectral analysis of random processes consistently in language that uses only time-averaging rather than averaging over the statistical ensemble of realizations. Moreover, this book also shows that such a treatment possesses some advantage over the traditional one. . .Professor Gardner's books are both valuable additions to the available literature on the theory of random processes."

Similarly, Professor Enders A. Robinson of Columbia University, author of thirty books on time-series analysis, states the following in his review [5]:

> "This book can be highly recommended to the engineering profession. Instead of struggling with many unnecessary concepts from abstract probability theory, most engineers would prefer to use methods that are based on the available data. This highly readable book gives a consistent approach for carrying out this task. In this work Professor Gardner has made a significant contribution to statistical spectral analysis, one that would please the early pioneers of spectral theory and especially Norbert Wiener."

Apparently, the distinction to be made is not Professor Hinich's distinction between engineers/scientists and statisticians, but rather it is the distinction between pragmatists (as defined by the American philosophers Charles Sanders Peirce and William James), who can adopt whatever conceptualization best serves the practical purpose at hand, and those others who believe in the sanctity of one particular system of conceptualization, regardless of its practical consequences. The nonpragmatists speak of a controversy over the stochastic and nonstochastic approaches to time-series analysis. But, there is really no basis for controversy. The only real issue is one of judgement—judgement in choosing for each particular time-series analysis problem the most appropriate of two alternative approaches [6].

Making inferences from available data is tricky business for anyone. For example, Professor Hinich makes the following inference about a population numbering in the hundreds of thousands on the basis of data from a single member of this population: "It is clear to me [Hinich] from reading this book that most engineers do not understand the need for careful thinking about statistical matters."

For more on this topic, see "Ensembles in Wonderland" published in the SP Forum section of *IEEE Signal Processing Magazine*, April 1994.

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