

Detection and Source Location of Weak Cyclostationary Signals: Simplifications of the Maximum-Likelihood Receiver

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Abstract—The problem of dual-receiver interception of low-SNR signals, which includes detection of the presence of a particular signal type and location of its source, is considered. In particular, source-location based on time-difference-of-arrival (TDOA) measurements is considered, and the low-SNR maximum-likelihood receiver for joint detection and TDOA estimation is taken as a starting point. By explicitly revealing the way in which this receiver exploits the spectral correlation properties of the cyclostationary signal, several partial implementations with optimality properties of their own are proposed. These greatly simplified implementations, which require only a one-dimensional search over the TDOA parameter, are shown by simulation to perform competitively with the relatively complicated maximum-likelihood receiver, which requires a two- or three-dimensional search for PSK signals.

I. INTRODUCTION

DURING the last decade, it has been established and is now generally agreed upon that the detection and analysis tasks of interception of low-SNR signals are often best accomplished by exploitation of *cyclic features*, which are features due to underlying periodicities such as sinewave carriers, pulse-trains, chipping and hopping operations, etc. The pioneering work in this area was carried out in the mid-to-late 1970's, and a review of a variety of ad hoc cyclic-feature detectors is given in the unpublished report [1]. Since that time, the cyclic-feature detection/estimation approach has been put on a firm theoretical foundation. The theory of spectral correlation in cyclostationary signals developed in [2] and [3] has been shown to provide a unifying conceptual and mathematical framework for signal detection based on cyclic-feature exploitation [4]. It has been shown that the great majority of cyclic-feature detectors/estimators, including ad hoc schemes such as delay-and-multiply chip-rate detectors, filter-squarer carrier-doublers, and various hop-rate detectors, as well as optimum detectors/estimators such as weak-signal maximum-likelihood receivers, maximum-SNR spectral-line regenerators, and maximum-deflection detectors, and also in-

cluding ambiguity plane and Wigner-Ville time-frequency plane techniques, can be understood, analyzed, and compared in terms of a general approach called either *cyclic spectral analysis* or *spectral correlation analysis* [2]. Moreover, this approach has led to an especially general and flexible cyclic-feature detector/estimator, called the *cyclic spectrum analyzer* or *spectral correlation analyzer*, that (with postprocessing) can emulate all of the above techniques as well as more general and powerful techniques for detection, classification according to modulation/coding type, and modulation/coding parameter estimation [5], [6].

In this paper, the spectral correlation approach is generalized from single-receiver interception to dual-receiver interception for which the primary tasks are not only to detect the presence of a particular signal type but also to estimate the location of its source.

To locate the source of a detected signal, the time-difference-of-arrival (TDOA) of that signal at two spatially separated receivers can be used to determine a hyperboloidal surface on which the source lies. If this is repeated for a second pair of receivers (or a single moving pair at a later time), then the intersection of the second hyperboloidal surface with the first further narrows down the location of the source to a curved line. If, in addition, the source lies on a surface of known location (e.g., the earth's surface), the intersection of the curved line and the known surface further narrows down the location to one or two points.

For this approach to source location, it has recently been shown that the spectral correlation properties of low-SNR signals can be used to great advantage by providing a basis for the design of signal-selective TDOA estimation algorithms that are highly tolerant not only to noise but also to cochannel interference from sources not of interest that are either near to or far from the source of interest [7]. In this paper, one of the best performing ad hoc spectral-correlation-exploiting TDOA-estimation algorithms presented in [7] is shown to be a partial but much simpler implementation of the relatively complicated weak-signal maximum-likelihood TDOA estimator. Similarly, the relatively simple single-receiver spectral-correlation-exploiting detector was shown in [4] to be a partial implementation of the single-receiver weak-signal maximum-likelihood detector. This same approach of decomposing the maximum-likelihood receiver for weak signals into a multicycle detector and considering the performance of single-cycle detectors obtained therefrom has been pursued in [13] for

Paper approved by the Editor for Data Communications of the IEEE Communications Society. Manuscript received April 4, 1991; revised March 25, 1992. This work was supported in part by E-Systems, Inc., Greenville Division; by the United States Army Research Office under Contract DAAL03-89-C-0035, sponsored by the Army Communications Electronics Command Center for Signals Warfare; and by the Office of Naval Research under Contract N00014-92-J-1218.

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IEEE Log Number 9210319.

the case of single-sensor reception in white non-Gaussian noise. Generalizing on this, it is shown in this paper that the weak-signal maximum-likelihood joint detector and TDOA estimator is made up of a multiplicity of components that can be partitioned into a variety of detectors and estimators, each of which is optimal in either a maximum-likelihood sense or a maximum-SNR sense. The results of extensive simulations that compare the performances of some of these alternative detectors and estimators are then presented, and it is concluded that the performance of the relatively complex maximum-likelihood joint detector/estimator, which requires a two- or three-dimensional search over phase and timing parameters (for PSK signals, these include the carrier phase [for BPSK only], chip timing, and time-difference-of-arrival), can be closely approximated in many cases (for data record lengths that are normally used in weak-signal detection and source location) by much simpler suboptimum detectors/estimators that avoid all parameter searches for detection, and reduce the search to one dimension for TDOA estimation.

In Section II, the low-SNR maximum-likelihood receiver for joint detection and TDOA estimation is presented and characterized in terms of spectral correlation measurements. This general result is then made more specific for the case of PSK signals. In Section III, several optimal detectors and TDOA estimators are presented and shown to exactly match individual terms in the sum of terms that comprise the maximum-likelihood receiver. Suboptimal implementations of these component detector/estimators are proposed as practical alternatives. In Section IV, performance evaluations obtained from extensive simulations are presented and used to draw conclusions about the relative performances of the complicated maximum-likelihood receiver, its optimum components, and their simplified implementations.

II. THE MAXIMUM-LIKELIHOOD RECEIVER

We consider the following model for reception at two separate locations indexed by q :

$$H_1: x_q(t) = s(t - \theta_q) + n_q(t), \quad |t| \leq T/2, \quad q = 1, 2 \quad (1a)$$

$$H_0: x_q(t) = n_q(t), \quad |t| \leq T/2, \quad q = 1, 2 \quad (1b)$$

where H_1 and H_0 are the signal-present and signal-absent hypotheses, T is the data collection time, θ_1 and θ_2 are times of arrival of the zero-mean real-valued random signal $s(t)$, relative to some arbitrary reference time, and $n_1(t)$ and $n_2(t)$ are independent white Gaussian noises (WGN) with power spectral density N_0 . For sufficiently low SNR, the log-likelihood ratio can be closely approximated by a quadratic functional of the data $\{x_q(t): |t| \leq T, q = 1, 2\}$. This functional can be derived by replacing the continuous-time model with its discrete-time equivalent, then expanding the likelihood ratio in a Taylor series, retaining only the first few terms (cf. [11]). Finally, the truncated Taylor series for the discrete-time model can be replaced by its continuous-time counterpart by simply replacing sums over discrete time

with integrals over continuous time. The resultant statistic is given by the sum of three terms

$$y_{LLR} = y_{12} + y_{11} + y_{22}, \quad (2)$$

where

$$y_{12} \triangleq \frac{2}{N_0^2} \iint_{-T/2}^{T/2} R_s(u - \theta_1, v - \theta_2) x_1(u) x_2(v) du dv \quad (3a)$$

$$y_{11} \triangleq \frac{1}{N_0^2} \iint_{-T/2}^{T/2} R_s(u - \theta_1, v - \theta_1) x_1(u) x_1(v) du dv \quad (3b)$$

$$y_{22} \triangleq \frac{1}{N_0^2} \iint_{-T/2}^{T/2} R_s(u - \theta_2, v - \theta_2) x_2(u) x_2(v) du dv \quad (3c)$$

and $R_s(u, v)$ is the autocorrelation for the zero-mean random signal $s(t)$

$$R_s(u, v) = E\{s(u)s(v)\}. \quad (4)$$

For a PSK signal, we have the model

$$s(t) = \sum_{n=-\infty}^{\infty} p(t - nT_0 - \theta_3) \cos(2\pi f_c t - \theta_4 - \phi_n), \quad (5)$$

where $p(t)$ is the keying envelope, T_0 is the keying interval, θ_3 is the keying clock phase, f_c is the carrier frequency, θ_4 is the carrier phase, and $\{\phi_n\}$ is the M -ary random message sequence (possibly spread encoded), which is modeled as an independent and identically distributed sequence. (θ_3 and θ_4 contain an additive component that is the arbitrary reference time for θ_1 and θ_2 .) We shall assume that the four parameters $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ are unknown but that the keying (or chip) rate $1/T_0$, carrier frequency f_c , and keying (or chip) envelope $p(t)$ are known. Nevertheless, we shall also consider some suboptimum detectors/estimators that require knowledge of only the keying rate or the carrier frequency. The autocorrelation for the model (5) is given by

$$R_s(u, v) = \frac{1}{2} \sum_{n=-\infty}^{\infty} p(u - nT_0 - \theta_3) p(v - nT_0 - \theta_3) \cdot (\cos[2\pi f_c(u + v) - 2\theta_4] + \cos[2\pi f_c(u - v)]) \quad (6)$$

for BPSK, and it is given by

$$R_s(u, v) = \frac{1}{2} \sum_{n=-\infty}^{\infty} p(u - nT_0 - \theta_3) \cdot p(v - nT_0 - \theta_3) \cos[2\pi f_c(u - v)] \quad (7)$$

for QPSK, as well as M -ary PSK with $M = 4z$ ($z = \text{integer}$).

Since $R_s(u, v)$ depends on the two unknown parameters θ_3 and θ_4 for BPSK (but only θ_3 for QPSK) and the other two unknown parameters θ_1 and θ_2 appear in (3), then in order to obtain the maximum-likelihood estimate of the TDOA $\theta_1 - \theta_2$,

we must jointly maximize y_{LLR} with respect to all four (three for QPSK) parameters $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$:

$$y_{ML} \triangleq \max_{\Theta} y_{LLR}, \quad (8)$$

and to detect the presence of the signal $s(t)$, we must compare (8) to a threshold γ

$$y_{ML} \underset{H_0}{\overset{H_1}{\geq}} \gamma. \quad (9)$$

Thus, (8) and (9) with (2), (3), and (6) or (7) substituted in is the weak-signal maximum-likelihood receiver.

In order to explicitly reveal the effects of the cyclostationarity of the signal, we can expand its autocorrelation function in a generalized Fourier series

$$R_s(t + \tau/2, t - \tau/2) = \sum_{\alpha} R_s^{\alpha}(\tau) e^{i2\pi\alpha t} \quad (10)$$

where

$$R_s^{\alpha}(\tau) \triangleq \lim_{Z \rightarrow \infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} R_s(t + \tau/2, t - \tau/2) e^{-i2\pi\alpha t} dt \quad (11)$$

is called the *cyclic autocorrelation function* [2], [3] and where, in general, the *cycle frequency* α ranges over all integer multiples of all fundamental frequencies $1/T_k$ corresponding to incommensurate periods T_k , for $k = 1, 2, 3, \dots$, of cyclostationarity. For BPSK, assuming that the period of the carrier and the keying period are incommensurate, we have only two fundamental cycle frequencies, $1/T_1 = 1/T_0$ and $1/T_2 = 2f_c$. For QPSK, there is only one fundamental cycle frequency $1/T_1 = 1/T_0$. Substituting (10) into (3a) by making the change of variables $u = t + \tau/2$, $v = t - \tau/2$, and then executing the integral over t yields

$$y_{12} = \frac{2T}{N_0^2} \sum_{\alpha} \int_{-T}^T R_s^{\alpha}(\tau)^* R_{12T}^{\alpha}(\tau + \theta_1 - \theta_2) d\tau e^{i\pi\alpha(\theta_1 + \theta_2)}, \quad (12)$$

where R_{12T}^{α} is the *cyclic cross-correlogram* [2], [3] for $x_1(t)$ and $x_2(t)$:

$$R_{12T}^{\alpha}(\tau) \triangleq \frac{1}{T} \int_{-(T-|\tau|)/2}^{(T-|\tau|)/2} x_1(t + \tau/2) x_2(t - \tau/2) e^{-i2\pi\alpha t} dt \quad (13)$$

for $|\tau| \leq T$, and $R_{12T}^{\alpha}(\tau) = 0$ for $|\tau| > T$. Applying Parseval's relation to (12) yields the alternative expression

$$y_{12} = \frac{2T}{N_0^2} \sum_{\alpha} \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{12T}^{\alpha}(f) \cdot e^{i2\pi f(\theta_1 - \theta_2)} df e^{i\pi\alpha(\theta_1 + \theta_2)}, \quad (14)$$

where

$$S_s^{\alpha}(f) \triangleq \int_{-\infty}^{\infty} R_s^{\alpha}(\tau) e^{-i2\pi f\tau} d\tau \quad (15)$$

is called the *cyclic spectral density function* and

$$S_{12T}^{\alpha}(f) \triangleq \int_{-\infty}^{\infty} R_{12T}^{\alpha}(\tau) e^{-i2\pi f\tau} d\tau \quad (16)$$

is called the *cyclic cross-periodogram* for $x_1(t)$ and $x_2(t)$ [2], [3]. The frequency domain expression (14) for the component y_{12} of the likelihood statistic y_{LLR} can be better understood by using the alternative expressions [2], [3], [6]

$$S_s^{\alpha}(f) = \lim_{W \rightarrow \infty} \lim_{Z \rightarrow \infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} \frac{1}{W} \cdot E\{S_W(t, f + \alpha/2) S_W^*(t, f - \alpha/2)\} dt \quad (17)$$

and

$$S_{12T}^{\alpha}(f) = \frac{1}{T} X_{1T}(f + \alpha/2) X_{2T}^*(f - \alpha/2) \quad (18)$$

where X_{qT} is the finite-time Fourier transform of the data $x_q(t)$,

$$X_{qT}(f) = \int_{-T/2}^{T/2} x_q(t) e^{-i2\pi ft} dt \quad (19)$$

for $q = 1, 2$, and similarly $S_W(t, f)$ is the sliding finite-time Fourier transform of $s(t)$,

$$S_W(t, f) = \int_{t-W/2}^{t+W/2} s(u) e^{-i2\pi fu} du. \quad (20)$$

We see from (17) that the cyclic spectral density $S_s^{\alpha}(f)$ is the limit, as the bandwidth $(1/W)$ approaches zero, of the time-average bandwidth-normalized correlation between the spectral components of $s(t)$ in the bands of approximate width $(1/W)$ centered at $f + \alpha/2$ and $f - \alpha/2$. Consequently, $S_s^{\alpha}(f)$ is also called the *spectral correlation density function* [2], [3], [6].

The other two components y_{11} and y_{22} of the statistic y_{LLR} can similarly be expressed as

$$y_{qq} = \sum_{\alpha} \frac{T}{N_0^2} \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{qqT}^{\alpha}(f) df e^{i2\pi\alpha\theta_q} \quad (21)$$

for $q = 1, 2$. Thus, the weak-signal ML (maximum-likelihood) detector/estimator can be expressed as

$$y_{ML} = \max_{\Theta} \left\{ 2 \sum_{\alpha} \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{12T}^{\alpha}(f) \cdot e^{i2\pi f(\theta_1 - \theta_2)} df e^{i\pi\alpha(\theta_1 + \theta_2)} + \sum_{q=1}^2 \sum_{\alpha} \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{qqT}^{\alpha}(f) df e^{i2\pi\alpha\theta_q} \right\}. \quad (22)$$

For the BPSK and QPSK signals, the spectral correlation density function is given by [2], [3], [6]

$$S_s^{\alpha}(f; \theta_3, \theta_4) = \frac{1}{4T_0} \left[P(f + f_c + \alpha/2) P^*(f + f_c - \alpha/2) + P(f - f_c + \alpha/2) \cdot P^*(f - f_c - \alpha/2) \right] e^{-i2\pi\alpha\theta_3}, \quad (23a)$$

$\alpha = k/T_0$

for $k = 0, \pm 1, \pm 2, \pm 3, \dots$, and for the BPSK signal only we also have

$$S_s^\alpha(f; \theta_3, \theta_4) = \frac{1}{4T_0} P(f \pm f_c + \alpha/2) \cdot P^*(f \mp f_c - \alpha/2) e^{\pm i 2\theta_4},$$

$$\alpha = \mp 2f_c, \quad (23b)$$

and

$$S_s^\alpha(f; \theta_3, \theta_4) = \frac{1}{4T_0} P(f \pm f_c + \alpha/2) P^*(f \mp f_c - \alpha/2) \cdot \exp\{-1[2\pi(\alpha \pm 2f_c)\theta_3 \mp 2\theta_4]\},$$

$$\alpha = \mp 2f_c + k/T_0 \quad (23c)$$

for $k = \pm 1, \pm 2, \pm 3, \dots$, where $P(f)$ is the keying envelope transform

$$P(f) = \int_{-\infty}^{\infty} p(t) e^{-i 2\pi f t} dt. \quad (24)$$

In (23a)–(23c), we have introduced the notation $S_s^\alpha(f; \theta_3, \theta_4)$ to explicitly denote the dependence of $S_s^\alpha(f)$ on θ_3 and θ_4 . The maximum-likelihood receiver for BPSK or QPSK signals is simply (22) with $S_s^\alpha(f)$ replaced by $S_s^\alpha(f; \theta_3, \theta_4)$, and the set Θ to maximize over is $\{\theta_1, \theta_2, \theta_3\}$ for QPSK and $\{\theta_1, \theta_2, \theta_3, \theta_4\}$ for BPSK. It is important to note that the dimensionality of these searches can be reduced by taking $\theta_1, \theta_2, \theta_3$, or θ_4 as the arbitrary timing reference referred to at the start of this section. This can be accomplished simply by setting, say, θ_1 equal to zero in the expression for the ML statistic. For instance, if we choose θ_1 as the timing reference, then the parameters to be estimated are transformed to $\theta'_1 = \theta_1 - \theta_1 = 0$, $\theta'_2 = \theta_2 - \theta_1$ (= the TDOA), $\theta'_3 = \theta_3 - \theta_1$, and $\theta'_4 = \theta_4 - 2\pi f_c \theta_1$. Then the search for BPSK signals is over 3 timing parameters, whereas for QPSK it is over 2 timing parameters. We retain the general form of the ML statistic throughout the paper for reasons of completeness. The simulations reported in Section IV, however, take advantage of this reduction in dimensionality.

We now turn to a discussion of the various components in the ML receiver (22) in terms of their optimality properties and suboptimal implementations.

III. COMPONENTS OF THE ML RECEIVER

Although it is not immediately obvious, each individual term in y_{12} can be used to optimally estimate the TDOA $\theta_2 - \theta_1$, and every individual term in the ML statistic (22) can be used to optimally detect the presence of the signal $s(t)$. Moreover, there are five groups of terms that are themselves each an ML statistic. The first and second groups of the ML statistic (22) that are actually ML statistics themselves are simply the $\alpha = 0$ terms of y_{11} and y_{22} ,

$$y_{RAD,q} = \int_{-\infty}^{\infty} S_s^0(f) S_{qqT}^0(f) df, \quad (25)$$

for which no maximization is necessary since the PSD $S_s^0(f)$ does not depend on Θ . This is the single-receiver weak-signal ML detector for a stationary signal model, obtained by

randomizing the phase θ_3 and θ_4 , and is called the *optimum radiometer* [4]. Both $y_{RAD,1}$ and $y_{RAD,2}$ are components of the third group,

$$y_{MLS} = \max_{\theta_1 - \theta_2} \left\{ 2 \int_{-\infty}^{\infty} S_s^0(f) S_{12T}^0(f) e^{i 2\pi f (\theta_1 - \theta_2)} df + \sum_{q=1}^2 y_{RAD,q} \right\}, \quad (26)$$

which is the dual-receiver weak-signal detector/estimator for a stationary signal. The fourth and fifth groups are y_{11} and y_{22} themselves,

$$y_{MCD,q} = \max_{\theta_1, \theta_3, \theta_4} \left\{ \sum_{\alpha} \int_{-\infty}^{\infty} S_s^\alpha(f)^* S_{qqT}^\alpha(f) df e^{i 2\pi \alpha \theta_q} \right\}. \quad (27)$$

These statistics are called *multi-cycle detectors* (MCD), and are the single-receiver weak-signal ML detectors for cyclostationary signals in WGN [4]. Let us now consider the optimality properties of the individual terms indexed by α in y_{11} , y_{22} , and y_{12} .

It is shown in [2], [3], and [9] that the quadratic processor that generates a maximum-power spectral line at frequency α from a signal $s(u - \theta_q)$, using data $x_q(u)$ over the sliding time interval $[t - T/2, t + T/2]$, subject to the constraint that the spectral density of the output noise at the frequency α , due to WGN at the input, not exceed some specified level, is given by (after down-converting the output from the center frequency α to zero)

$$z_{qq}^\alpha = \int_{-\infty}^{\infty} S_s^\alpha(f)^* S_{qqT}^\alpha(t, f) df e^{i 2\pi \alpha \theta_q} \quad (28)$$

where $S_{qqT}^\alpha(t, f)$ is the sliding cyclic auto-periodogram

$$S_{qqT}^\alpha(t, f) \triangleq \frac{1}{T} X_{qT}(t, f + \alpha/2) X_{qT}^*(t, f - \alpha/2). \quad (29)$$

We see that for $t = 0$, this maximum-SNR spectral line statistic (28) is exactly the same as each single-cycle statistic in y_{11} and y_{22} (2) in the likelihood statistic (2). Similarly, it can be shown that the quadratic processor for $x_1(t)$ and $x_2(t)$ [which uses the crossproduct $x_1(t - u)x_2(t - v)$ instead of the autoprodut $x_1(t - u)x_1(t - v)$] that generates the maximum-SNR spectral line produces the statistic

$$z_{12}^\alpha = \int_{-\infty}^{\infty} S_s^\alpha(f)^* S_{12T}^\alpha(t, f) e^{i 2\pi f (\theta_1 - \theta_2)} df e^{i \pi \alpha (\theta_1 + \theta_2)}, \quad (30)$$

where $S_{12T}^\alpha(t, f)$ is the sliding cyclic cross-periodogram for $x_1(t)$ and $x_2(t)$. Again, we see that for $t = 0$, the statistic (30) is exactly the same as each single-cycle statistic in y_{12} in the likelihood statistic (2). It can be seen from (23a)–(23c) that for the cycle frequencies of interest for BPSK and QPSK, the dependence of $S_s^\alpha(f) = S_s^\alpha(f; \theta_3, \theta_4)$ on the keying phase θ_3 and carrier phase θ_4 is just a frequency-independent phase factor, which can be factored out of the integral over frequency in (28) and (30). Thus, neither of these

single-cycle statistics requires maximization over θ_3 or θ_4 . Detection using the single-cycle detector (SCD) (28) requires no maximization at all, and detection or TDOA estimation using the cross single-cycle detector/estimator (CSCD/E) (30) requires a maximization over the TDOA $\theta_2 - \theta_1$ only.

Because each of the terms composing y_{12} is optimal in the sense of maximum-SNR, the statistic y_{12} is itself a good candidate for further investigation. We call the detection/estimation statistic

$$y_{CMCD/E} \triangleq \max_{\Theta} \{y_{12}\} \quad (31)$$

the *cross multi-cycle detector/estimator* (CMCD/E). We suspect that it may have advantages over the MCD and ML because it does not contain terms that correspond to a single sensor; such terms (especially the $\alpha = 0$ term) are particularly susceptible to variable broadband background noise and interference.

As shown in [4] and [8], a single $\alpha \neq 0$ term such as (28) can be superior to the $\alpha = 0$ term for detection when variable noise and/or cochannel interference is present. Similar results are obtained for TDOA estimation performance in [12], except that in this case the single- α term in y_{12} is approximated by replacement of the assumed-known spectral correlation density function $S_s^\alpha(f; \theta_3, \theta_4)$ with the estimate

$$S_{11T}^\alpha(f)_{\Delta f} = \left[\frac{1}{T} X_{1T}(f + \alpha/2) X_{1T}^*(f - \alpha/2) \right] \otimes W(f) \quad (32)$$

(where \otimes denotes convolution), which converges to $S_s^\alpha(f; \theta_3, \theta_4) e^{-i2\pi\alpha\theta_1}$ as $T \rightarrow \infty$ (and the width Δf of the smoothing window $W(f)$ is then allowed to approach zero) assuming only that $x_1(t) = s(t - \theta_1) + n_1(t)$ where $n_1(t)$ is any signal or noise that does not possess the cycle frequency α [2]. The substantial advantage of using only a single-cycle statistic is that the two- or three-dimensional search over Θ can be reduced to a one-dimensional search over the TDOA $\theta_2 - \theta_1$ (using one term from y_{12}) or no search at all for signal detection (using the magnitude of one term from y_{11} or y_{22}).

Another advantage of deleting the $\alpha = 0$ statistics is that they are much more strongly corrupted by a variable noise level (in y_{11} and y_{22}) and by cochannel interferences (in y_{12}), which can exhibit TDOA's of their own that can interfere with the estimation of the TDOA of the signal of interest. Of course, the reason that the likelihood statistic can be improved on is that neither variable noise-level nor cochannel interference is included in the model (1) from which the likelihood statistic is derived. Furthermore, generalizing the model to include cochannel interference greatly complicates the problem (typically ruling out an analytical solution) and requires that the receiver have knowledge of the interference type.

An advantage of modifying the likelihood statistic, by replacing in either y_{12} or y_{11} or y_{22} the assumed-known spectral correlation density of the signal $s(t)$ with an estimate such as (32), is that the only knowledge about the signal that is then required by the detector and/or TDOA estimator is one

(or more) cycle frequencies, such as the keying rate or the doubled carrier frequency.

As explained in [2] and [4], when $S_s^\alpha(f)$ is not known, it can be replaced with the estimate $S_{qqT}^\alpha(t, f)_{\Delta f}$ for $q = 1$ or 2 [cf. (32)]. In this case, the optimum single-cycle statistic (28) is approximated by (letting $t = 0$)

$$|\tilde{z}_{qq}^\alpha| = \int_{-\infty}^{\infty} |S_{qqT}^\alpha(f)_{\Delta f}|^2 df, \quad (33)$$

which we call the *spectral correlation magnitude detector* (SCMD). For a known cycle frequency α , the SCMD is a useful detector that is tolerant to all noise and interference that does not possess the cycle frequency α . When graphed versus f and α , the integrand of the SCMD is useful for modulation recognition, as well as detection [2], [4]. Since using the estimate (32) correctly matches all three phases θ_3 , θ_4 , θ_q in the y_{qq} term (21), we can perform the sum over all α in (33) without a search over any parameters. This yields what is called the *multi-cycle SCMD* (MCSCMD)

$$\tilde{z}_{qq} = \sum_{\alpha} \int_{-\infty}^{\infty} |S_{qqT}^\alpha(f)_{\Delta f}|^2 df. \quad (34)$$

A simpler version of the CMCD/E statistic (31) can be constructed by setting θ_1 equal to zero (cf. Section II) and replacing each ideal spectral correlation function $S_s^\alpha(f)$ with the estimate $S_{22T}^\alpha(f)_{\Delta f}$ and each cyclic cross-periodogram $S_{12T}^\alpha(f)$ with the frequency-smoothed version $S_{12T}^\alpha(f)_{\Delta f}$ (this is equivalent to letting θ_1 be the arbitrary reference phase referred to in Section II). In this case, we obtain the *multi-cycle SCPD/E* (MCSCPD/E) receiver

$$y_{MCSCPD/E} = \max_{\theta_2} \operatorname{Re} \left\{ \sum_{\alpha} \int_{-\infty}^{\infty} S_{22T}^\alpha(f)_{\Delta f}^* S_{12T}^\alpha(f)_{\Delta f} \cdot e^{i2\pi f \theta_2} df e^{i\pi \alpha \theta_2} \right\}. \quad (35)$$

The addition in (35) is coherent (unlike the MCSCMD) because when the signal is present and θ_2 is equal to the TDOA, each statistic in the sum tends to a positive real number with increasing T , whereas when the signal is absent this is not the case.

A single term from (35) is

$$\tilde{z}_{12}^\alpha(\theta_2) = \operatorname{Re} \left\{ \int_{-\infty}^{\infty} S_{22T}^\alpha(f)_{\Delta f}^* S_{12T}^\alpha(f)_{\Delta f} \cdot e^{i2\pi f \theta_2} df e^{i\pi \alpha \theta_2} \right\}, \quad (36)$$

which is called the *spectral correlation product detector/estimator* (SCPD/E). By maximizing \tilde{z}_{12}^α over the unknown TDOA parameter θ_2 , we obtain a useful detection statistic. Moreover, the maximal value of θ_2 is a powerful TDOA estimate as shown in [7] and [12], where this same estimate is derived from an ad hoc least-squares optimality criterion, and is shown with simulations to perform very well in a variety of severe noise and interference environments.

To simplify detection prior to TDOA estimation, we can mimic the auto-SCM detector to obtain the *cross-SCMD* (CSCMD) detector

$$\tilde{w}_{12}^{\alpha} = \int_{-\infty}^{\infty} \left| S_{12T}^{\alpha}(f)_{\Delta f} \right|^2 df, \quad (37)$$

or the *cross-MCSCMD* (CMCSCMD) detector

$$\tilde{w}_{12} = \sum_{\alpha} \int_{-\infty}^{\infty} \left| S_{12T}^{\alpha}(f)_{\Delta f} \right|^2 df, \quad (38)$$

which require no maximization over the TDOA parameter.

The preceding components of the ML receiver and their approximate implementations can now be summarized in a compact form by using the following definitions:

$$y_{pq} = \sum_{\alpha} z_{pq}^{\alpha} \quad (39)$$

$$z_{pq}^{\alpha} = \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{pqT}^{\alpha}(f)_T e^{i2\pi f(\theta_p - \theta_q)} df e^{i\pi\alpha(\theta_p + \theta_q)} \quad (40)$$

$$\tilde{y}_{pq} = \sum_{\alpha} \tilde{z}_{pq}^{\alpha} \quad (41)$$

$$\tilde{z}_{pq}^{\alpha} = \int_{-\infty}^{\infty} S_{qqT}^{\alpha}(f)^* S_{pqT}^{\alpha}(f)_{\Delta f} e^{i2\pi f(\theta_p - \theta_q)} df e^{i\pi\alpha(\theta_p + \theta_q)} \quad (42)$$

$$\tilde{w}_{pq}^{\alpha} = \int_{-\infty}^{\infty} \left| S_{pqT}^{\alpha}(f)_{\Delta f} \right|^2 df. \quad (43)$$

In terms of these definitions, the thirteen detectors and joint detector/estimators derived in this section can be expressed as follows:

Maximum Likelihood

$$y_{ML} = \max_{\theta_2, \theta_3, \theta_4} \{2y_{12} + y_{11} + y_{22}\}_{\theta_1=0} \quad (44)$$

Maximum Likelihood for Stationary Signal

$$y_{MLS} = \max_{\theta_2} \{2z_{12}^0 + z_{11}^0 + z_{22}^0\}_{\theta_1=0} \quad (45)$$

Cross Multicycle Detector/Estimator

$$y_{CMCD/E} = \max_{\theta_2, \theta_3, \theta_4} \{y_{12}\}_{\theta_1=0} \quad (46)$$

Multicycle Detector

$$y_{MCD} = \max_{\theta_2, \theta_3, \theta_4} \{y_{qq}\} \quad (47)$$

Cross Single-Cycle Detector Estimator

$$y_{CSCD/E} = \max_{\theta_2} \text{Re}\{z_{12}^{\alpha}\}_{\theta_1=0} \quad (48)$$

Single-Cycle Detector

$$y_{SCD} = |z_{qq}^{\alpha}| \quad (49)$$

Radiometer

$$y_{RAD} = z_{qq}^0 \quad (50)$$

Multicycle Spectral Correlation Product Detector/Estimator

$$y_{MCSPD/E} = \max_{\theta_2} \text{Re}\{\tilde{y}_{12}\}_{\theta_1=0} \quad (51)$$

Spectral Correlation Product Detector/Estimator

$$y_{SCPD/E} = \max_{\theta_2} \text{Re}\{z_{12}^{\alpha}\}_{\theta_1=0} \quad (52)$$

Multicycle Spectral Correlation Magnitude Detector

$$y_{MCSCMD} = \sum_{\alpha} |\tilde{z}_{qq}^{\alpha}| \quad (53)$$

Cross Multicycle Spectral Correlation Magnitude Detector

$$y_{CMCSCMD} = \sum_{\alpha} \tilde{w}_{12}^{\alpha} \quad (54)$$

Cross Spectral Correlation Magnitude Detector

$$y_{CSCMD} = \tilde{w}_{12}^{\alpha} \quad (55)$$

Spectral Correlation Magnitude Detector

$$y_{SCMD} = |\tilde{z}_{qq}^{\alpha}|. \quad (56)$$

The y_{pq} and \tilde{y}_{pq} are multicycle (MC) statistics, whereas the z_{pq}^{α} and \tilde{z}_{pq}^{α} are single-cycle (SC) statistics. Those with subscript pq with $p \neq q$ are cross (C) statistics, whereas those with pp or qq are autostatistics. Those with the tilde, \tilde{y}_{pq} and \tilde{z}_{pq}^{α} , use estimated spectral correlation (SC) functions, whereas those without the tilde, y_{pq} and z_{pq}^{α} , use ideal spectral correlation functions. With the addition of the symbols M for magnitude, P for product, D for detector, and D/E for joint detector/estimator, we can form all the acronyms in (44)–(56), except for ML (for maximum likelihood), MLS (for ML for the stationary model), and RAD (for the radiometer, which is MLS for a single receiver). For simplicity of notation, we use the designation ML , from this point forward, for the receiver described by (39), (40), and (44), even if the sum over α in (39) contains only a subset of all the cycle frequencies of the signal.

In summary, every individual term for $\alpha \neq 0$ in the likelihood statistic in (22) is an optimum detection and/or TDOA estimation statistic in its own right; each of these single-cycle statistics circumvents the search over three or all four of the parameters in Θ ; and each of these single-cycle statistics for $\alpha \neq 0$ admit approximations that avoid the need

for any knowledge of the spectral correlation function $S_s^\alpha(f)$, other than the value of the cycle frequency α . Furthermore, the multicycle counterparts of these approximated single-cycle statistics avoid the search over either three or all four of the parameters in Θ . Also, for $\alpha = 0$, the sum of the three terms from y_{12} , y_{11} , and y_{22} is the likelihood statistic for joint detection/estimation using the stationary model of the signal, and the $\alpha = 0$ term from either y_{11} or y_{22} alone is the likelihood statistic (namely, the optimum radiometer) for detection for the stationary model of the signal at a single receiver. In addition, the $\alpha = 0$ term from y_{12} alone is the likelihood statistic for TDOA estimation for the stationary model of the signal, namely, the optimum generalized cross correlator [10], which filters the data with the low-SNR Wiener filter prior to cross correlation. These results motivate a comparative simulation study of the detection and TDOA estimation performance of the maximum-likelihood receiver (22) and the various simpler single-cycle and multicycle receivers.

IV. COMPARATIVE SIMULATION STUDY

In this section, we present the results of a computer simulation study designed to compare the relative performances of the ML receiver, some of its optimal components, and their suboptimal implementations. Since the computational complexity of the suboptimal components is much less than that for the ML receiver itself (due to the avoidance of the search over multiple unknown timing parameters), it is of practical interest to determine the point, in terms of data collect length, at which these components attain performance comparable to that of the ML receiver. A second objective of the simulation study is to determine the relative importance of the various terms indexed by α that make up each detector/estimator. That is, we want to determine which values of α in the sum dominate performance. The results obtained may be different for the different detector/estimators and for different signal environments.

Two signal environments are simulated: signal plus white Gaussian noise (WGN) with a fixed spectral density N_0 , and signal plus WGN with variable N_0 having a coefficient of variation (variance divided by squared mean) of 1/10. Results obtained for the latter environment are roughly representative of the performance for many environments containing broadband or multiple narrowband interference. Although the results are not generally representative for a small number of strong isolated narrowband interferences, this type of interference is easily removed using spectral excision techniques.

For fixed N_0 , the performance ordering between the detector/estimators is relatively clear for it is in this environment that the ML and MCD are optimal in the maximum-likelihood sense when *all* terms indexed by α are included. Hence, we are concerned primarily with the collection-time crossover point mentioned previously, and secondarily with the relative influences of the different terms indexed by α that compose the estimator. For variable N_0 , none of the estimators is optimal in the ML sense, so the performance ordering is unknown and therefore of interest. Since the $\alpha = 0$ terms are more

TABLE I
 α -SETS USED IN SIMULATIONS

i	A_i
1	$\{0\}$
2	$\{1/T_0\}$
3	$\{2f_c\}$
4	$\{0, 1/T_0, 2f_c\}$
5	$\{1/T_0, 2f_c\}$
6	$\{0, 1/T_0, 2f_c \pm 1/T_0\}$
7	$\{0, 1/T_0, 2f_c, 2f_c \pm 1/T_0\}$

heavily influenced by a variable N_0 than the $\alpha \neq 0$ terms, the primary interest here is the relative performance of each of the terms composing the estimator. Some prediction of the nature of the dependence of performance on the particular set of cycle frequencies used is possible on the basis of the relative strengths of the corresponding cyclic features, as measured by the maximum attainable SNR which is realized by the single-cycle detector for each cycle frequency. These maximum SNR's, which are proportional to

$$\eta^\alpha \triangleq \int_{-\infty}^{\infty} |S_s^\alpha(f)|^2 df, \quad (57)$$

are given in [8] for BPSK, QPSK, and other PSK signals. Nevertheless, comparisons of signal detection performance, as measured by either probability of detection (P_D) for a given probability of false alarm (P_{FA}), or by the root-mean-squared error (RMSE) for TDOA estimation, cannot always be made by comparing SNR's; thus, we must in the final analysis rely on the simulations performed.

With these objectives in mind, we turn now to a description of the simulation parameters. This is followed by a graphical presentation of the results, which are the P_D for a P_{FA} of 1/10 for the detection problem, and for the TDOA estimation problem. The results are discussed in terms of the goals of the study.

To begin with, each (possibly infinite) sum over the cycle frequency α in the simulated detector/estimators is replaced by a finite sum over a selected set A_i of cycle frequencies α . The seven sets of cycle frequencies used are shown in Table I, and include only the cycle frequencies corresponding to the five largest η_α , which in order of descending η_α are $\alpha = 0, 2f_c, 1/T_0, 2f_c + 1/T_0, 2f_c - 1/T_0$; the first two are of equal strength and the last three are of equal strength. Negative values of α are not considered because they add no new information to the statistic. This can be deduced from the symmetries exhibited by both ideal and measured spectral correlation functions

$$\begin{aligned} S_s^{-\alpha}(f) &= S_s^\alpha(f)^*, \\ S_{X_T}^{-\alpha}(f) &= S_{X_T}^\alpha(f)^*. \end{aligned}$$

Thus, for every term in the ML receiver (or in the MCD or CMCD/E or MCSCPD/E), there is a conjugate term, which renders all these receivers real-valued, which in turn validates the maximization operation. This operation is therefore equivalent to using only nonnegative α 's and maximizing the real part of the resultant statistic.

Since the first three α -sets are singletons, the detectors are single-cycle detectors there. The signal of interest (SOI) is BPSK with rectangular pulse envelope, a symbol rate $1/T_0$ equal to $1/16$ the sampling rate, and a carrier frequency f_c of $1/4$ the sampling rate. WGN is added to the SOI to obtain an SNR (in the SOI band) of 0 dB. Collect times ranging from 8 to 128 symbols are used for detection; the longest of these collect times is typical for weak-signal detection. For TDOA, collect times up to 256 symbols are used.

For the MCSCMD and MCSCPD/E receivers, spectral-smoothing window-widths are equal to $1/10$ of the symbol rate. Monte Carlo simulations were performed with 500 trials for each case of signal present and signal absent, except for the ML receiver for which 250 trials were used in each case because of the severe computational requirements of simulating this receiver. For the collect time of 256 symbols, only the CMCD/E and MCSCPD/E are simulated. To ensure reliability of the estimated RMSE's for this larger collect, it is necessary to increase the number of trials used to 1500 for the CMCD/E and 2000 for the MCSCPD/E. The grid of carrier phase search points is made finer (CMCD/E only) by increasing the number of values of carrier phase from 36 to 72. For the collects of 512 and 1024 symbols, only the MCSCPD/E is simulated (2000 trials).

For the ML and CMCD/E receivers, maximization over the carrier phase θ_4 , the symbol timing θ_3 , and the TDOA $\theta_2 - \theta_1$ is necessary. This is accomplished by an exhaustive search over 36 values of the carrier phase evenly spaced over the range $[0, 2\pi)$, and 32 values of the symbol timing parameter evenly spaced over the range $[-T_0/2, T_0/2)$. The TDOA search parameter is restricted to kT_s , where k ranges from 0 to 75, and T_s is the sampling increment. For the MCSCPD/E, only the search over the TDOA is necessary; and for the MCSCMD, no search is necessary. For each of the ML, MCSCPD/E, and CMCD/E, the result of each Monte Carlo run is a detection statistic and a TDOA estimate, whereas for the MCSCMD and MCD it is only a detection statistic.

The detection performance is measured by the value of P_D where $P_{FA} = 1/10$. By normalizing the set of Monte Carlo statistics and choosing a large set of thresholds, many (P_D, P_{FA}) pairs can be computed. Only the pair with $P_{FA} = 1/10$ is reported herein. The TDOA performance is measured by computing the RMSE of the collection of estimates for the signal-present case, normalized by the true TDOA, which is $\Delta = 21T_s$. The results are graphed versus the α -set number (i corresponding to A_i is the abscissa) for both detection and TDOA estimation results.

A. Detection Performance

1) *No Noise Variability*: To be absolutely clear, noise variability equal to zero means that the value of N_0 from trial to trial is constant. The detection performance for this case is shown in Figs. 1–5, which correspond to collect times ranging from 8 symbols to 128 symbols in powers of 2, respectively. From Fig. 1, we see that the performance ordering for the shortest collect can be symbolically denoted by

$$ML \geq CMCD/E \geq MCD \geq MCSCPD/E \geq MCSCMD$$

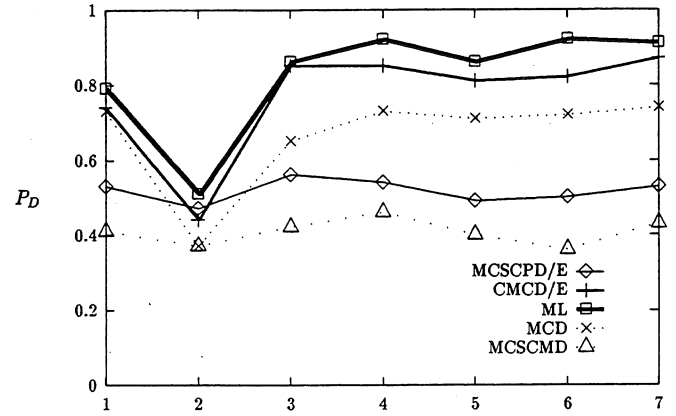


Fig. 1. P_D for $P_{FA} = 0.1$, $T = 8T_0$, noise variability = 0.

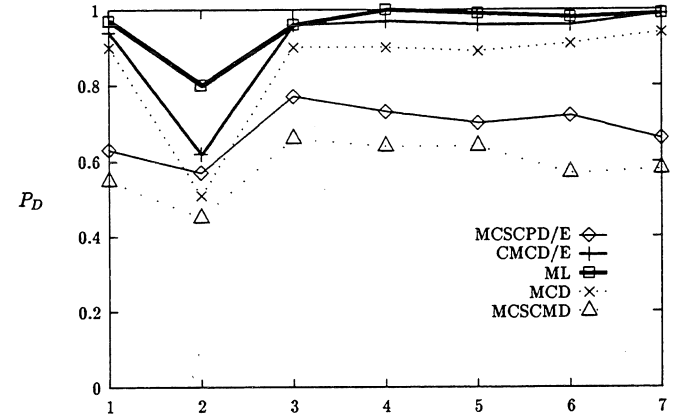
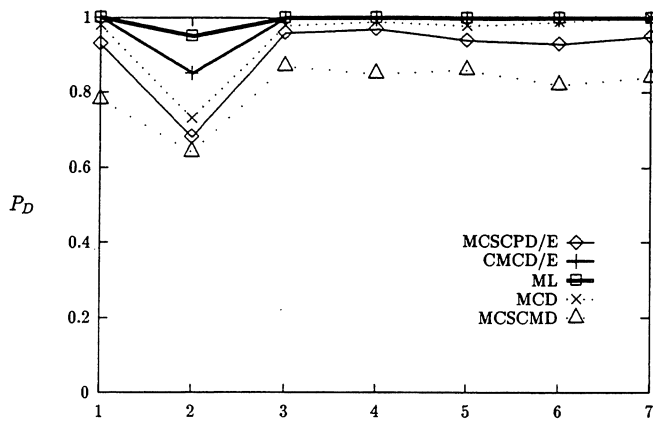
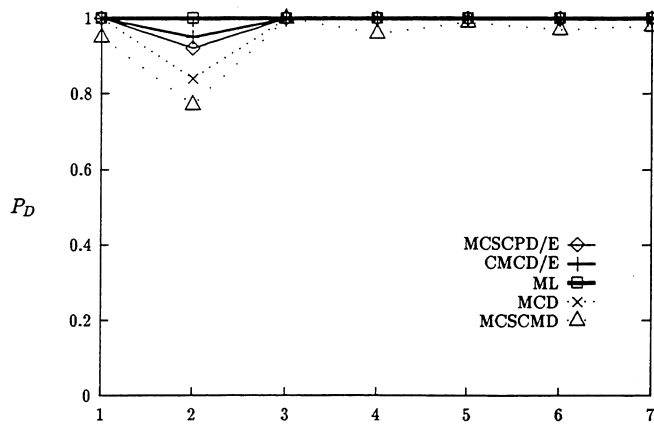
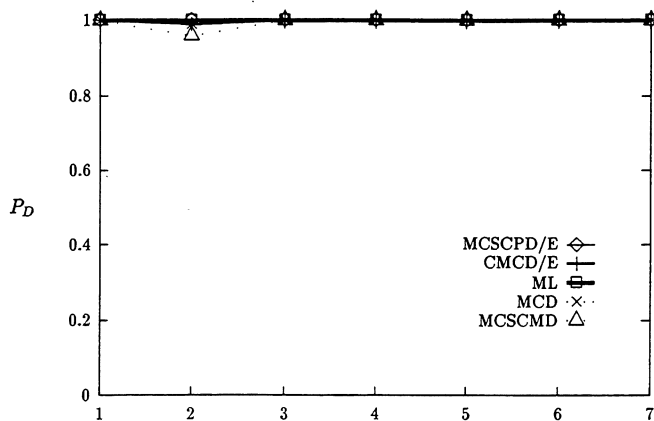


Fig. 2. P_D for $P_{FA} = 0.1$, $T = 16T_0$, noise variability = 0.

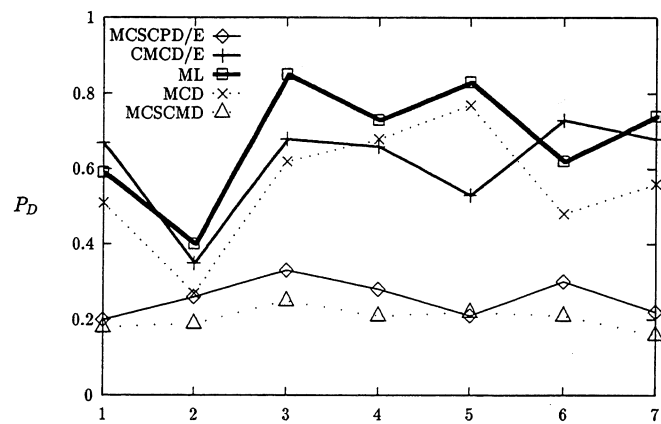
for all A_i except A_2 (which includes only the symbol rate $\{1/T_0\}$). The performance is poorest for each detector for A_2 , and MCSCPD/E performs slightly better than the MCD and the CMCD/E for A_2 . This can be explained by the fact that A_2 includes only the weakest cyclic feature (smallest η_α). The performance of each detector increases for all A_i as the collect time is increased, but the performance ordering remains unchanged. For the collect time $64T_0$, detection is nearly perfect for all detectors and for all A_i except A_1 and A_2 ; and for $T = 128T_0$, all methods perform equally except the MCSCMD, which nevertheless delivers very good performance.

The most useful cycle frequencies are $\alpha = 0$ (A_1) and $\alpha = 2f_c$ (A_3). These correspond to the strongest cyclic features (largest η_α). The best performance for all devices typically occurs for A_3 or A_4 . Thus, for the case of zero noise variability, the performance of the ML detector using a large α -set can be closely approximated by using the ML detector modified to exclude all but a small subset of values of α , which in turn can be closely approximated by the MCD or by the MCSCMD for collect times that are typically used in weak-signal detection (large T). For very small collect times (e.g., $T = 8T_0$), the ML detector provides a substantial detection-performance advantage over all of the other detectors, especially the MCSCMD and the MCSCPD/E, for all α -sets except A_2 , which results in poor detection for all

Fig. 3. P_D for $P_{FA} = 0.1$, $T = 32T_0$, noise variability = 0.Fig. 4. P_D for $P_{FA} = 0.1$, $T = 64T_0$, noise variability = 0.Fig. 5. P_D for $P_{FA} = 0.1$, $T = 128T_0$, noise variability = 0.

methods, and except for CMCD/E for A_3 . We recall the fact that for singleton α -sets, the MCD requires no maximization over any parameters—it is the single-cycle detector (SCD) [4] and its magnitude is the detection statistic. Thus, the best ML performance can be closely approximated by the SCD with A_3 (or A_1) for large collect.

2) *Noise Variability of 1/10*: Here the value of N_0 is chosen at random for each Monte Carlo trial as a sample of the square of a Gaussian random variable designed such that the variance of N_0 divided by its squared mean is equal to 1/10. The performance of the detectors for this

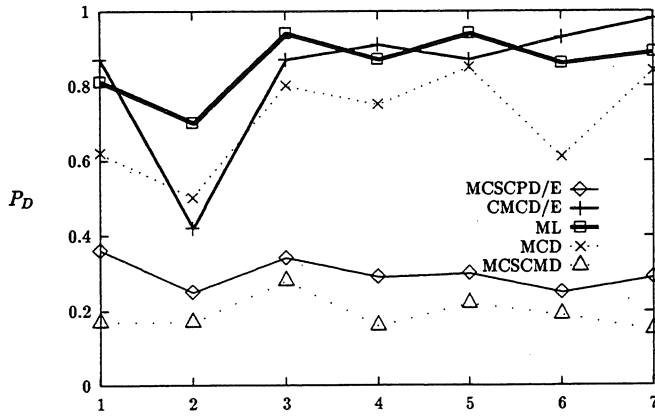
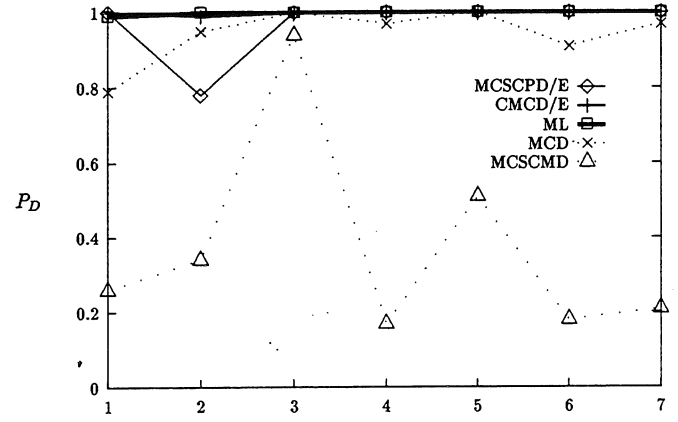
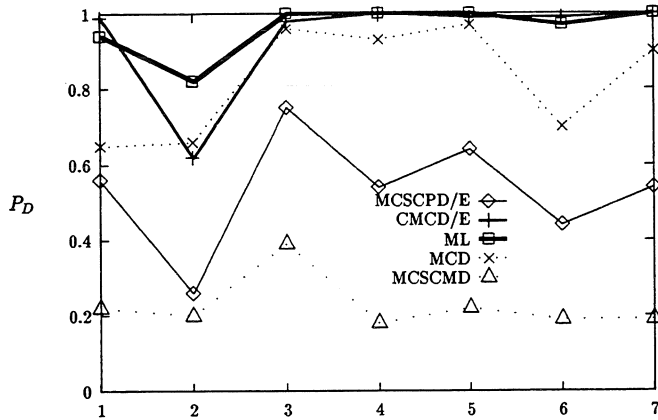
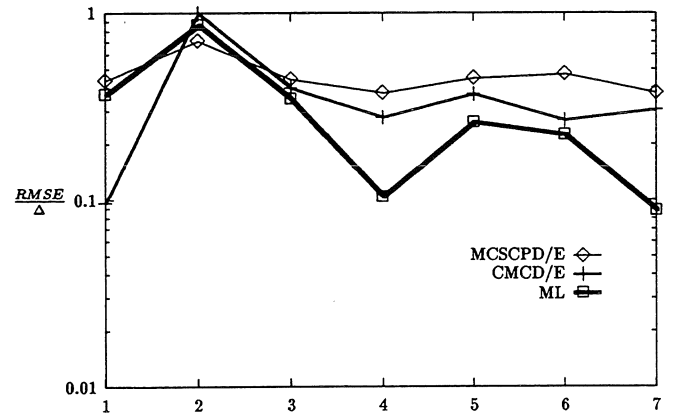
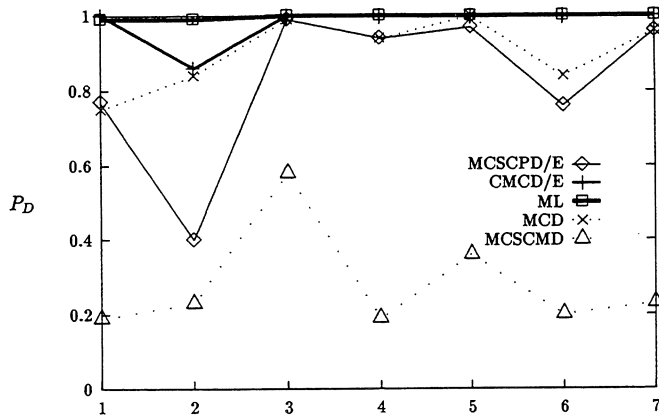
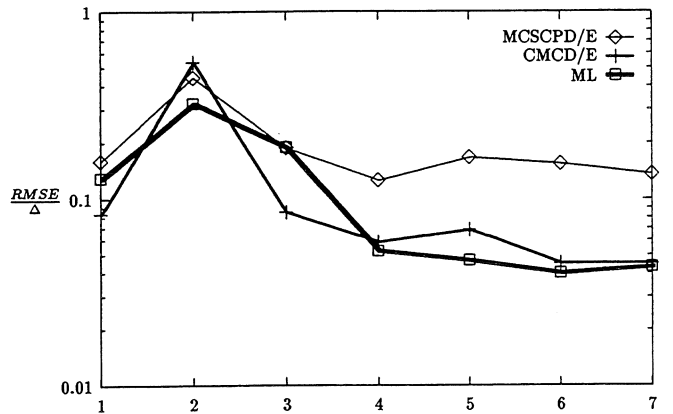
Fig. 6. P_D for $P_{FA} = 0.1$, $T = 8T_0$, noise variability = 0.1.

signal environment is shown in Figs. 6–10. In contrast to the previous case, we find here that the performance ordering of the detectors is dependent upon the α -sets. For the ML and MCD detectors, the best performance is for α -sets A_3 and A_5 , whereas for the CMCD/E it is best for A_6 and A_7 . For the MCSCMD and MCSCPD/E, the performance is very poor for the smaller collects, but their performance patterns match with those of the ML detector. The main influence here is the presence of an *estimate of an autospectrum* in the detection statistic, that is, the presence of $\alpha = 0$ in the α -set combined with the presence of a single-receiver term in the detection statistic. These measured autospectra are heavily influenced by the variable WGN, whereas both cyclic cross spectra and cyclic autospectra, as well as the noncyclic cross spectrum, are not. There are no such autospectral estimates in the CMCD/E, and so the presence of the $\alpha = 0$ term does not cause its performance to degrade. All of the other detectors do contain measured autospectra and, therefore, deliver weak performance for all A_i containing $\alpha = 0$. For all α -sets except A_1 , this problem can be overcome with large enough collect time because although the conditional variances of the detection statistic continue to be influenced by the variability in the noise as the collect is increased, the conditional means move apart, allowing good detection to occur (Fig. 10). However, the MCD is affected adversely even for T as large as $128T_0$. So, in the case of nonzero noise variability, all the detectors are effective for α -sets containing $2f_c$ and not containing 0, and the CMCD/E can be very effective for α -sets $i = 3$ and above. The best performance of any of the detectors in this environment can be closely approximated with a single-cycle statistic without any search over unknown parameters, as is also true in the case of zero noise variability.

B. TDOA Estimation Performance

1) *No Noise Variability*: The TDOA estimation performance of the ML, CMCD/E, and MCSCPD/E receivers for $T = 32T_0$, $64T_0$, and $128T_0$ is shown in Figs. 11–13, respectively. The general performance ordering is

$$ML \geq CMCD/E \geq MCSCPD/E$$

Fig. 7. P_D for $P_{FA} = 0.1$, $T = 16T_0$, noise variability = 0.1.Fig. 10. P_D for $P_{FA} = 0.1$, $T = 128T_0$, noise variability = 0.1.Fig. 8. P_D for $P_{FA} = 0.1$, $T = 32T_0$, noise variability = 0.1.Fig. 11. Normalized RMSE of TDOA estimate, $T = 32T_0$, noise variability = 0.Fig. 9. P_D for $P_{FA} = 0.1$, $T = 64T_0$, noise variability = 0.1.Fig. 12. Normalized RMSE of TDOA estimate, $T = 64T_0$, noise variability = 0.

with the performances of the ML and CMCD/E being close, and that of the MCSCP/E being far behind. The dominant cycle frequencies are 0 and $2f_c$; A_4 and A_6 or A_7 deliver the best performance for the ML and MCSCP/E, whereas any A_i containing $\alpha = 0$ delivers good performance for CMCD/E. Fig. 13 reveals that for long collect, the CMCD/E and ML are very good estimators for A_4 and A_6 , whereas the MCSCP/E delivers relatively poor estimates for all A_i . The MCSCP/E needs about twice the collect of the ML or CMCD/E to attain a similar performance mark.

Unlike detection, the ML performance is not closely approximated by a much simpler partial implementation for the relatively short collects considered here: the performance advantage of the ML over the MCSCP/E is substantial for all A_i and all collects considered.

2) *Noise Variability of 1/10* The basic performance patterns of the estimators is the same in this case as in the previous case (Figs. 14–18). The MCSCP/E is barely affected by the variability in the noise. The CMCD/E is affected most

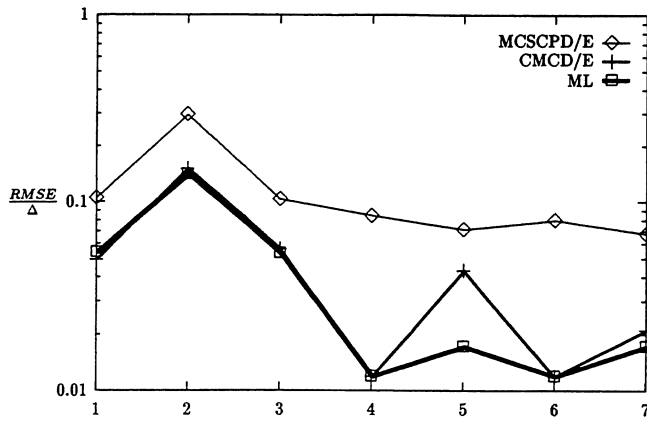


Fig. 13. Normalized RMSE of TDOA estimate, $T = 128T_0$, noise variability = 0.

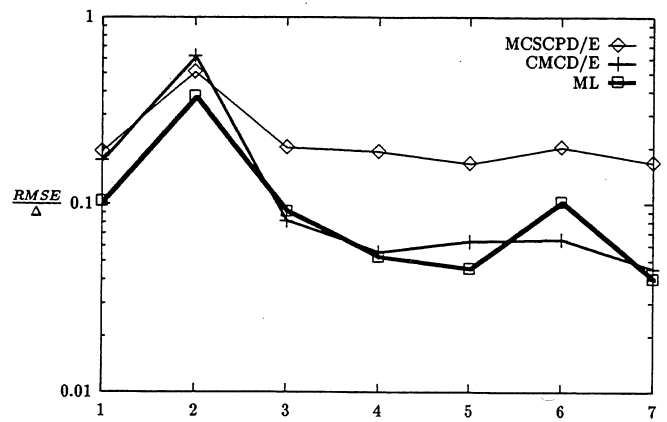


Fig. 15. Normalized RMSE of TDOA estimate, $T = 64T_0$, noise variability = 0.1.

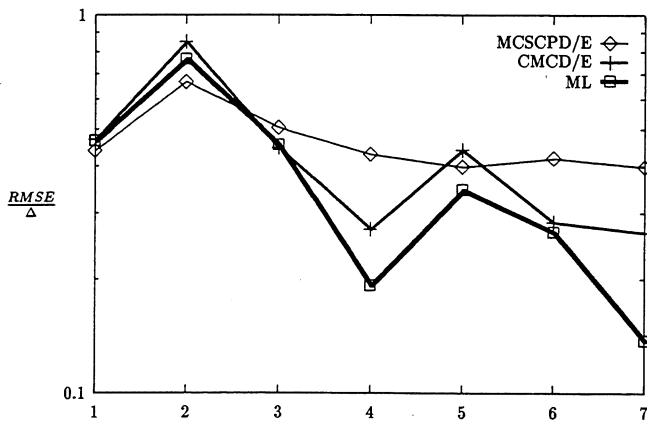


Fig. 14. Normalized RMSE of TDOA estimate, $T = 32T_0$, noise variability = 0.1.

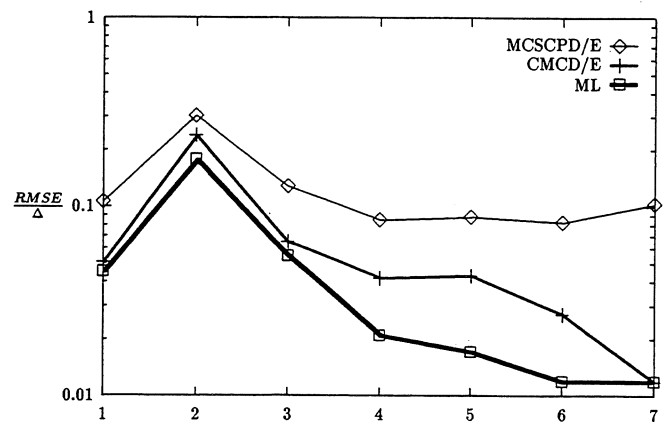


Fig. 16. Normalized RMSE of TDOA estimate, $T = 128T_0$, noise variability = 0.1.

strongly for the A_i that contain 0, and in particular for A_4 and A_6 . However, the performance of both the CMCD/E and ML estimators is very good for $i = 4$ and above for long collect (Fig. 16). As in the case of zero variability, the performance of the ML is not approached by the MCSCPD/E even for the longest collect considered. Nevertheless, as Figs. 17 and 18 illustrate, the performance of the MCSCPD/E, which requires only a single-parameter search, continues to improve at roughly the same rate as the CMCD/E which, like the ML, requires at least a two-dimensional and in some cases a three-dimensional search. (In Fig. 17, the CMCD/E produces no errors for A_7 , thus the curve points straight down at A_6 , since the log rmse is $-\infty$ for A_7 .)

To conclude, TDOA estimation is enhanced by adding values to the α -set for all three estimators considered here; larger α -sets deliver better performance in general. both the ML and the CMCD/E perform well in both environments, and the MCSCPD/E has an RMSE ranging from about 5 to 10 times that of the ML even for the longest collect considered. For example, for the specific signal and noise model considered here, if a normalized RMSE in the range of 1/10 to 1/100 is acceptable, then the computational simplicity of the MCSCPD/E renders it competitive with the ML and CMCD/E, even though a relatively long collect is required to achieve this performance. The same conclusion holds for lower

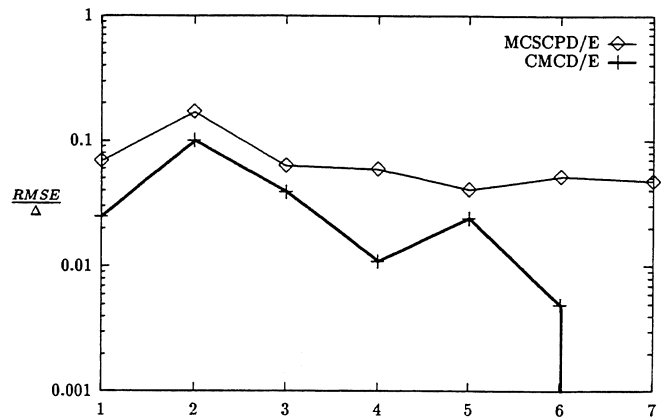


Fig. 17. Normalized RMSE of TDOA estimate, $T = 256T_0$, noise variability = 0.1.

RMSE provided that the required collect length is acceptable. On the other hand, the added complexity of the CMCD/E can be warranted when the collect length is limited.

V. CONCLUSIONS

It is shown that the weak-signal maximum-likelihood joint detector and TDOA estimator for dual-receiver signal interception can be decomposed into a variety of optimal detec-

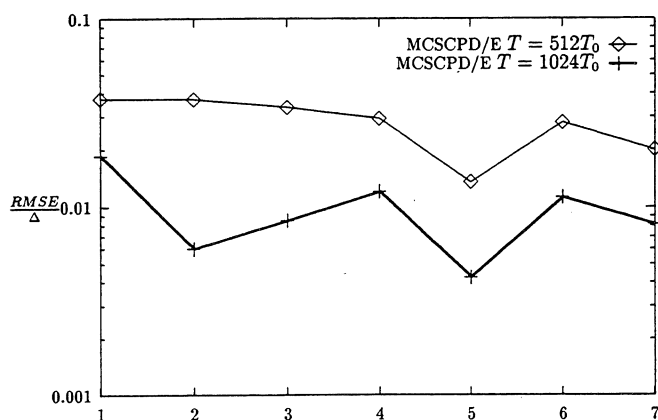
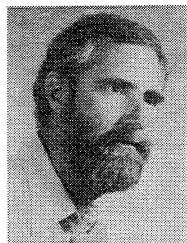


Fig. 18. Normalized RMSE of TDOA estimate, noise variability = 0.1.

tors and TDOA estimators. It is also shown that suboptimal implementations of the component detectors and estimators can be substantially less computationally intensive than the maximum-likelihood detector/estimator. The results of extensive Monte Carlo simulations show that the performance of the relatively complicated maximum-likelihood detector/estimator can in many cases be closely approximated by the performances of the much simpler suboptimum detectors and estimators, and for short collects with variable noise level some suboptimum detector/estimators actually perform better than the ML detector/estimator (which actually provides maximum likelihood only for fixed noise level).

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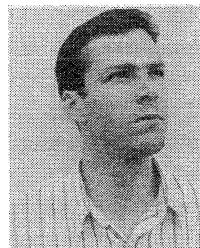


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