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Identification of polyperiodic nonlinear systems ☆

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Abstract

A method for identifying the Fourier–Volterra kernels of polyperiodic nonlinear systems by crosscorrelating the system output with prescribed nonlinear transformations of a prescribed random input is introduced. A frequency-domain counterpart of this method, which can utilize a computationally efficient FFT algorithm to compute cross-spectra of the system output and prescribed nonlinear transformations of the input, is also introduced. With complex inputs, the methods apply to infinite-order systems with infinite memory. For real inputs, they apply only to finite-order systems.

Zusammenfassung

Eine Methode zur Identifikation des Fourier–Volterra-Kerns mehrfach-periodischer nichtlinearer Systeme wird eingeführt; sie arbeitet mit der Kreuzkorrelation des Systemausgangs mit nichtlinearen Transformationen einer vorgeschriebenen Zufallserregung. Auch ein Frequenzbereichs-Gegenstück dieser Methode wird eingeführt; es kann einen recheneffizienten FFT-Algorithmus zur Berechnung von Kreuzspektren des Systemausgangs und vorgeschriebenen nichtlinearen Eingangstransformationen nutzen. Mit komplexen Eingangssignalen lassen sich die Verfahren auf Systeme unendlichen Grades mit unbegrenztem Gedächtnis anwenden, mit reellwerrtigen Eingangssignalen nur auf Systeme endlicher Ordnung.

Résumé

On introduit une méthode d'identification des noyaux de Fourier–Volterra pour les systèmes nonlinéaires polypériodiques en intercorrelant la sortie du système avec des transformations nonlinéaires prédéfinies d'une entrée aléatoire donnée. Une contre-partie fréquentielle de cette méthode, qui peut utiliser un algorithme de TFR efficace en termes de calcul pour trouver les inter-spectres de la sortie du système et des transformées nonlinéaires prédéfinies de l'entrée, est également introduite. Dans le cas de données complexes, les méthodes s'appliquent aux systèmes d'ordre infini avec une mémoire infinie. Dans le cas de données réelles, elles ne s'appliquent qu'aux systèmes d'ordre fini.

Keywords: System identification; Nonlinear; Time-variant systems; Fourier–Volterra series

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1. Introduction

The class of nonlinear systems whose characteristics change with time according to one or more (possibly incommensurate) periodicities, namely the polyperiodic nonlinear (PPN) systems, is appropriate for describing the relationships between input/output measurements on natural systems subject to seasonal variations. Examples can be found in meteorology, atmospheric science, oceanography, hydrology, econometrics, and so on. This class of systems is also appropriate for manmade systems subject to single or multiple periodic variation, collectively called polyperiodic variation. Examples include systems subject to vibration from rotating machinery, radio communication systems, and other signal processing systems where periodic signal processing operations (e.g., sampling, scanning, modulating, multiplexing, coding, and so on) are used.

A great deal of work has gone into development of theory and methods for identifying, by input/output measurement, time-invariant nonlinear (TIN) systems [3, 9–13]. Recently, new methods, which are based on using system inputs that are cyclostationary [5], have been introduced for both polyperiodic linear [4, 5] and time-invariant nonlinear systems that admit Volterra-series representation [7]. Nevertheless, there is (to our knowledge) no general approach to PPN system identification reported in the literature.

In this paper, the methods proposed in [7] for the class of real TIN-Volterra systems are generalized to real PPN systems. More specifically, for the class of PPN systems that can be represented by a generalized form of Volterra series, called a Fourier–Volterra series, we have developed a class of methods for identifying the Fourier–Volterra kernels. The extension of the results from time-invariant to polyperiodically time-variant nonlinear systems is based on representing a PPN Fourier–Volterra system as a multiplicity of TIN Volterra systems whose outputs are frequency shifted (by the cycle frequencies of the PPN system) and summed. In fact, this representation suggests how to specify a class of cyclostationary complex-valued time-series inputs for PPN systems that enables the analytical specification of a set of operators on the input that are orthonormal over all time to the Fourier–Volterra operators. Such operators are used to obtain an input/output type of crosscorrelation formula for identifying the individual Fourier–Volterra kernels of arbitrary order of a PPN system of possibly infinite order and possibly infinite memory. A class of cyclostationary real-valued time-series inputs is also introduced for which the same sets of specified operators apply. However, the orthogonality for different orders for these real inputs holds only for Fourier–Volterra operators of order less than that of the specified operator. Thus, these real inputs can be used to identify Fourier–Volterra kernels only for finite-order systems.

To our knowledge, the methods introduced here for complex inputs are the first methods to be able to identify an arbitrary-order Fourier–Volterra kernel for an infinite-order system. Unfortunately, these methods can be used only when a complete description of the input–output rule (such as a mathematical model) that defines the system for complex inputs is available, e.g., for simulation on a computer. Complex inputs cannot, of course, be applied to physical systems. Nevertheless, methods for identifying the Fourier–Volterra kernels from mathematical models have potentially important practical applications such as the calculation of the performance of a communication system with nonlinear components (cf. [1, 2]). Also the methods for the real counterparts of the complex inputs, which require that the system be accurately approximable by a finite-term Fourier–Volterra series, appear to be computationally attractive when finite-state inputs are used (cf. [7]).

2. Polyperiodic nonlinear systems

To introduce PPN systems that can be represented by Fourier–Volterra series, let us first consider the class of real TIN systems with possibly complex-valued input $x(k)$ and output $y(k)$ whose members admit the Volterra series representation

$$y(k) = \sum_{j_{1_1}} h_1(j_{1_1})x(k-j_{1_1}) + \sum_{j_{2_1},j_{2_2}} h_2(j_{2_1},j_{2_2})x(k-j_{2_1})x(k-j_{2_2})$$

$$+ \sum_{j_{3_1}, j_{3_2}, j_{3_3}} h_3(j_{3_1}, j_{3_2}, j_{3_3}) x(k - j_{3_1}) \times x(k - j_{3_2}) x(k - j_{3_3}) + \cdots. \quad (1)$$

In shorthand notation, we have

$$y(k) = \sum_n \left[\sum_{j_n} h_n(\boldsymbol{j}_n) \lambda_n(k, \boldsymbol{j}_n, x(\cdot)) \right], \quad (2)$$

where k represents discrete time, $\boldsymbol{j}_n = [j_{n_1}, j_{n_2}, \ldots, j_{n_n}]$, $h_n(\boldsymbol{j}_n)$ is the nth-order Volterra kernel [14] and $\lambda_n(k, \boldsymbol{j}_n, x((\cdot))$ is the nth-order lag product of $x(k)$,

$$\lambda_n(k, \boldsymbol{j}_n, x((\cdot)) \triangleq \prod_{r=1}^{n} x(k - j_{n_r}). \quad (3)$$

This class of real TIN system can be generalized to a class of polyperiodic nonlinear systems in the same way as the class of time-invariant linear systems is generalized to that of polyperiodic linear systems. That is, we allow the kernels $h_n(k, \boldsymbol{j}_n)$ to be polyperiodic functions of time k. Under the assumption that the Fourier series associated with each polyperiodic kernel converges, one has

$$h_n(k, \boldsymbol{j}_n) = \sum_{\alpha \in A_n} h_n^{\alpha}(\boldsymbol{j}_n) \mathrm{e}^{\mathrm{i}2\pi\alpha k}, \quad (4)$$

where

$$h_n^{\alpha}(\boldsymbol{j}_n) \triangleq \langle h_n(k, \boldsymbol{j}_n) \mathrm{e}^{-\mathrm{i}2\pi\alpha k} \rangle, \quad (5)$$

$\langle \cdot \rangle$ denotes average over all time, namely

$$\langle f(k) \rangle \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} f(k), \quad (6)$$

and A_n is the set of frequencies for which $h_n^{\alpha}(\cdot) \not\equiv 0$. Substituting the generalized kernels (4) in place of the Volterra kernels in the Volterra series (2) yields the system representation

$$y(k) = \sum_{\alpha \in A} \left\{ \sum_n \left[\sum_{j_n} h_n^{\alpha}(\boldsymbol{j}_n) \lambda_n(k, \boldsymbol{j}_n, x(\cdot)) \right] \right\} \mathrm{e}^{\mathrm{i}2\pi\alpha k}$$

$$= \sum_{\alpha \in A} y_{\alpha}(k) \mathrm{e}^{\mathrm{i}2\pi\alpha k}, \quad (7)$$

where A is the union of all sets A_n for $n = 1, 2, 3, \ldots$

We shall use the abbreviation PPN to denote the class of real polyperiodic nonlinear systems whose members admit the joint Fourier–Volterra series representation (7). Also, we shall refer to the frequencies α in (7) as the cycle frequencies of the system. We can see from (7) that a PPN system is equivalent to a multiplicity of TIN Volterra systems (with Volterra kernels $h_n^{\alpha}((\cdot))$ whose outputs $y_{\alpha}(k)$ are frequency shifted (by α) and summed. Thus, a PPN system is composed of multidimensional convolution operators (operating on the lag-products of the input) followed by frequency-shift operators and a summing operator.

The kernel $h_n^{\alpha}(\cdot)$ of the operator

$$\sum_{j_n} h_n^{\alpha}(\boldsymbol{j}_n) \lambda_n(k, \boldsymbol{j}_n, x(\cdot)) \mathrm{e}^{\mathrm{i}2\pi\alpha k}, \quad (8)$$

operating on $x(t)$, shall be called the nth-order Fourier–Volterra kernel (which reduces to the Volterra kernel for $\alpha = 0$), and the corresponding operator shall be called the nth-order Fourier–Volterra operator (which reduces to the Volterra operator for $\alpha = 0$). However, the elementary operator

$$\lambda_n^{\alpha}(k, \boldsymbol{j}_n, x(\cdot)) \triangleq \lambda_n(k, \boldsymbol{j}_n, (\cdot)) \mathrm{e}^{\mathrm{i}2\pi\alpha k} \quad (9)$$

also shall be called the nth-order Fourier–Volterra operator.

3. The identification methods

We are interested in the problem of identifying PPN systems using input/output measurements. That is, we want to determine the cycle frequencies α and the Fourier–Volterra kernels $h_n^{\alpha}(\cdot)$. We are interested in two versions of this problem. In one version we have a mathematical model of the operator that maps input sequences into output sequences $y(k)$ and we want to calculate α and $h_n^{\alpha}(\cdot)$ in the representation (7) for this model. In the other version of the problem, we have a physical system that maps $x(k)$ into $y(k)$, but we have no mathematical model and we want to fit the model (7) to this system by appropriately selecting α and $h_n^{\alpha}(\cdot)$. In the first version of the problem, we are allowed to use complex-valued input sequences (if this will facilitate the calculations), but in the second version we are restricted to real input sequences.

We shall consider only random input sequences $x(k)$ and we shall employ as in [7] the temporal probability framework [5, 6, 8] to describe the statistical characteristics of the inputs. We begin by reviewing the necessary concepts and definitions.

A *stationary* time-series $z(k)$ is one for which infinite-time averages of lag-products (and all other well-behaved time-invariant (or k-invariant) functions of the time series) are all finite and are not all identically zero, and for which the sinusoidally weighted lag-products (and all other sinusoidally weighted well-behaved time-invariant functions) of the time-series have infinite-time averages that are identically zero for all noninteger sinewave frequencies [6]. That is, for a stationary time series the cyclic moments

$$R_z^\alpha(\boldsymbol{j}_n) = \langle \lambda_n(k, \boldsymbol{j}_n, z(\cdot))\mathrm{e}^{-\mathrm{i}2\pi\alpha k}\rangle \tag{10}$$

are zero for all noninteger real cycle frequencies α, all lag variables $\boldsymbol{j}_n$, and all positive integers n. It follows that if $z(k)$ is stationary, then every well-behaved k-invariant function of $z(k - k_1)$, $z(k - k_2)$, $\ldots, z(k - k_n)$, say

$$\begin{aligned} f(k, \boldsymbol{k}_n, z(\cdot)) &= f(z(k-k_1), \ldots, z(k-k_n)) \\ &= f(0, \boldsymbol{k}_n - \mathbf{1}_n k, z(\cdot)) \end{aligned} \tag{11}$$

(where $\mathbf{1}_n$ is the n-dimensional row vector with all elements equal to unity), is also stationary. If (10) is nonzero for some noninteger α, then $z(k)$ exhibits *cyclostationarity* and α is called a *cycle frequency*. Specifically, when the set of all cycle frequencies is not integrally divisible by a single fundamental cycle frequency, then $x(k)$ exhibits *polycyclostationarity* [6].

If the joint fraction-of-time probability density functions of a time-series factor into products of the individual fraction-of-time densities, then the time-series is said to be a sequence of statistically (temporally) independent variables [6]. Under such an assumption for $z(k)$ its joint moments factor as follows:

$$\left\langle \prod_{q=1}^{p} z^{n_q}(k - j_{m_q}) \right\rangle = \prod_{q=1}^{p} \langle z^{n_q}(k - j_{m_q})\rangle \tag{12}$$

for all distinct lags $j_{m_1}, j_{m_2}, \ldots, j_{m_p}$.

We shall now briefly describe a method for identifying a cycle frequency α of the PPN system under consideration. Since $\lambda_n(k, \boldsymbol{j}_n, x((\cdot)) = \lambda_n(0, \boldsymbol{j}_n - \mathbf{1}_n k, x(\cdot))$, then the operator that defines $y_\alpha(\cdot)$ in (7) is a k-invariant operator and, therefore, transforms any stationary sequence $x(\cdot)$ into a stationary sequence $y_\alpha(\cdot)$. Therefore, the cyclic mean of the system output is given by

$$M_y^\beta \triangleq \langle y(k)\mathrm{e}^{-\mathrm{i}2\pi\beta k}\rangle = \sum_\alpha \langle y_\alpha(k)\rangle \delta_{\alpha\beta}, \tag{13}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta. Thus, as long as $\langle y_\alpha(k)\rangle \neq 0$ for each cycle frequency of the system, all these frequencies can be determined since the spectrum of the output $y(k)$ will contain spectral lines of strength $|M_y^\alpha|^2$ at each cycle frequency α and only at these frequencies. If the fraction-of-time probability density functions of $x(k)$ ar even functions, then all odd-order moments are zero, and, therefore, only even-order nonlinearities (even n) in the system can contribute to $\langle y_\alpha(k)\rangle$. In order to detect cycle frequencies associated with odd-order, as well as even-order nonlinearities, we can use the spectral lines present in the spectrum of one or more quadratic transformations of $y(k)$.

Regarding the identification of the Fourier–Volterra kernels, the PPN system representation (7) suggests deriving methods for their identification by extending and generalizing the approach proposed in [7] for identifying the Volterra kernels of TIN systems. More specifically, let us consider as in [7] the two classes of cyclostationary inputs:

$$x(k) = z(k)\mathrm{e}^{\mathrm{i}\omega k}, \tag{14a}$$

$$x(k) = \mathrm{Re}\{z(k)\mathrm{e}^{\mathrm{i}\omega k}\}, \tag{14b}$$

where $z(k)$ is a real- or complex-valued stationary time-series and ω is any real number such that

$$\frac{n\omega}{2\pi} + \alpha - \beta \neq \text{integer} \tag{15}$$

for all differences $\alpha - \beta$ of cycle frequencies of the PPN system of interest and for all nonzero integers n. (The reason for this constraint on ω is explained in Appendix A.) Moreover, the following result (also utilized in [7]) holds: For some stationary time-series $z(k)$ we can find a set of k-invariant operators

$\varphi_n(k, \boldsymbol{k}_n, z(\cdot))$ for a given $z(k)$ that is orthonormal (reciprocal) to the set of the lag-product operators

$$\lambda_n(k, \boldsymbol{j}_n, z((\cdot)) = \prod_{r=1}^{n} z(k - j_{n_r}) \tag{16}$$

in the sense that

$$\langle \lambda_n(k, \boldsymbol{j}_n, z((\cdot))\varphi_n(k, \boldsymbol{k}_n, z((\cdot))^* \rangle = \delta_{\tilde{\boldsymbol{j}}_n \boldsymbol{k}_n}, \tag{17}$$

where $\tilde{\boldsymbol{j}}_n$ is any permutation of $\boldsymbol{j}_n$, and $\delta_{\tilde{\boldsymbol{j}}_n \boldsymbol{k}_n}$ is the n-dimensional Kronecker delta (which is the product of n one-dimensional Kronecker deltas: $\delta_{j_{n_r}, k_{n_s}} = 1$ for $j_{n_s} = k_{n_s}$ and $\delta_{j_{n_r}, k_{n_s}} = 0$ for $j_{n_r} \neq k_{n_s}$).

The two following theorems and associated corollaries provide the results on identifying Fourier–Volterra kernels for both complex and real signals.

Theorem 1. *Let the operators γ_n^β be defined by*

$$\gamma_n^\beta(k, \boldsymbol{k}_n, z((\cdot)) \triangleq \mathrm{e}^{-\mathrm{i}\omega \boldsymbol{1}_n(\boldsymbol{k}_n - \boldsymbol{1}_n k)'} \varphi_n(k, \boldsymbol{k}_n, z((\cdot))\mathrm{e}^{\mathrm{i}2\pi\beta k}, \tag{18}$$

where φ_n satisfies (17), and let $x(k)$ be given by (14a) and (15). Then these operators γ_n^β are orthonormal over the orders n, the lag sets $\boldsymbol{k}_n$, and the cycle frequencies β to the Fourier–Volterra operators λ_m^α:

$$\langle \lambda_m^\alpha(k, \boldsymbol{j}_m, x((\cdot))\gamma_n^\beta(k, \boldsymbol{k}_n, z((\cdot))^* \rangle = \delta_{nm}\delta_{\tilde{\boldsymbol{j}}_n \boldsymbol{k}_n}\delta_{\alpha\beta}. \tag{19}$$

A proof of this theorem is given in Appendix A. (In (18), prime denotes the transpose operation.) As a corollary to this theorem, we have the following result on identifying Fourier–Volterra kernels.

Corollary. *Given operators φ_n satisfying (17) and system input specified by (14a) and (15), the operators γ_n^β defined by (18) can be used to identify the symmetrized Fourier–Volterra kernels in (7) by performing the crosscorrelation operation*

$$h_n^\alpha(\boldsymbol{k}_n) = \frac{1}{P(\boldsymbol{k}_n)} \langle y(k)\gamma_n^\alpha(k, \boldsymbol{k}_n, z((\cdot))^* \rangle, \tag{20}$$

where $P(\boldsymbol{k}_n)$ is the number of distinct permutations of the elements $\boldsymbol{k}_n$. Specifically, if $\boldsymbol{k}_n = \{k_{m_1}, \ldots, k_{m_1}, k_{m_2}, \ldots, k_{m_2}, \ldots, k_{m_p}, \ldots, k_{m_p}]$ where k_{m_q} is repeated n_q times and $n_1 + n_2 + \cdots + n_p = n$, then

$$P(\boldsymbol{k}_n) = \frac{n!}{n_1! n_2! \cdots n_p!}. \tag{21}$$

(The kernels of any system can be symmetrized, by averaging $h_n^\alpha(\boldsymbol{j}_n)$ over all permutations $\tilde{\boldsymbol{j}}_n$, without affecting the output of the system, cf. [13] for the case of $\alpha = 0$.)

For real inputs we have

$$x(k) = \frac{1}{2} z(k)\mathrm{e}^{\mathrm{i}\omega k} + \frac{1}{2} z^*(k)\mathrm{e}^{-\mathrm{i}\omega k} \tag{22}$$

and, parallel to Theorem 1, we have the following theorem (which is also is proved in Appendix A).

Theorem 2. *Let the operators γ_n^β be defined by (18) modified to include the factor 2^n where φ_n satisfies (17), and let $x(k)$ be given by (22). Then these operators γ_n^β are orthonormal over the orders n, the lag sets $\boldsymbol{k}_n$, and the cycle frequencies β to the Fourier–Volterra operators λ_m^α of orders $m \leqslant n$:*

$$\langle \lambda_m^\alpha(k, \boldsymbol{j}_m, x(\cdot))\gamma_n^\beta(k, \boldsymbol{k}_n, z(\cdot))^* \rangle = \delta_{nm}\delta_{\tilde{\boldsymbol{j}}_n \boldsymbol{k}_n}\delta_{\alpha\beta} \quad \text{for } m \leqslant n. \tag{23}$$

Thus, parallel to the corollary, for an Nth-order Fourier–Volterra system (for which $h_m^\alpha \equiv 0$ for all $m > N$ in (7)), (20) holds for $n = N$. After (20) is used to identify the Nth-order Fourier–Volterra kernels for all cycle frequencies, α, the Nth term in (7) can be subtracted off to produce the output of an $(N-1)$th-order system whose highest-order kernels $h_{N-1}^\alpha(\boldsymbol{j}_{N-1})$ can then be identified for all α. However, since small estimation errors can compound, a poor accuracy can result when the strength of kernels of different order is very different.

The following lemma, proved in [7], provides the operators $\varphi_n(k, \boldsymbol{k}_n, z(\cdot))$ satisfying (17).

Lemma. *Let $z(k)$ be a stationary sequence of statistically (temporally) independent variables so that the powers $z^m(k)$ are also linearly independent. In general, there exists a set of univariate orthogonal* *operators* $\psi_n(z(k-k_0))$ *that are orthonormal to the particular lag-product operators* $\lambda_m(k, \boldsymbol{j}_m, z(\cdot)) = z^m(k-j_0)$ *for* $m \leqslant n$,

$$\langle z^m(k-j_0)\psi_n^*(z(k-k_0))\rangle = \delta_{mn}\delta_{j_0k_0} \quad \text{for } m \leqslant n, \tag{24}$$

and we can define operators to satisfy the orthonormality property (17) and, for $\omega = 0$ *in (18), to also satisfy the orthonormality property (19) for* $m \leqslant n$, *as follows:*

$$\varphi_n(k, \boldsymbol{k}_n, z(\cdot)) \triangleq \prod_{q=1}^{p} \psi_{n_q}(z(k-k_{m_q})), \tag{25}$$

where n_q *is the number of times the lag* k_{m_q} *is repeated.*

By comparing the results, here obtained, for identifying PPN Fourier–Volterra systems with those of TIN Volterra systems derived in [7], one can see that the differences can be summarized as follows:

(i) the frequency value $\omega/2\pi$ of both possible input signals (14) not only must be irrational as in [7] but has to satisfy (15) for $\alpha - \beta \neq 0$;
(ii) the operators γ_n^β differ from those used for TIN system identification by the presence of the factor $e^{i2\pi\beta k}$, which renders them time-variant;
(iii) the operators γ_n^β are orthonormal to the Fourier–Volterra operators not only over the orders n and the lag sets $\boldsymbol{k}_n$ but also over the cycle frequencies β.

According to these observations, both the examples and the discussion reported in Section III of [7] apply also to PPN system identification. Specifically, since PM method in both its complex- and real-valued versions revealed to be a computationally attractive (relatively speaking) method for determining the Volterra kernels of TIN Volterra systems, here we shall consider such a method also for identifying PPN Volterra kernels.

Let $z(k) = \sigma e^{i\theta(k)}$ be a purely stationary sequence of statistically independent (white) variables having an M-ary discrete uniform circular temporal-probability distribution (fraction-of-time distribution [5, 6, 8]), which results from $\theta(k)$ having an M-ary discrete uniform distribution in the interval $[-\pi, \pi]$. In this case, we can make the choice

$$\psi_n(z(k-r)) \triangleq \sigma^{-n} z^n(k-r) \tag{26}$$

and satisfy (24) provided that $n - m$ is not a nonzero integer multiple of M. Therefore, a sufficient condition for identification formula (20) to be valid is that n_q in (25), which is used in (20), be less than M. That is, the alphabet size M must exceed the number of times any lag value is repeated at the point in the domain of the kernel at which the kernel is to be identified. (This restriction is a result of the fact that $z^m(k)$ are linearly independent only for $1 \leqslant m \leqslant M$.) This can be guaranteed for arbitrarily high-order kernels, by letting $M \to \infty$: i.e., by using a continuous uniform distribution for $\theta(k)$.

It follows from (26) that for the complex phase-modulated (PM) input, we have

$$\varphi_n(k, \boldsymbol{k}_n, z(\cdot)) = \sigma^{-n}\lambda_n(k, \boldsymbol{k}_n, z(\cdot)) \tag{27}$$

and, therefore,

$$\gamma_n^\beta(k, \boldsymbol{k}_n, z(\cdot)) = \sigma^{-n}\lambda_n(k, \boldsymbol{k}_n, x(\cdot))e^{i2\pi\beta k}. \tag{28}$$

The real counterpart (14b) of the input in this example is the phase-modulated sinewave

$$x(k) = \sigma\cos(\omega k + \theta(k)). \tag{29}$$

It is important to recognize that for $\omega \neq 0$, both the complex and real PM inputs take on values throughout a continuum (a disk of diameter 2σ for complex PM and an interval length of 2σ for real PM) even though the random sequence $\theta(k)$ has only a finite number (M) of states for each k (the same finite set of states for all k). For $\omega = 0$, the PM inputs take on only a finite number (M) of values. Thus, for $\omega \neq 0$, the PM input is much richer than it is for $\omega = 0$. Specifically, it is a white input with a fraction-of-time amplitude density equal to that of a sinusoid.

Regarding computational complexity, it can be seen from the proof of Theorem 1 in Appendix A, that the averaging time used in practice must not only be long enough to obtain statistically reliable estimates of the infinitely long averages used in the theory, but also be long enough to adequately reduce bias that results from sinewaves with frequencies $n\omega/2\pi$ not being orthogonal over all finite-length intervals to sinewaves with frequencies $\alpha - \beta$ for all cycle frequencies α of the PPN system, where β is the cycle frequency of the kernel $h_n^\beta(\cdot)$ being identified. Thus, ideally for each β, ω should be chosen to maximize the minimum (over all $\alpha \in A$

and all n less than or equal to the order of the system) of the noninteger part of $n\omega/2\pi + \alpha - \beta$. This follows from (A.3) in Appendix A. Moreover, to exploit the computational efficiency of FFT algorithms, the frequency-domain counterparts of the time-domain methods, which directly identify the multidimensional Fourier transforms of the Fourier–Volterra kernels (i.e., the Fourier–Volterra transfer functions $H_n^\alpha(\boldsymbol{f}_n)$) can be considered. All the equations derived in Section IV of [7] still apply to the identification of the PPN Fourier–Volterra transfer-functions provided that the operator γ_n is replaced with the operator γ_n^α given by (18), $w(\boldsymbol{k}_n)$ is replaced with

$$w_n^\alpha(\boldsymbol{k}_n) = \frac{1}{P(\boldsymbol{k}_n)} \gamma_n^\alpha(0, \boldsymbol{k}_n, z(\cdot)), \tag{30}$$

and the truncated (to intervals of length T) output $y_T(k)$ is replaced with its α-frequency-shifted version so that the estimate of $H_n^\alpha(\boldsymbol{f}_n)$ obtained from the Fourier transform of (20), by reducing the averaging time for $\langle\cdot\rangle$ from ∞ to T, can be closely approximated for $T \gg K$ by

$$\hat{H}_n^\alpha(\boldsymbol{f}_n) \cong \sum_{\boldsymbol{0} \leqslant \boldsymbol{k}_n \leqslant \boldsymbol{K}} \left[\frac{1}{T} \sum_{k=-\infty}^{+\infty} y_T(k) w_T^\alpha(\boldsymbol{k}_n - \boldsymbol{1}_n k)^* \times e^{-i2\pi\alpha k} \right] e^{-i2\pi \boldsymbol{f}_n \boldsymbol{k}_n'}$$

$$= \left[\frac{1}{T} W_T^\alpha(-\boldsymbol{f}_n)^* Y_T(\boldsymbol{f}_n \boldsymbol{1}_n' - \alpha) \right] \otimes A_K(f_1) \otimes \cdots \otimes A_K(f_n), \tag{31}$$

where

$$A_K(f) = \sum_{k=-K/2}^{K/2} e^{-i2\pi fk} = \frac{\sin[\pi f(K+1)]}{\sin(\pi f)}, \tag{32}$$

$$Y_T(f) = \sum_{|k| \leqslant T/2} y(k) e^{-i2\pi kf}, \tag{33}$$

$$W_T^\alpha(-\boldsymbol{f}_n) = \sum_{|\boldsymbol{k}_n| \leqslant T/2} w^\alpha(\boldsymbol{k}_n) e^{i2\pi \boldsymbol{f}_n \boldsymbol{k}_n'}, \tag{34}$$

and $\boldsymbol{0} \leqslant \boldsymbol{k}_n \leqslant \boldsymbol{K}$ means $0 \leqslant k_{n_i} \leqslant K$ for $i = 1, 2, \ldots, n$, where K should be chosen to exceed the system memory length L (but not excessively) for which $h_n^\alpha(\boldsymbol{k}_n) = 0$ for $k_{n_i} > L$ for every i.

4. Simulations

A simulation experiment was carried out to verify that the proposed system identification method assures that the Fourier–Volterra kernels estimates converge to the correct values. Specifically, we considered a second-order PPN system consisting of an LTI causal transformation described by the first-order difference equation

$$-0.89897\, z(k-1) + 1.89897\, z(k) = x(k), \tag{35}$$

followed by the nonlinear periodic system

$$y(k) = z(k)^2 [1 + 0.3\cos(1.37k)]. \tag{36}$$

From (35) and (36) it follows that the system is characterized by the kernels $h_2^0(k_1, k_2) = h(k_1)h(k_2)$ and $h_2^{\pm 1.37}(k_1, k_2) = 0.15 h_2^0(k_1, k_2)$, with $h(\cdot)$ denoting the impulse response of system (35).

System input of PM types with sinewave frequencies of $\omega = 0$ and $\omega = 1$, and alphabet sizes M equal to 3, 6, 9 and 12 were considered. Both methods, namely real and complex versions of PM, were tested. A number K of 1000 Monte Carlo trials were used in correspondence of an input-data record length of 10 000, whereas for 100 000 samples $K = 100$ trials were considered. The bias and the coefficient of variation defined as

$$|\text{bias}| \triangleq \frac{\frac{1}{K} \sum_{i=1}^{K} |\hat{h}_2^\alpha(i) - h_2^\alpha(k_1, k_2)|}{|h_2^\alpha(k_1, k_2)|}, \tag{37}$$

$$\text{coef. var} \triangleq \frac{\frac{1}{K} \sum_{i=1}^{K} [\hat{h}_2^\alpha(i) - h_2^\alpha(k_1, k_2)]^2}{[h_2^\alpha(k_1, k_2)]^2}, \tag{38}$$

were evaluated for the estimates of the 2nd-order kernels $h_2^0(k_1, k_2)$, $h_2^{1.37}(k_1, k_2)$ computed in correspondence of $k_1 = 0$ and $k_2 = 1$. In (37) and (38) $\hat{h}_2^\alpha(\mathrm{i})$ is the ith estimate of $h_2^\alpha(k_1, k_2)$.

The results are reported in Tables 1 and 2 where the first letter (C or R) in the abbreviations denotes complex or real method. Since the variations in bias and coefficient of variation with respect to the alphabet size M were considered to be statistically insignificant, only the average values of these parameters (over the set of alphabet sizes) are reported

Table 1
Bias and coefficient of variation for Fourier–Volterra kernel $h_2^0(0,1)$ estimates obtained using various methods

Method	Record length = 10 000		Record length = 100 000	
	|bias|	coef. var	|bias|	coef. var
CPM				
$\omega = 0$	1%	2×10^{-4}	0.3%	1×10^{-5}
$\omega = 1$	1%	2×10^{-4}	0.3%	2×10^{-5}
RPM				
$\omega = 0$	3%	1×10^{-3}	0.9%	1×10^{-4}
$\omega = 1$	3%	1×10^{-3}	0.9%	1×10^{-4}

Table 2
Bias and coefficient of variation for Fourier–Volterra kernel $h_2^{1.37}(0,1)$ estimates obtained using various methods

Method	Record length = 10 000		Record length = 100 000	
	|bias|	coef. var	|bias|	coef. var
CPM				
$\omega = 0$	5%	4×10^{-3}	1%	3×10^{-4}
$\omega = 1$	5%	4×10^{-3}	1%	3×10^{-4}
RPM				
$\omega = 0$	15%	4×10^{-2}	5%	4×10^{-3}
$\omega = 1$	15%	4×10^{-2}	5%	3×10^{-3}

in the tables. The results show that both bias and coefficient of variation can be made arbitrarily small by increasing the record length. In fact, for a 10-fold increase in averaging time, there is a 10-fold decrease in coefficient of variation. Finally, it is worthwhile to note that the lower accuracy of the estimates of $h_2^{1.37}(0,1)$ with respect to those of $h_2^0(0,1)$ can be justified taking into account the considerable difference in strength between the two kernels.

5. Conclusion

The class of real polyperiodic nonlinear systems that admit a new double series representation, called the Fourier–Volterra series, is considered, and a class of cyclostationary complex-valued random inputs that enables the analytical specification of sets of operators on the input that are orthonormal over all time to the Fourier–Volterra operators is defined. The reciprocal operators are used to obtain a crosscorrelation formula for identifying the Fourier–Volterra kernels of PPN systems of possibly infinite order and memory. A class of cyclostationary real-valued time-series inputs for which the same sets of operators apply is also introduced. However, in this case the orthogonality for different orders holds only for Fourier–Volterra operators of order less than that of the specified operator. Then, this real inputs can be used to identify Fourier–Volterra kernels only for finite order systems.

The computational load of the cross-correlation method can be substantial not only because of averaging time but also because of the number of terms required in the double series representation. To significantly reduce such a computational load the frequency-domain counterpart of the cross-correlation method is introduced, which is based on frequency-smoothed cyclic cross-periodograms that can be evaluated by FFT algorithms.

Appendix A. Proofs of Theorems 1 and 2

Proof of Theorem 1. Substituting (14a) into (3), (3) into (9), and the result together with (18) into the left-hand side of (19) yields

$$(19) = \langle \lambda_m^\alpha(k, \boldsymbol{j}_m, z(\cdot)) \mathrm{e}^{\mathrm{i}\omega(mk - \mathbf{1}_m \boldsymbol{j}_m')} \times \mathrm{e}^{\mathrm{i}\omega(\mathbf{1}_n \boldsymbol{k}_n' - nk)} \varphi_n(k, \boldsymbol{k}_n, z(\cdot))^* \mathrm{e}^{-\mathrm{i}2\pi\beta k} \rangle$$
$$= \langle \lambda_m(k, \boldsymbol{j}_m, z(\cdot))\, \varphi_n(k, \boldsymbol{k}_n, z(\cdot))^* \times \mathrm{e}^{\mathrm{i}\omega(m-n)k} \mathrm{e}^{\mathrm{i}2\pi(\alpha-\beta)k} \rangle \mathrm{e}^{\mathrm{i}\omega(\mathbf{1}_n \boldsymbol{k}_n' - \mathbf{1}_m \boldsymbol{j}_m')}. \quad (A.1)$$

Since $\lambda_m(k, \boldsymbol{j}_m, z(\cdot))$ and $\varphi_n(k, \boldsymbol{k}_n, z(\cdot))^*$ are k-invariant functions of $[z(k - j_1), \ldots, z(k - j_m)]$ and $[z(k - k_1), \ldots, z(k - k_n)]$, respectively, and $z(k)$ is stationary, then

$$\lambda_m(k, \boldsymbol{j}_m, z(\cdot))\, \varphi_n(k, \boldsymbol{k}_n, z(\cdot))^* \quad (A.2)$$

is stationary. Consequently, the average (A.1) is zero for

$$(n - m)\omega/2\pi + \alpha - \beta \neq \text{integer}, \quad (A.3)$$

which is satisfied by (15), provided that $m \neq n$. For $m = n$ and $\alpha = \beta$ we have

$$\langle \lambda_n(k, \boldsymbol{j}_n, z(\cdot))\, \varphi_n(k, \boldsymbol{k}_n, z(\cdot))^* \rangle \mathrm{e}^{\,\mathrm{i}\omega \mathbf{1}_n(\boldsymbol{k}_n - \boldsymbol{j}_n)'}, \tag{A.4}$$

and (17) guarantees that this average is zero for $\boldsymbol{j}_n \neq \boldsymbol{k}_n$ and is unity for $\boldsymbol{j}_n = \boldsymbol{k}_n$. Thus, we have verified (19). □

Proof of Theorem 2. Substituting (22) into (4), and (4) into (9), and the result together with (18) (modified to include the factor 2^n) into the left-hand side of (23) yields the same result as (A.1) except that the factor

$$\lambda_m^\alpha(k, \boldsymbol{j}_m, z(\cdot))\, \mathrm{e}^{\,\mathrm{i}\omega(mk - \mathbf{1}_m \boldsymbol{j}_m')} \tag{A.5}$$

is replaced with

$$\sum_{\substack{p+q=m \\ 0 \leqslant p,q \leqslant m}} \lambda_p^\alpha(k, \boldsymbol{j}_p, z(\cdot))\, \lambda_q(k, \boldsymbol{j}_q, z(\cdot))^* \times \mathrm{e}^{\mathrm{i}\omega([p-q]k + \mathbf{1}_q \boldsymbol{j}_q' - \mathbf{1}_p \boldsymbol{j}_p')}, \tag{A.6}$$

where $\lambda_0(k, \boldsymbol{j}_0, (\cdot)) \triangleq 1$. Using the same reasoning as in the proof of Theorem 1, we obtain the desired result (the right-hand side of (23)) provided that $p - q - n = 0$ only if $p = n$ and $q = 0$. The only way to guarantee this is to impose the restriction $m \leqslant n$. □

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