An Overview of Recent Developments in Cyclostationary Signal Processing

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Abstract

This paper is divided into two parts. In the first part, the results of a search of the literature for recent papers dealing with the theory and application of cyclostationarity are presented and discussed. In the second part, the recently developed theory of higher-order cyclostationarity is briefly reviewed and an application of this theory is described in detail. The application is that of detecting and determining the modulation type of each of an unknown number of spectrally and temporally overlapping random cyclostationary signals present in a given set of data.

I Introduction

The importance of using the property of cyclostationarity, which is exhibited by virtually all communication signals, in the design and analysis of communications and signal-processing systems has recently been explicitly recognized by both academicians and practicing engineers. This recognition is reflected in an increased number of published journal papers, the addition of an *IEEE Transactions on Signal Processing* EDICS number, the appearance of cyclostationary signal processing as a suggested topic in the calls for papers of prominent professional conferences (e.g., ICASSP), and an increasing amount of associated commercial and military product development. As a result of this recognition of the topic, the first *Workshop on Cyclostationary Signals*, organized and chaired by Professor William A. Gardner of the University of California at Davis, was held in August of 1992. This first workshop was deemed a success by its participants and resulted in the first edited volume of theoretical and applied research articles devoted exclusively to cyclostationarity [1].

The second *Workshop on Cyclostationary Signals* was held in August of 1994. This second workshop was organized by Professor Stephan V. Schell of The Pennsylvania State University with some help from the author. The goals of the second workshop were similar to those of the first: (i) to provide a forum for researchers to discuss their work with other qualified workers, (ii) to promote a sense of community among researchers, workers, and sponsors, and (iii) to provide a means for rapid dissemination of recent results and developments in the field. The events of the two-day workshop appear to have met the first two goals, and this proceedings is designed to meet the third goal. The contributions of this paper toward meeting the third goal are described next.

In Section II, recent additions to the literature on cyclostationary signals are reviewed and the major topics and trends are identified. In Section III, the theory of higher-order cyclostationarity is briefly reviewed, and in Sections IV–VI, this theory is applied to a specific problem in the area of signal detection and modulation recognition. Finally, in Section VII, conclusions are drawn and the paper is summarized.

II Recent Literature

The results of a search of the open literature for papers dealing with cyclostationary signals and cyclostationarity-exploiting signal-processing algorithms are presented in this section. The time period searched over is January 1992 to June 1994, although not all journals can keep up with their stated publication schedules. Thus, only sources that were actually available by July 1994 were included in the search. The following major international journals were searched for articles on cyclostationarity: EURASIP's *Signal Processing, IEEE Transactions on Communications, Signal Processing*, and *Information Theory, IEEE Signal Processing Letters*, and the *IEEE Signal Processing Magazine.* In addition, several major international conference proceedings were searched: the *Proceedings of the International Conference on Acoustics, Speech, and Signal Process-*

ing, 1993, the *Proceedings of the 27th Asilomar Conference on Signals, Systems, and Computers*, 1993, and the *Proceedings of the 28th Annual Conference on Information Sciences & Systems*, 1994. Papers from other conference proceedings were included as well, but the searches of these proceedings were not as thorough. The University of California library database `Melvyl` was also used to confirm that the journal search was sufficiently thorough.

The complete set of citations from the search make up the reference list for this paper. The result of sorting the papers by topic is summarized in Table 1. As can be seen from the table, there are eight major topics on which papers have been recently published. Two of these topics, higher-order cyclostationarity and blind channel identification and equalization, have received a large amount of attention compared to previous years. The proceedings that you are reading contains contributions from several researchers to the study of the latter topic, but there were no submissions for regular papers on the former topic. For this reason, the second part of this paper presents some recent work on the application of higher-order cyclostationarity to the problem of detection, sorting, and modulation recognition of multiple spectrally and temporally overlapping signals, which will be referred to as cochannel signals henceforth. Before describing this research, descriptions of the eight major research topics in Table 1 and some highlights of the associated published work are presented.

Paper Topic	**Journal Papers**	**Conference Papers**	**Total**
Channel Equalization	5	8	**13**
Time-Delay Estimation	5	0	**5**
Direction Finding	5	2	**7**
Spatial Filtering	2	5	**7**
Detection/ Interception	4	0	**4**
Signal Separation	1	3	**4**
Hardware/ Measurement	4	2	**6**
HOCS	7	8	**15**
Totals	33	28	**61**

Table 1: Topics and numbers of papers cited in the reference list of this paper. HOCS stands for higher-order cyclostationarity.

The first topic is blind-channel identification and equalization (BCIE) [1]–[14]. In simplest terms, the problem here is to undo the effects of a (usually linear time-invariant) channel on a transmitted digital communication signal without requiring a training sequence. The major deleterious effect of a linear time-invariant channel on such signals is to produce intersymbol interference. Thus, the main job of the BCIE algorithm is to remove intersymbol interference. This job can be done, conceptually speaking, by estimating the channel impulse response and applying its inverse to the received data. However, in practice this straightforward approach is rarely taken because of the substantial noise amplification that results from the amplification of weak spectral components of the signal. Thus, a trade-off must be made between the amount of noise amplification and the amount of signal equalization that is done by the equalizer. The identification of the magnitude response of the channel can be accomplished by using the second-order statistics of a stationary model of the output of the channel together with some knowledge of the second-order statistics of a stationary model of the input, but since the channels in question are usually nonminimum phase, the phase response of the channel cannot be estimated using these models. In the past, it was thought that this difficulty could only be surmounted by the use of higher-order statistics, which—even for stationary models of the input and output signals—can identify the magnitude and phase responses of a nonminimum-phase channel. Recently, however, following a suggestion of W. A. Gardner's, it has been demonstrated that nonminimum-phase channels can be identified using second-order statistics if the inherent cyclostationarity of the output signal is not ignored. For digital communication signals, this typically requires sampling the baseband signal at a rate larger than the symbol rate. Various algorithms for accomplishing the BCIE task have been introduced, and further work and algorithm development is reported in the cited references.

The second topic is time-delay estimation, which is sometimes called time-difference-of-arrival (TDOA) estimation [15]–[19]. The problem here is to estimate the relative delay between a signal's wavefronts that arrive at two physically separated sensors. TDOA estimates can be used, for example, to determine the location of the emitter of the signal. Traditional approaches to TDOA estimation have focussed on the cross-correlation function between the outputs of the two sensors. In the absence of interfering signals and noise, the magnitude of this cross-correlation is maximum at the TDOA. However, in the presence of interference, there are multiple peaks in the cross-correlation function. In addition, these peaks can have oscillatory envelopes and, therefore, can interfere with each other in a complicated manner if they are not spaced sufficiently far from each other. To mitigate the effects of noise and interference, a class of TDOA estimators, in which the sensor outputs are filtered before the cross-correlation is estimated (the generalized cross-correlation algorithms), was developed. Such filtering can result in greatly enhanced performance if the interferers do not substantially spectrally overlap the signal of interest. A class of estimators (originally introduced by W. A. Gardner) which exploit cyclostationarity circumvent the interference problem because they are signal selective. For example, by using the cyclic cross-correlation instead of the cross correlation, only the desired signals (those exhibiting the chosen cycle frequency) contribute to the measurement (asymptotically). Another interesting fact is that TDOAs can be estimated by using only cyclic autocorrelations (and no cross correlations—cyclic or otherwise), although there is an inherent ambiguity in the resulting TDOA estimate [17].

The third topic is array processing for direction-finding, which is sometimes called direction-of-arrival estimation or bearing estimation [20]–[26]. Given an array of sensors upon which a number of signals is impinging, the problem is to determine the directions from which the signals arrive. A slightly more general version of this problem that is of considerable interest in practice is to determine the direction of arrival for only a specific subset of the received signals. This subset can be those signals, for example, that exhibit cyclostationarity at a specific cycle frequency. As originally proposed by W. A. Gardner, the directions of arrival of the signals in the subset can be estimated without substantial degradation due to the presence of the other signals because of the signal selectivity of the statistics of cyclostationary signals. This approach allows the estimation of the directions of arrival for more signals than sensors by applying the algorithm sequentially for desired values of the cycle-frequency parameter. New algorithms are studied in [22, 23, 24, 26], and important performance analyses are provided in [20, 21].

The fourth topic is array processing for waveform estimation, which is sometimes referred to as beamforming [27]–[30]. The problem here is to determine the weights to apply to the output of each of an array of sensors such that the sum of weighted outputs contains only the signal of interest and no contributions from the interferers. There is considerable interest in determining these weights by processing only the received data, that is, in determining the weights with little or no prior information about the signal environment, or even about the physical characteristics of the sensors. As originally proposed by W. A. Gardner, by recognizing that the signal of interest has a unique cycle frequency with respect to the interferers, the array weights can be determined by—in a suitably general sense—maximizing the amount of cyclostationarity at that cycle frequency that is exhibited by the sum of weighted outputs. There are various techniques for accomplishing this maximization, some of which are studied in the cited references, and among them is an interesting new technique called *programmable canonical correlation analysis* (PCCA) [32]. While this technique can be used to blindly adapt array weights by exploiting cyclostationarity, it is considerably more general in the sense that it can exploit many other properties of the signal and interferers as well or instead. Some of the signal properties that PCCA can exploit in order to form a beam in the direction of the signal of interest and nulls in the directions of the interferers include cyclostationarity, bandwidth, temporal structure such as periodic gating, location of spectral support, and constancy of the modulus.

The fifth topic is weak-signal detection, which is called interception when the signal(s) to be detected is intended for a receiver other than the one to be used [18], [33]–[35]. The problem here is to detect the presence of a signal that is heavily corrupted by noise and, possibly, interference. One way to conceptualize the approaches taken in the cited references is in terms of sine-wave generation. Because the signal of interest is assumed to be cyclostationary (e.g, a direct-sequence spread-spectrum signal), additive sine-wave components can be generated by nonlinearly transforming the received data. If enough data is available, the presence of these generated sine waves can be detected, which results in detection of the signal itself. Since accurate estimates of the amplitudes and phases of the generated sine waves are not required here, such detection can be accomplished for relatively low signal-to-noise ratios (SNRs) with less data than that required for parameter-estimation applications.

The sixth topic is single-sensor signal separation [36]–[39]. The problem here is to process a single data record that contains multiple signals in order to extract from it a desired signal. This desired signal can be completely temporally and spectrally overlapped by interferers. This problem is difficult because linear time-invariant filtering cannot be used to separate such signals. Therefore, any method of separating the signals must be nonlinear or time-variant or both. A special kind of linear, time-variant filtering, called linear polyperiodic time-variant filtering, is especially appropriate for filtering cyclostationary signals. Because such filtering is equivalent to independently filtering a set of frequency-shifted versions of the input and then summing, it is also called frequency-shift (FRESH) filtering. According to W. A. Gardner's and W. A. Brown's original discovery, the frequency-shift operators enable the filter to exploit the spectral redundancy that is inherent in cyclostationary signals and to thereby separate cochannel signals. Linear polyperiodically time-variant filtering has been generalized to nonlinear polyperiodically time-variant filtering; that is, linear FRESH filtering has been generalized to nonlinear FRESH filtering. This latter kind of filtering can be useful when the signals in question have little or no second-order cyclostationarity to exploit. Both kinds of FRESH filtering are studied in the cited references.

The seventh topic is implementation methods for cyclostationarity-exploiting algorithms and measurement of cyclostationarity [40]–[45]. The first problem that falls into the seventh category is to determine efficient architectures for the realization of spectral correlation analyzers and other, more special-purpose, cyclostationarity-exploiting signal processors. A nice contribution is [41], which contains a thorough complexity analysis of digital implementations of spectral correlation analyzers. The second problem is to mathematically characterize the properties of standard estimators, such as estimators of the spectral correlation function and cyclic autocorrelation [44].

Finally, the eighth topic is higher-order cyclostationarity (HOCS), which is a broader category than the previous categories [46]–[58]. Higher-order cyclostationarity is the study of the higher-order statistics of cyclostationary signals and their applications. These statistics can be moments or cumulants or other kinds of statistics, but emphasis is on cumulants. The eighth topic has two distinct subtopics, theory and application, and has largely been studied by two research groups. Each group has a distinct mathematical framework in which it carries out its studies. The first group is headed by the author and Professor William A. Gardner of the University of California at Davis and Statistical Signal Processing, Incorporated, and the second is headed by Professor Georgios B. Giannakis of the University of Virginia at Charlottesville. The former group uses the fraction-of-time probabilistic framework (temporal averaging), whereas the latter group uses the stochastic probabilistic framework (ensemble averaging). In addition, the former group *derived* the cumulant as the solution to a key problem in the application of higher-order cyclostationarity [46], whereas the latter group adopted the cumulant because of the potential utility of some of its well-known properties.

There are several papers on cyclostationarity that do not fall into any of the eight categories described above [59]–[62]. The first two report on a study of linear time-invariant filter implementation, the third discusses autoregressive periodically time-varying time-series models, and the fourth discusses the identification of nonlinear systems (see also [49]).

III Cyclostationarity

The theory of higher-order cyclostationarity is the theory of the strengths of the amplitudes, frequencies, and phases of the finite-strength additive sine-wave components that exist in the outputs of certain nonlinear transformations of time-series that do not themselves contain such sine-wave components, and the measurement thereof. In fact, a signal (or time-series) is said to exhibit nth-order cyclostationarity if there exists an nth-order homogeneous polynomial transformation of the signal that contains at least one finite-strength additive sine-wave component with nonzero frequency. This existence requirement is equivalent to a requirement on the moment of the signal obtained by using fraction-of-time probability: the moment must contain at least one additive sine-wave component. Alternatively, a stochastic process is said to exhibit higher-order cyclostationarity if some higher-order moment of the process (obtained by using a stochastic expectation over a hypothetical ensemble of time-series) contains periodic or polyperiodic components. If the stochastic process is cycloergodic for the order of interest and the moments of this order are periodic or polyperiodic (i.e., the process is cyclostationary or polycyclostationary), then the moments and cumulants obtained by stochastic expectation are equal (with probability one) to those obtained using the fraction-of-time expectation. Since there are relatively few situations of interest for which an ensemble of data records is actually available, it is often more appropriate to work with fraction-of-time probability rather than stochastic probability. In any case, the words signal, moment, cumulant, and expectation are used henceforth without regard to whether they arise from the fraction-of-time probabilistic framework or from the stochastic-process framework. Keeping this in mind, let us turn to a description of the fundamental parameters of the theory of higher-order cyclostationarity.

The *nth-order temporal moment function* is the expected value of a product of n delayed and possibly conjugated versions of the signal. If the signal is polycyclostationary (abbreviated henceforth to cyclostationary), this moment is polyperiodic for some value of n. Some types of signals have time-invariant second-order moments, but polyperiodic moments for orders greater than two. These kinds of signals are second-order stationary, but are higher-order cyclostationary. Other types of signals have polyperiodic second-order moments and, therefore, they must also have polyperiodic moments for some or all orders greater than two, because the polyperiodic components of the lower-order delay products multiply each other in the higher-order delay product. For these signals, the question arises as to how much of the polyperiodic higher-order moment is due to such multiplications, and how much is not. That is, what is new in the higher-order moment? i.e., how much is purely nth-order polyperiodicity? The answer to this question turns out to be the *nth-order temporal cumulant function*, which is defined in terms of the logarithm of the characteristic function of the set of n delayed versions of the signal [46, 47]. Thus, cumulants, which are known for their tolerance to Gaussian contamination, their linearity with respect to statistically independent signals and variables, and their ability to convey information about the phase (absolute time reference) of their inputs, also exhibit the property we call purity.

By purity we mean that the nth-order temporal cumulant function, which is polyperiodically time-variant, is equivalent to the pure nth-order polyperiodic component of the nth-order moment [46], and which is a nonlinear function of temporal moment functions of order 1 through n. The purity property exists for cumulants regardless of whether the signal is stationary (of any order) or cyclostationary (of any order), or whether the fraction-of-time or stochastic probabilistic frameworks are used. It happens to be most obvious in the case of cyclostationary signals and the fraction-of-time framework, which is the most natural choice for studying sine-wave generation.

Since both the temporal moments and cumulants of a cyclostationary signal are polyperiodic functions of time, they can be represented as Fourier series. For the moment functions, the Fourier coefficients are called *cyclic temporal moment functions*, and the Fourier frequencies are called *impure cycle frequencies*, or just cycle frequencies. For the cumulant functions, the Fourier coefficients are called *cyclic temporal cumulant functions*, or just cyclic cumulants, and the Fourier frequencies are called *pure cycle frequencies*, or just cycle frequencies. What is important to understand about the cyclic cumulants is that if a pure nth-order cycle frequency is unique to a signal of interest in a data set that also contains signals of no interest, then the corresponding nth-order cyclic cumulant estimated from the data asymptotically approaches the cyclic cumulant of the signal of interest. The same is not true (in general) for cyclic moments: they do not exhibit the property of purity or, equivalently, they are not signal selective.

The frequency-domain parameters of higher-order cyclostationarity are defined as limiting versions (as the frequency resolution width approaches zero) of moments and cumulants of the complex envelopes of narrowband components of the signal. These spectral moments and cumulants can be expressed in terms of multidimensional Fourier transforms of the cyclic moments and cumulants, respectively [46, 47]. The spectral cumulant has support only on certain $(n-1)$-dimensional hyperplanes contained in its n-dimensional domain of definition. Each of these hyperplanes is associated with a distinct value of the cycle-frequency parameter. The values of the spectral cumulant on each hyperplane is characterized by the *nth-order cyclic polyspectrum* for the corresponding cycle frequency.

The relationships between the probabilistic parameters of cyclostationary signals are illustrated in Figure 1. In the case of $n = 2$, if there are no finite-strength additive sine-waves in the signal itself, then the temporal moments are equal to the temporal cumulants, which are characterized by the cyclic autocorrelation functions, and the spectral moments are equal to the spectral cumulants, which are characterized by the spectral correlation functions. If there are sine-wave components in the signal, then the temporal cumulant differs from the temporal moment in the same way that the mean-square value of a random variable differs from its variance. In general, the inner diamond of Figure 1 represents the relationships between the Fourier coefficients of the temporal moments and cumulants and their transforms, whereas the outer diamond represents the relationships between the temporal and spectral moments and cumulants themselves. In the case of a strict-sense stationary signal, the inner and outer diamonds represent the same quantities; thus only one diamond is necessary to represent the parameters.

In subsequent sections, an application of the theory is discussed. This application is based on the signal-selectivity property of cyclic cumulants. To

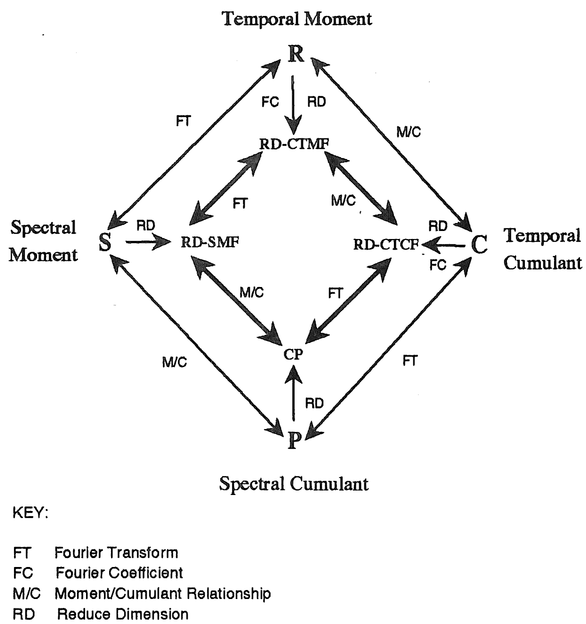

Figure 1: The parameters of higher-order cyclostationarity. The quantities at the vertices of the outer diamond are defined in the text. The acronyms at the vertices of the inner diamond represent (clockwise from top): reduced-dimension cyclic temporal moment function, reduced-dimension cyclic temporal cumulant function, cyclic polyspectrum, reduced-dimension spectral moment function.

reiterate, this property states that if the data is the sum of a number of statistically independent signals that possess unique cycle frequencies, then the cyclic cumulants for each signal can be obtained (asymptotically) from the cyclic cumulants of the data.

IV General Signal Search

The problem considered here is simply stated: Determine the number of cyclostationary signals present, if any, in a given data record, and determine their modulation types. The signals can be completely temporally and spectrally overlapping and can have low SNR.

If the signals in the data are spectrally disjoint, or only partially spectrally overlapping, then energy detection followed by linear time-invariant filtering can be used to achieve some degree of separation. Thus, this case is of less interest than the case of cochannel signals because the theory of higher-order cyclostationarity is not needed. Nevertheless, the method to be described can be used to handle the simpler problem of detecting and classifying non-cochannel signals as well as the more difficult problem of cochannel signals.

V Algorithms

Because of limitations on available space, the mathematical descriptions of the algorithms are not given here. Instead, they are described qualitatively. Some important details are left out, but enough are given to allow the reader to understand the approach, and to evaluate the potential of the methods for application to actual problems in the field.

The basic idea behind the proposed solution to the general signal search problem is that of cyclic cumulant estimation. The proposed solution can be divided into the following steps:

1. Estimate the pure cycle frequencies and cyclic cumulants of the given data for orders 1 through N,
2. Group these estimates according to harmonic relations among the cycle frequency estimates,
3. Define features consisting of each group's cycle frequencies and cyclic cumulants,
4. Compare the measured features to stored features for the set of signals of interest to perform detection and classification.

The first of these steps is accomplished by the *general search algorithm*, the second and third by the *grouping algorithm*, and the fourth by the *classification algorithm*. These algorithms are described next.

V.A The General Search Algorithm

The *general search algorithm* (GSA) can be thought of as a blind cyclic cumulant estimator. It is blind in the sense that estimation of an nth-order cyclic cumulant requires knowledge of all lower-order cycle frequencies, and the GSA estimates these required lower-order cycle frequencies from the data. It starts with a processing order of one, and progresses up to the desired order N. Its input parameters include the sets of delays to use in forming the delay products, the sets of optional conjugations to apply to each of the delay products, the maximum order of processing N to use, and the number A of cycle frequency estimates to output for each order, set of conjugations, and delay set. The latter parameter defines an effective threshold for cycle frequency estimation.

The GSA is based on the algebraic relationship between the nth-order moment and the cumulants of orders 1 to n. This relationship, first derived by Shiryaev and Leonov (see Chapter 2 in [1]), expresses the nth-order moment as the nth-order cumulant plus a nonlinear function of the cumulants of orders 1 to $n-1$. This relationship is used to devise a recursive cumulant estimator, which is the GSA.

The output of the GSA consists of a sequence of lists of ordered pairs $(\hat{C}, \hat{\alpha})$, where $\hat{C}$ is the magnitude of the cyclic cumulant estimate and $\hat{\alpha}$ is the corresponding cycle frequency estimate. This sequence of lists is indexed by three things: the order of processing n, the number of conjugations m, and the delay vector $\boldsymbol{\tau} = [\tau_1 \cdots \tau_n]$. Ideally, this sequence of lists contains only cycle frequencies of signals that are present in the data. Because the estimation of these cycle frequencies and their strengths is blind, and because of the presence of noise and interfering signals, the estimates are not exact. Thus, the sequence of lists contains some entries with $\hat{\alpha}$ that are not close to actual cycle frequencies of signals in the data. The A parameter effectively controls the number of true and false cycle frequency estimates.

V.B The Grouping Algorithm

The output of the GSA, as described in the previous section, consists of a sequence of lists of cycle frequency estimates for all the signals in the data, plus some false cycle frequencies. However, the association of each cycle frequency estimate to a particular signal remains to be done, and this is the job of the *grouping algorithm* (GA). In order to associate elements of the sequence of lists with each other and, eventually, with a signal in the data, some general relationships between cycle frequencies obtained by processing a given signal using various orders and numbers of conjugations must be known. At the time of this writing, the grouping algorithm has been completed only for the class of signals whose complex envelopes can be represented as complex-valued pulse-amplitude-modulated signals. This class of signals shall be referred to as digital quadrature-amplitude modulation, or digital QAM. Digital QAM signals include all amplitude-shift-keyed (ASK) and phase-shift-keyed (PSK) signals, some partial-response signals like duobinary and modified duobinary signals, and those digital signals commonly referred to as QAM signals. The latter signals are those amplitude- and phase-shift-keyed signals for which the symbols in the symbol constellation do not all lie on a single circle (PSK) or on a single radial line (ASK).

By deriving and examining the cyclic cumulants of this relatively broad class of communication signals ([46, 47]), relationships between the cycle frequencies and the two modulation parameters, carrier frequency f_c and symbol rate T_0^{-1}, can be determined for arbitrary n and m. These relationships are given

		Bits per Symbol			
n	m	2	4	8	>
2	0,2	$\frac{k}{T_0} \pm 2f_c$	$\emptyset$	$\emptyset$	$\emptyset$
2	1	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$
4	0,4	$\frac{k}{T_0} \pm 4f_c$	$\frac{k}{T_0} \pm 4f_c$	$\emptyset$	$\emptyset$
4	2	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$
6	0,6	$\frac{k}{T_0} \pm 6f_c$	$\emptyset$	$\emptyset$	$\emptyset$
6	1,5	$\frac{k}{T_0} \pm 4f_c$	$\frac{k}{T_0} \pm 4f_c$	$\emptyset$	$\emptyset$
6	3	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$
8	0,8	$\frac{k}{T_0} \pm 8f_c$	$\frac{k}{T_0} \pm 8f_c$	$\frac{k}{T_0} \pm 8f_c$	$\emptyset$
8	2,6	$\frac{k}{T_0} \pm 4f_c$	$\frac{k}{T_0} \pm 4f_c$	$\emptyset$	$\emptyset$
8	4	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$
10	0,10	$\frac{k}{T_0} \pm 10f_c$	$\emptyset$	$\emptyset$	$\emptyset$
10	1,9	$\frac{k}{T_0} \pm 8f_c$	$\frac{k}{T_0} \pm 8f_c$	$\frac{k}{T_0} \pm 8f_c$	$\emptyset$
10	3,7	$\frac{k}{T_0} \pm 4f_c$	$\frac{k}{T_0} \pm 4f_c$	$\emptyset$	$\emptyset$
10	5	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$	$\frac{k}{T_0}$

Table 2: Potential cycle frequencies for the analytic signals corresponding to some PSK signals. There are no cycle frequencies for the values of m that are not shown in the table. The + sign is associated with the first value of m, and the > sign means a power of two greater than eight.

by

$$\begin{aligned} n = 2m \quad &\Rightarrow \quad \alpha = \pm k/T_0 \\ n \neq 2m \quad &\Rightarrow \quad \alpha = (n-2m)f_c \pm k/T_0, \end{aligned} \tag{1}$$

and are used to associate elements in the sequence of lists with each other. The cycle frequencies for $n = 2m$ are called *lower* cycle frequencies, whereas those for $n \neq 2m$ are called *upper* cycle frequencies. The following is a qualitative description of the grouping algorithm:

1. Input the GSA output sequence (indexed by n, m, and $\boldsymbol{\tau}$) of lists: $(\hat{C}, \hat{\alpha})_{n,m,\boldsymbol{\tau}}$.

2. Identify and cluster the $n = 2m$ subsequence of lists.

3. Group resulting clusters according to fundamental frequency.

4. Identify and cluster the $n \neq 2m$ subsequence of lists.

5. Associate upper clusters with lower clusters to form distinct groups.
6. For each group, form a feature by outputting the peak value of $\hat{C}$ for each cluster in the group.

The term *cluster* in the above description means to use an unsupervised learning algorithm to partition the list elements into subsets. This clustering results in a set of clusters, or subsets, each of which can contain multiple elements (multiple ordered pairs $(\hat{C}, \hat{\alpha})$), but for which the $\hat{\alpha}$ values are, in some sense, close to each other. The important parameters of a cluster are its mean (average over the $\hat{\alpha}$) and its peak (maximum over the $\hat{C}$).

The output of the GA is a set of numbers, which are the peak values of the cyclic-cumulant estimates, corresponding to each of the potential cycle frequencies listed in (1). For example, using Table 2 and knowledge of the cyclic cumulant functions for BPSK signals [46, 47], it can be shown that the cyclic cumulants for all the upper and lower cycle frequencies are nonzero for some τ for this signal type. Furthermore, the value of τ for which each nth-order cyclic cumulant is maximum can be found analytically. An ideal feature for detection and classification can be defined as the set of the maxima of all the upper and lower cyclic cumulants for cycle frequencies associated with a given signal. These maxima can be arranged in a matrix for which the row corresponds to the harmonic number (k in (1)) and the columns correspond to the distinct ordered pairs (n, m) (see Figure 2). The value of the matrix element is the maximum value (over all values of τ) of the cyclic cumulant with cycle frequency $(n - 2m)f_c \pm k/T_0$. The ideal feature for BPSK is shown in Figure 3 for

$\vdots$	$\vdots$	$\vdots$	$\vdots \ \dots$
$k = 2$	$2f_c \pm 2/T_0$	$0f_c \pm 2/T_0$	$\dots$
$k = 1$	$2f_c \pm 1/T_0$	$0f_c \pm 1/T_0$	$\dots$
$k = 0$	$2f_c$	$0f_c$	$\dots$
	$(n, m) = (2, 0)$	$(n, m) = (2, 1)$	$\dots$

Figure 2: Definition of a feature matrix for detection and classification. An element of the feature matrix corresponds to the maximum of the magnitude of the cyclic cumulant for cycle frequency $\alpha = (n-2m)f_c \pm k/T_0$, where k is uniquely specified by the row and the values of n and m are uniquely specified by the column.

orders $n = 2p$ for $p = 1, 2, 3$, and 4. For each order n, each of the $n+1$ possible values of m are included. Thus, the matrix has

$$(2 + 1) + (4 + 1) + (6 + 1) + (8 + 1) = 24$$

columns. The maximum value of k is set equal to 5 for this ideal feature. Note that this feature representation suppresses the values of the symbol rate and carrier frequency: every rectangular-pulse BPSK signal has the same feature regardless the particular values of T_0 and f_c. The ideal feature for QPSK is shown in Figure 4 (cf. column four in Table 2). Notice that there are four dark cells in the ideal feature matrix for BPSK, and that these same cells are dark in the ideal feature matrix for QPSK. These cells are set equal to zero purposely because they correspond to cycle frequencies of zero. Cyclic cumulants for cycle frequencies of zero always contain contributions from all signals present in the data. These cyclic cumulants can be signal selective only when there is a single signal in the data, which is not the problem of interest here. Thus, zero cycle frequencies are never included in a feature.

The ideal feature for any digital QAM signal for which the symbols in the baseband PAM representation are independent and identically distributed can be computed analytically from existing formulas. The grouping algorithm outputs estimates of these ideal features.

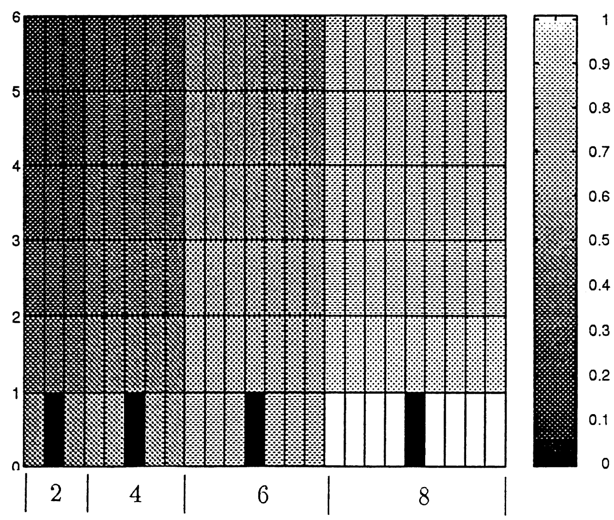

Figure 3: The ideal feature matrix for a BPSK signal with rectangular keying envelope.

V.C The Classification Algorithm

Once the GSA and GA have done their work, each of the resulting features can be classified by comparing them to stored features that were computed (or measured) for a set of signals of interest. Development of the classification algorithm (CA) is ongoing, but the general idea is to compute the distance (using an appropriate metric) between the measured feature and

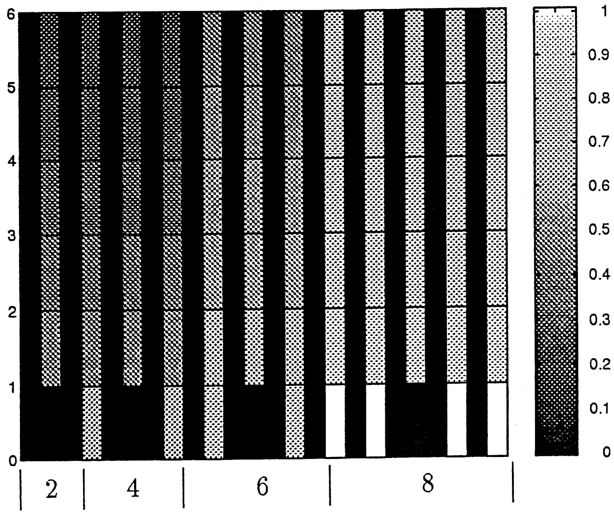

Figure 4: The ideal feature matrix for a QPSK signal with rectangular keying envelope.

the stored features, and then to declare the modulation type of the signal to be that of the stored feature that is closest to the measured feature. A significant difficulty to overcome is that the ideal (stored) features depend on the signal power, which is unknown. Nevertheless, a normalization procedure that allows proper computation of the distance between the measured and ideal features has been developed. Experiments are underway to measure the performance of the classification algorithm.

VI Simulation Results

In this section, some typical results from the GSA/GA combination are presented. The first set of measurements are for PSK signals with rectangular keying envelopes. Using the parameters in Table 3, feature matrices were measured for simulated BPSK, QPSK, 8PSK, and offset QPSK (OQPSK) signals. The results are shown in Figures 5–8. By comparing with the entries in Table 2 and the ideal features for BPSK and QPSK shown in Figures 3 and 4, it is evident that the first three of these measured features are correct. Also, it is interesting to note that the ideal feature for OQPSK cannot (at present) be determined theoretically because its baseband PAM representation does not have independent symbols. Nevertheless, the portion of the ideal feature for $n = 2$ can be computed, and this matches with measurement. Thus, the GSA/GA combination can be used to quantify a signal's higher-order cyclostationarity in cases for which no mathematical results exist. Note that the four features are all distinct, indicating that these measured features do have potential

Collect	16,384 samples
SNR	∞
Delays	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 4 0 0 0 0 0 0 0 7 0 0 0 0 0 0 0 9 0 0 0 0 0 0 0 12
Orders	2–8, even
Conjugations	all
T_0	23 samples
f_c	0.004
A	15

Table 3: Parameters for the first set of measured feature matrices.

for classification. Note also that whereas the first three columns of the QPSK and 8PSK features are identical, the remainder of these two features are distinct. This observation provides one motivation for using higher-order cyclostationarity for the purpose of signal classification: the need to classification of signals with similar or identical second-order cyclostationarity. To understand another motivation, let us turn to the next set of simulations.

The focus of the next set of results is on bandwidth-efficient signals such as duobinary signals. Feature matrices were measured using the parameters listed in Table 4 for simulated duobinary (0% excess bandwidth [EBW]), 25% EBW BPSK, 50% EBW BPSK, and 100% EBW BPSK signals. The results are shown in Figures 9–12. As is predicted by theory, the number of harmonics of the symbol rate that correspond to cycle frequencies for

Collect	16,384 samples
SNR	∞
Delays	0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 4 0 0 0 0 0 0 0 8 0 0 0 0 0 0 0 16
Orders	2–8, even
Conjugations	all
T_0	16 samples
f_c	0.0
A	12

Table 4: Parameters for the second set of measured feature matrices.

which the cyclic cumulant is not identically zero increases with increasing order n. Thus, even signals with no second-order cyclostationarity (such as duobinary signals) can yield feature matrices with substantial content provided that the maximum order of processing is large enough. Note also that the feature matrices could be used to estimate the excess bandwidth of the signal by comparing to ideal feature matrices for signals with various EBWs. That is, by including sufficiently many ideal feature matrices in the catalog of signals of interest, the signal can be classified not only as a BPSK signal, but as a BPSK signal with a specific amount of excess bandwidth.

A point that is worth repeating is that the power of the methods presented here is that the features can be measured for each signal even in the case in which the signal is completely spectrally and temporally overlapped by one or more interfering signals. This is illustrated with a final simulation example.

For the final example, two independent BPSK signals with equal power are added together. One of these signals has symbol interval length of 23 samples and a carrier offset of 0.004, whereas the other has symbol interval length of 15 samples and a carrier offset of -0.005. Thus, these two signals are almost completely spectrally overlapping. The feature matrices for each of these signals were measured using the simulation parameters in Table 3. The results are shown in Figures 13 and 14, where the symbol-interval-length and carrier-frequency estimates for the former feature are 23 and 0.004, respectively, and those for the latter are 15 and -0.005. A substantial number of harmonics are missing from both features, which is understandable because the total number of cycle frequency estimates output by the GSA was not increased relative to the first set of simulations (A is the same for both simulations).

VII Conclusions

In this paper, the literature on cyclostationary signal processing that was published during the past two years is briefly reviewed. The literature review reveals that substantial work has been published on eight topics, and the nature of the work on these eight topics is briefly described. The reference list contains the sixty-two citations found in the literature search. To fill a perceived gap in this proceedings, the remainder of the paper describes an application of the theory of higher-order cyclostationarity. First, the fundamental parameters of the theory of higher-order cyclostationarity are described. Following this, the theory is applied to the problem of enumerating and classifying the modulation type of each of an unknown number of random signals that are present in a given data set.

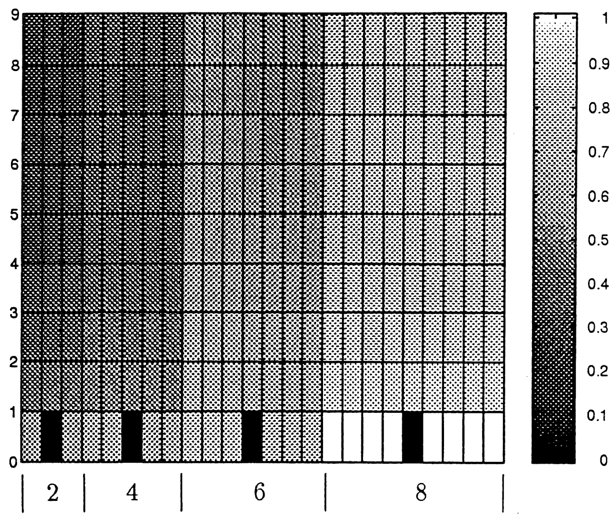

Figure 5: Measured feature matrix for a BPSK signal with rectangular keying envelope.

VIII Acknowledgements

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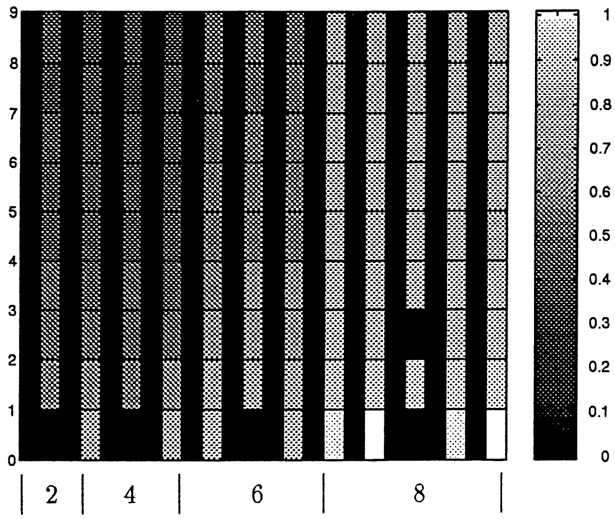

Figure 6: Measured feature matrix for a QPSK signal with rectangular keying envelope.

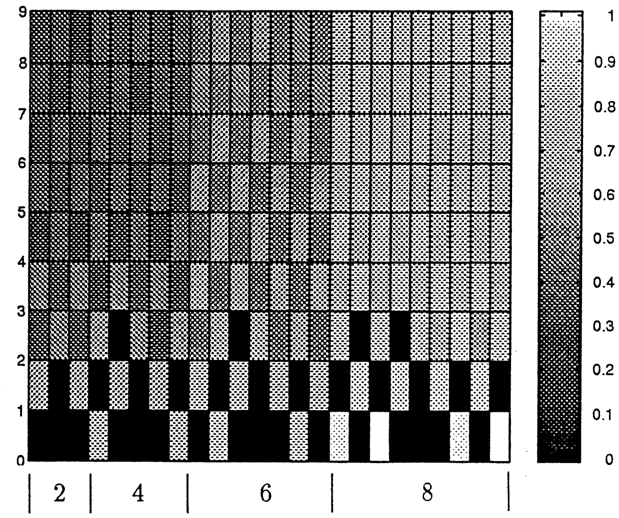

Figure 8: Measured feature matrix for an OQPSK signal with rectangular keying envelope.

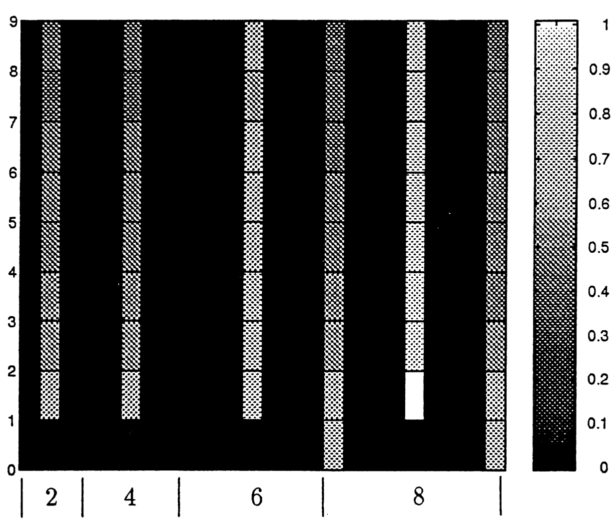

Figure 7: Measured feature matrix for an 8PSK signal with rectangular keying envelope.

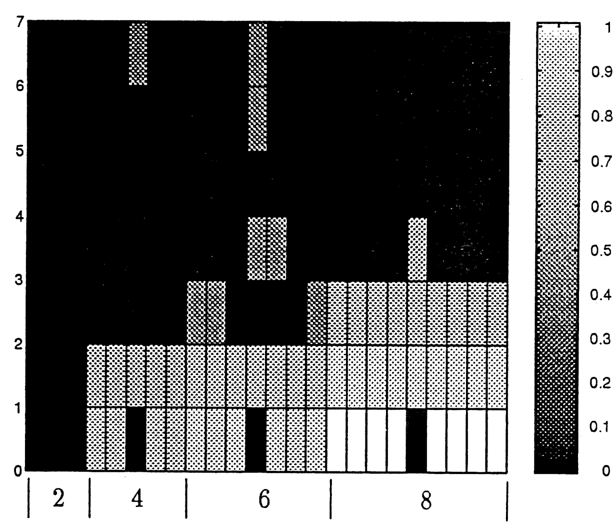

Figure 9: Measured feature matrix for a duobinary signal. The gray cells with large harmonic numbers are false cycle-frequency detections.

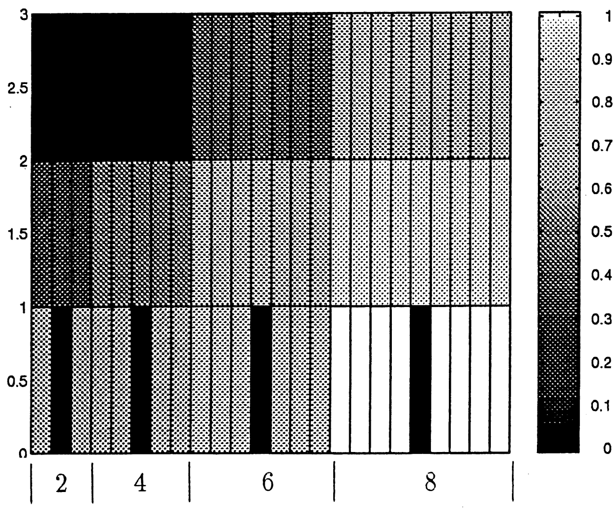

Figure 10: Measured feature matrix for a 25% excess-bandwidth BPSK signal.

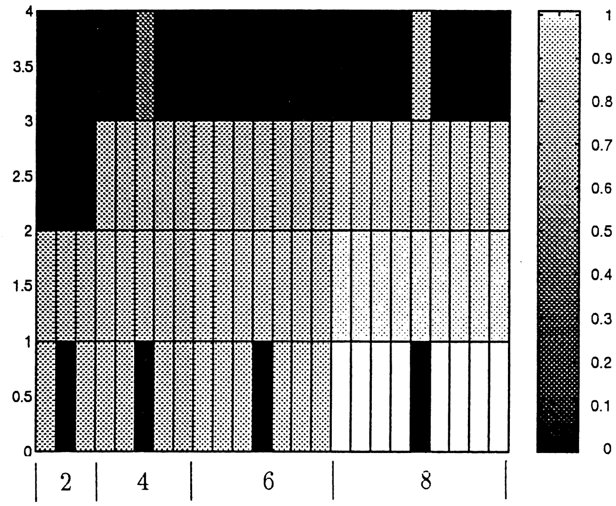

Figure 12: Measured feature matrix for a 100% excess-bandwidth BPSK signal.

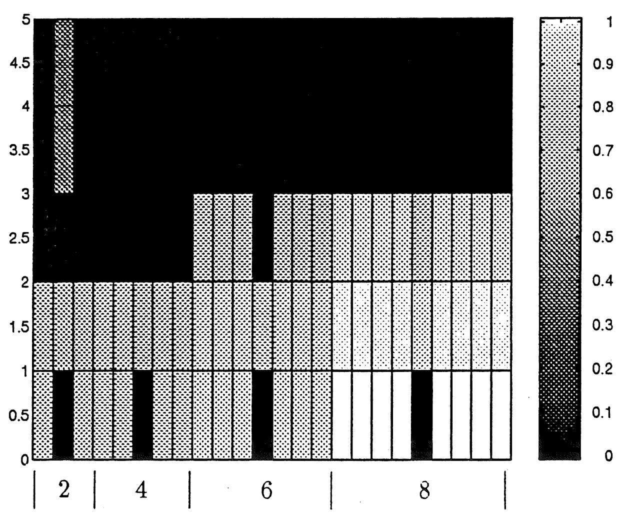

Figure 11: Measured feature matrix for a 50% excess-bandwidth BPSK signal. The gray cells with large harmonic numbers (for $n = 2$) are false cycle-frequency detections.

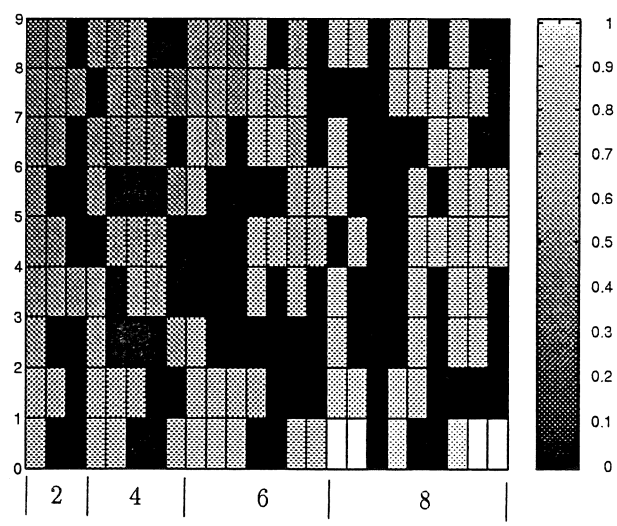

Figure 13: Measured feature matrix for the first of the groups output by the GA. The carrier offset and symbol rate were correctly estimated as 0.004 and 23. The input to the GSA consisted of two equipower BPSK signals.

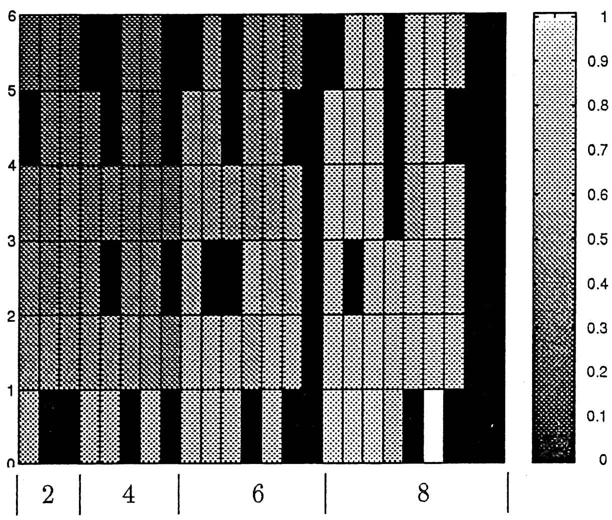

Figure 14: Measured feature matrix for the second of the groups output by the GA. The input to the GSA consisted of two equipower BPSK signals. The carrier offset and symbol rate were correctly estimated as -0.005 and 15.

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