An Algorithm for Further Improvements in Signal-Selective Time-Difference Estimation

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Abstract

A generalization of the SPECCOA algorithm for time-difference-of-arrival (TDOA) estimation for cyclostationary signals is studied. Similar to SPECCOA, this algorithm possesses the desirable signal-selectivity property that eliminates the ambiguity problem faced by conventional generalized cross correlation (GCC) algorithms in the presence of cochannel interfererence. Moreover, this generalized algorithm shows promise for outperforming the high-performance SPECCOA algorithm. The results of computer simulations are presented to illustrate the superior performance of this new algorithm relative to that of the GCC and SPECCOA algorithms. In addition, the performance of the cyclic cross-correlation (CCC) method, which is equivalent to the cross ambiguity function (CAF) method—except that the differential-Doppler-shift frequency used in the CAF is replaced with a differential-Doppler-shifted cycle frequency of the signal of interest—is also evaluated and compared.

I Introduction

Time-difference-of-arrival (TDOA) estimation is the problem of estimating the time difference between the same signal impinging upon two spatially separated receivers. A set of such measurements taken at different locations can be used to passively establish the position of the transmitter of the signal. The conventional approach to TDOA estimation consists of measuring a generalized cross correlation (GCC) [1] between the two data sets, and searching for a peak in this measurement. However, limitations to this method exist when noise and interference that spectrally overlap the signal of interest (SOI) are present. To address these limitations, methods of TDOA estimation based on a cyclostationary model of the SOI have been developed and have been shown to produce superior performance relative to the conventional GCC method. One of the most successful TDOA algorithms based on the cyclostationary model is referred to as SPECtral COherence Alignment (SPECCOA) [2]. By exploiting the cyclostationary nature of the SOI, this algorithm possesses a signal-selectivity property that provides it with substantial tolerance to noise and interference. In this paper, we describe a generalization of the SPECCOA algorithm that also enjoys the beneficial effects of exploiting the cyclostationarity of the SOI. For the purposes of comparison, another cyclostationarity-exploiting algorithm, called the cyclic cross-correlation (CCC) algorithm, is also included in the simulation study.

We begin in Section II by providing background material on the various algorithms. This leads to a derivation of the new algorithm in Section III. Computer simulation results comparing this new algorithm, SPECCOA, CCC, and GCC are provided in Section IV, and concluding remarks are made in Section V.

II Background

A mathematical model for the received signal in the TDOA estimation problem is specified by the following relations between complex envelopes

$$y_m(t) = A_m s(t - t_m) + n_m(t), \qquad m = 1, 2 \quad (1)$$

where $y_m(t)$ is the signal arriving at receiver m, $s(t)$ is the transmitted signal, $n_m(t)$ is noise and interference at receiver m, and A_m is a complex-valued amplitude parameter that represents gain and phase shifts caused by the propagation medium, the propagation distance, or the antenna and receiver. The TDOA parameter to be estimated is $D \triangleq t_2 - t_1$. It is assumed that $s(t)$ is uncorrelated with $n_1(t)$ and $n_2(t)$, that the noise components in $n_1(t)$ and $n_2(t)$ are uncorrelated, and that the interference components are correlated.

Consider the case with no noise or interference. In this idealized case, TDOA estimation can be performed by a simple cross-correlation between the two received data sets,

$$R_{21}(\tau) = E\{y_2(t) y_1^*(t - \tau)\} = A_1^* A_2 R_s(\tau - D) \quad (2)$$

where $E\{\cdot\}$ denotes the expectation operation. Since the autocorrelation function $R_s(\tau)$ peaks at $\tau = 0$, the cross correlation function $R_{21}(\tau)$ peaks at $\tau = D$. In the more realistic case of two finite-length data sets, the cross correlation function can be estimated by time averaging, and the TDOA parameter can be subsequently estimated by searching for a peak.

When noise and interference are present, however, the cross correlation method falters. Multiple interfering signals in the environment manifest themselves as multiple peaks in the cross correlation measurement, causing an ambiguity problem in determining the TDOA of the SOI. This problem is worsened when emitters (either interferers or SOIs) are located closely together so that the separation between their TDOAs does not exceed the widths of their correlation peaks. In this case, a resolvability problem does not permit an accurate estimation of the TDOA of any of the signals; indeed, it may not even be possible to discern the presence of the other signals. To alleviate this problem, the received data sets can be filtered prior to cross correlation, resulting in a generalized cross correlation (GCC) measurement. This can be expressed in the frequency domain using the Fourier transform of (2) weighted by $W(f)$ so that the TDOA estimate becomes

$$\hat{D} = \arg\max_{\tau} \left| \int_B \hat{S}_{21}(f)\, W(f)\, e^{j2\pi f\tau}\, df \right|, \qquad (3)$$

where the range of integration, B, is selected to coincide with the region of spectral support of $S_s(f)$. When prior knowledge of the SOI band is not available, the weighting function can be chosen to be the estimated power spectrum $W(f) = \hat{S}_1(f)$. This selects all signals and emphasizes the stronger ones. When the weighting function is unity, $W(f) \equiv 1$, (3) degenerates to the abovementioned cross correlation. Other GCC methods simply use other weighting functions in an attempt to reduce the effects of noise and interference. However, because the GCC involves only prefiltering the received data, when interferers are spectrally overlapping the SOI, the GCC is ineffective in discriminating against them.

Motivated by this shortcoming of the GCC, methods based on a cyclostationary model of the SOI have been derived. By virtue of exploiting the cyclostationary nature of the SOI, it is possible to design signal-processing algorithms that possess a signal-selectivity property making them more tolerant to noise and interference. This property holds in spite of the fact that the noise and interference may completely spectrally overlap the SOI. Armed with this ability, TDOA algorithms have been developed that overcome the ambiguity and inaccuracy problems caused by spectrally overlapping (co-channel) interference. One of the most successful of these algorithms is the SPECCOA algorithm which estimates the TDOA by

$$\hat{D} = \arg\max_{\tau} \left| \int \hat{S}_{21}^{\alpha}(f)\, \hat{S}_{11}^{\alpha}(f)^*\, e^{j2\pi f\tau}\, df \right|, \qquad (4)$$

where $\hat{S}_{21}^{\alpha}(f)$ and $\hat{S}_{11}^{\alpha}(f)$ are the estimates of the spectral correlation functions given by

$$\begin{aligned} \hat{S}_{21}^{\alpha}(f) &= S_{21_{\Delta t}}^{\alpha}(t, f)_{\Delta f} && (5) \\ &= \frac{1}{\Delta f} \int \frac{1}{\Delta t} X_2\Big(t, \nu + \frac{\alpha}{2}\Big)_{\Delta t} X_1^*\Big(t, \nu - \frac{\alpha}{2}\Big)_{\Delta t} d\nu, \end{aligned}$$

and

$$\begin{aligned} \hat{S}_{11}^{\alpha}(f) &= S_{11_{\Delta t}}^{\alpha}(t, f)_{\Delta f} && (6) \\ &= \frac{1}{\Delta f} \int \frac{1}{\Delta t} X_1\Big(t, \nu + \frac{\alpha}{2}\Big)_{\Delta t} X_1^*\Big(t, \nu - \frac{\alpha}{2}\Big)_{\Delta t} d\nu \end{aligned}$$

and

$$X(t, f)_{\Delta t} = \int_{t-\Delta t/2}^{t+\Delta t/2} x(w) e^{-j2\pi f w}\, dw. \qquad (7)$$

The frequency smoothing integrals in (5) and (6) are evaluated from $f - \Delta f/2$ to $f + \Delta f/2$ (see [4]) and α is a cycle frequency of the signal of interest (e.g., baud rate or doubled-carrier frequency) [4, 5, 6]. It should be noted that the GCC method with $W(f) = \hat{S}_1(f)$ can be interpreted as a special case of SPECCOA obtained by setting $\alpha = 0$.

The signal-selectivity advantage of the SPECCOA algorithm over the GCC algorithm is graphically illustrated in Figure 1. The GCC and SPECCOA

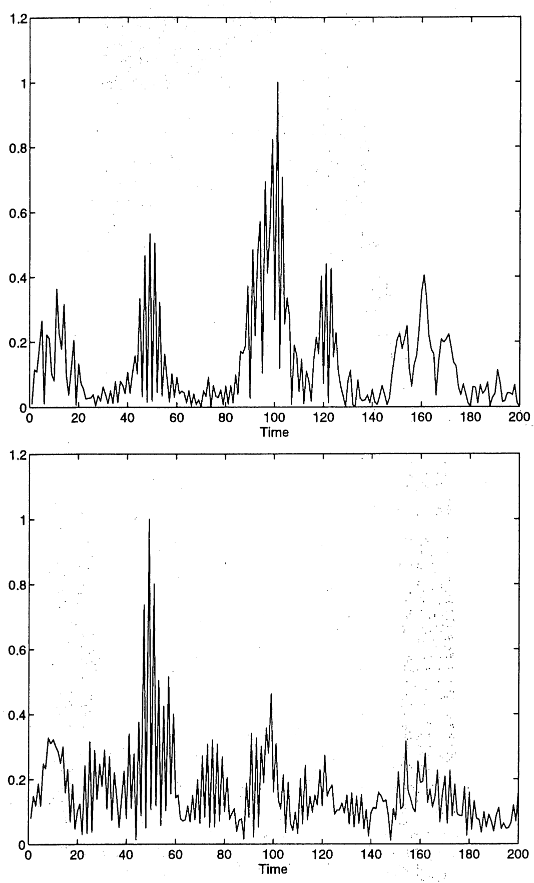

Figure 1: GCC and SPECCOA functions for a collection time of 8192 samples. Five BPSK interferers are present along with the BPSK SOI. The TDOA of the SOI is 48. The TDOAs of the interferers are 10, 95, 100, 120, and 160. The ambiguity problem is clearly evident in the GCC statistic (in the upper plot). The signal-selectivity property of the SPECCOA algorithm (in the lower plot) allows it to reject the interferers and select the desired TDOA.

functions for one sample path with collection time of 8192 samples are plotted for a case in which five equal power BPSK interferers are present in addition to the SOI. The SOI is normalized to have the same power as each of the interferers and no noise is present in this simulation. The baud periods of the SOI and interferers are 8, 5, 5, 15, 10, and 10, with TDOAs of 48, 120, 100, 160, 10, and 95 samples, respectively. The ambiguity problem with the GCC is clearly evident as there are several peaks in the function. In fact, by choosing the largest peak, the GCC incorrectly selects one of the interferers as the SOI. The GCC plot also illustrates the resolvability problem. Two interferers exist with TDOAs of 95 and 100, respectively. However, the GCC plot does not indicate any distinction between the two interferers; rather, the plot suggests that only one signal is present with TDOA between 95 and 100. On the other hand, the SPECCOA algorithm with α equal to the baud rate of the SOI does not suffer from any ambiguity or resolvability problem. The signal-selectivity property of SPECCOA has allowed it to reject all interferers TDOAs and highlight only the TDOA of the SOI. The SPECCOA statistic shows only a single strongly dominant peak centered at the correct TDOA of 48.

Another cyclostationarity-exploiting algorithm, the CCC, searches for the argument that maximizes the magnitude of the estimated cyclic cross-correlation function

$$\hat{R}_{21}^{\alpha}(\hat{D}) = \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} y_2(t) y_1^*(t - \hat{D}) e^{-j2\pi\alpha t}\, dt.$$

However, there is a complication that must be resolved. The cyclic autocorrelation function, $R_s^{\alpha}(\tau)$, which is a scaled and time-lag-shifted version of $R_{21}^{\alpha}(\tau)$ [2] is not unimodal for all signals of interest. In fact, for digital QAM signals with rectangular pulse envelopes and α equal to the keying (baud) rate, the magnitude of the cyclic autocorrelation function peaks at both $\tau = T_0/2$ and $\tau = -T_0/2$. However, for signals with Nyquist-shaped pulses, the magnitude of the cyclic autocorrelation peaks only at $\tau = 0$. Therefore, the determination of TDOA can become more involved than a simple peak search routine since the CCC function can exhibit a single peak or a pair of peaks depending on the pulse shape, as illustrated in Figure 2. For the simulations presented in Section IV, all the signals have rectangular pulse envelopes. The CCC is implemented for this case by searching for two peaks and taking the location of their midpoint as the estimate of the TDOA. An alternative that has not yet been evaluated seeks the median of the the function $\hat{R}_{21}^{\alpha}(\hat{D})$ for the TDOA estimate.

The success of SPECCOA has spawned the search for an algorithm with yet further improvements in TDOA estimation. To this end, a generalization of SPECCOA was sought. The derivation of such a generalization is provided in the next section.

III Generalized SPECCOA

Given the mathematical model (1), it can be shown [4] that the cross spectral correlation function between two received signals $y_m(t)$ and $y_k(t)$ can be expressed by

$$S_{mk}^{\alpha}(f) = A_m A_k^* S_s^{\alpha}(f) e^{-j2\pi(f+\alpha/2)t_m} e^{j2\pi(f-\alpha/2)t_k}, \tag{8}$$

provided that α is not a cycle frequency of either $n_m(t)$ or $n_k(t)$. Solving for $S_s^{\alpha}(f)$, we obtain

$$\begin{aligned} F_{mk}(f) &\triangleq \frac{1}{A_m A_k^*} S_{mk}^{\alpha}(f) e^{j2\pi(f+\alpha/2)t_m} e^{-j2\pi(f-\alpha/2)t_k} \\ &= S_s^{\alpha}(f). \end{aligned}$$

Therefore, it can be observed that $F_{mk}(f) = F_{pq}(f)$ for all $m, k, p, q \in \{1, 2\}$. However, since the ideal functions $F_{mk}(f)$ or $F_{pq}(f)$ are not available, estimates of these functions must be used, and the equivalence between the two estimated functions is only approximate; that is, $\hat{F}_{mk}(f) \cong \hat{F}_{pq}(f)$. The unknown parameters in $\hat{F}_{mk}(f)$ and $\hat{F}_{pq}(f)$ are the two cross spectral correlation functions $S_{mk}^{\alpha}(f)$ and $S_{pq}^{\alpha}(f)$, which can be estimated using smoothed cyclic periodograms [4], the complex-valued amplitude parameters A_m, A_k, A_p, A_q, and the time parameters t_m, t_k, t_p, and t_q. Although the amplitude parameters require estimation, the TDOA problem is concerned only with the estimated values of the time parameters.

In order to estimate the desired parameters, a least-squares problem for minimizing the sum (over f) of squared differences between $\hat{F}_{mk}(f)$ and $\hat{F}_{pq}(f)$ can be formulated, where the minimization is performed over the amplitude and time parameter estimates, $\hat{A}_m$, $\hat{A}_k$, $\hat{A}_p$, $\hat{A}_q$, $\hat{t}_m$, $\hat{t}_k$, $\hat{t}_p$, and $\hat{t}_q$. It can be shown (cf. [7], [2]) that the solution to this minimization problem for certain choices of m, k, p, and q yields the SPECCOA algorithm as given in (4).

As a generalization, another problem can be formulated to *jointly* fit all pairs of estimates $\hat{F}_{mk}(f)$ and $\hat{F}_{pq}(f)$. An appropriate formulation of such a minimization problem is

$$\min \int \left\| \hat{\boldsymbol{F}}(f) - \mathbf{1}\hat{S}_s^{\alpha}(f) \right\|^2 df, \tag{9}$$

where $\hat{\boldsymbol{F}}(f) = [\hat{F}_{11}(f)\ \hat{F}_{12}(f)\ \hat{F}_{21}(f)\ \hat{F}_{22}(f)]^T$, and $\mathbf{1} = [1\ 1\ 1\ 1]^T$. The minimization is performed over

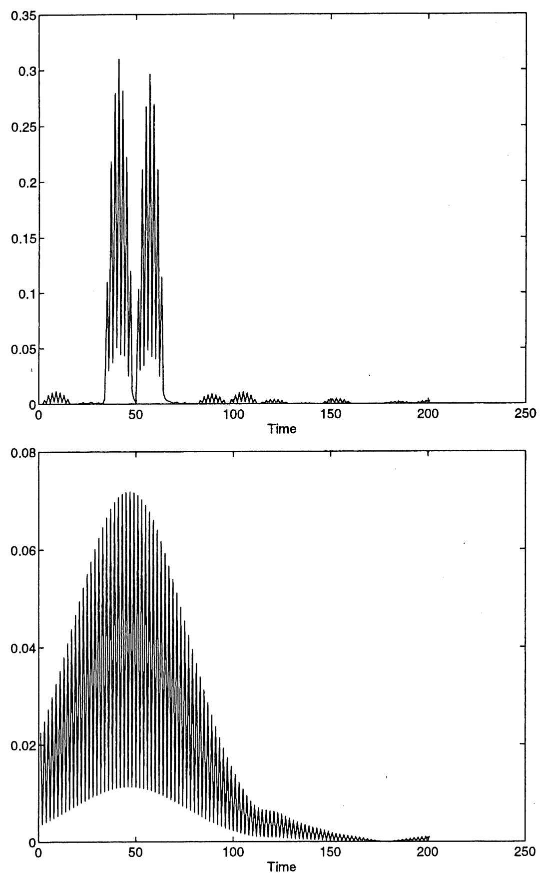

Figure 2: The CCC functions evaluated at the baud rate of a BPSK signal with rectangular pulse shape (upper plot) and Nyquist pulse shape (lower plot) with 50 % excess bandwidth.

all complex-valued amplitude parameters ($\hat{A}_1$, $\hat{A}_2$) and all time parameters ($\hat{t}_1$, $\hat{t}_2$); furthermore, since $S_s^\alpha(f)$ is unknown, it must also be included in the set of minimization parameters.

It can be shown [7] that the general solution to this problem reduces to a three-dimensional minimization of a nontrivial function over a magnitude parameter, $\hat{M}$, a phase parameter, $\hat{\phi}$, and the desired TDOA parameter, $\hat{D}$. The magnitude and phase parameters represent the relative complex-valued gain between the two receivers, $\hat{M}e^{j\hat{\phi}} = \hat{A}_2/\hat{A}_1$. The solution to this nonlinear three-dimensional mininization problem is difficult to obtain analytically and, without simplification, is impractical for implementation.

Nevertheless, insight into the capability of the approach can be gained by considering the simplified case in which $M \cong 1$ and $\phi \cong 0$ and, therefore, where $\hat{M} = 1$ and $\hat{\phi} = 0$ are adequate estimates; in other words, where it can be assumed that there is negligible mismatch between the gains and phases of the two channels and receivers. For this simplified case, the solution to the minimization problem (9) requires a search over only the TDOA parameter. The resultant algorithm is

$$\begin{aligned}\max_{\hat{D}} \Re \Bigg\{ & \int \hat{S}_{11}^\alpha(f)\hat{S}_{12}^\alpha(f)^* e^{j2\pi(f-\alpha/2)\hat{D}}\, df \\ & + \int \hat{S}_{21}^\alpha(f)\hat{S}_{11}^\alpha(f)^* e^{j2\pi(f+\alpha/2)\hat{D}}\, df \\ & + \int \hat{S}_{21}^\alpha(f)\hat{S}_{22}^\alpha(f)^* e^{j2\pi(f-\alpha/2)\hat{D}}\, df \\ & + \int \hat{S}_{22}^\alpha(f)\hat{S}_{12}^\alpha(f)^* e^{j2\pi(f+\alpha/2)\hat{D}}\, df \\ & + \int \hat{S}_{21}^\alpha(f)\hat{S}_{12}^\alpha(f)^* e^{j4\pi f\hat{D}}\, df \\ & + \int \hat{S}_{22}^\alpha(f)\hat{S}_{11}^\alpha(f)^* e^{j2\pi\alpha\hat{D}}\, df \Bigg\}, \qquad (10)\end{aligned}$$

and is called the G–SPECCOA algorithm (for generalized-SPECCOA) [8][1], because retaining any one of the first four terms in (10) and deleting the other five terms yields the SPECCOA algorithm (cf. [2]). In (10), $\Re\{\cdot\}$ denotes the real-part operation.

IV Simulation Results

In this section, the results of computer simulations are presented to illustrate the performance characteristics of the G-SPECCOA algorithm. Results are presented in terms of plots of normalized mean squared error (MSE normalized by the square of the true TDOA) in dB as a function of data collection time.[2] In each case, white Gaussian noise (WGN) is added with an SNR defined over the entire sampling frequency band, $[-f_s/2, f_s/2]$ where f_s is the sampling frequency. For the CCC, SPECCOA, and G-SPECCOA algorithms, the baud rate of the SOI was chosen to be the cycle frequency α. To estimate $S_{mk}^\alpha(f)$, frequency smoothing is applied to measured cyclic periodograms [4].

In the first simulation, we consider the case in which co-channel interference is present. The SOI is BPSK with a normalized carrier frequency of 0.25, a baud period of 16 samples, and a rectangular keying envelope. One BPSK co-channel interferer is present with carrier frequency of 0.31, baud period of 12 samples, and a rectangular keying envelope. The TDOA of the SOI is 48.1 samples and that of the interferer is 10 samples. Both the total-band SNR and the SIR are 0 dB, resulting in an overall SINR of –3 dB. For these parameters, the SOI and interferer have nearly the same bandwidth, with the interferer spectrally overlapping the SOI by approximately 66%. Furthermore, for the chosen baud periods and TDOAs, sufficient separation exists between peaks in the GCC measurement so as to remove the resolvability problem. The results of 1000 Monte Carlo trials are illustrated in Figure 3 where the NMSE is plotted as a function of collection time which varies from 512 samples (32 SOI bauds) to 16,384 samples (1024 SOI bauds). It should be noted that as the collection time increases and the NMSE becomes very small, the reliability of the results diminishes due to the fact that for 1000 trials, since time is quantized, most of the estimates are exact. Moreover, because time is quantized and the true TDOA is not an integer, the minimum attainable NMSE for this case is –53.6 dB; that is, an error floor exists at –53.6 dB in the NMSE plot.

The results from this simulation indicate the relative performance among GCC, CCC, SPECCOA, and G-SPECCOA. As expected, in this case of co-channel interference, all of the cyclostationarity-exploiting algorithms outperform GCC. The signal-selectivity advantage of CCC, SPECCOA and G-SPECCOA allow them to reject the interference, whereas the GCC suffers from an ambiguity caused by the presence of both the SOI and the interference. This fact is illustrated by an analysis of the errors in the TDOA estimates that are produced by

[1] A typographical error appears in [8] for the G-SPECCOA equation. In [8], the subscripts for $\hat{S}_{21}^\alpha(f)$ in the fourth integral are transposed. The correct expression, as given above, is $\hat{S}_{12}^\alpha(f)$.

[2] The simulation results reported in [8] are incorrectly scaled; therefore, the performance numbers stated therein are inaccurate, although the relative performance and general conclusions drawn are still valid.

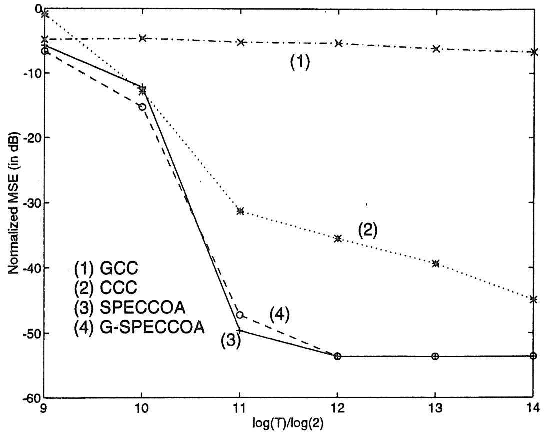

Figure 3: Performance comparison of GCC, SPECCOA, and G-SPECCOA as a function of collection time from 512 samples to 16,384 samples. Wideband inteference environment with approximately 66% spectral overlap and SIR = SNR = 0 dB.

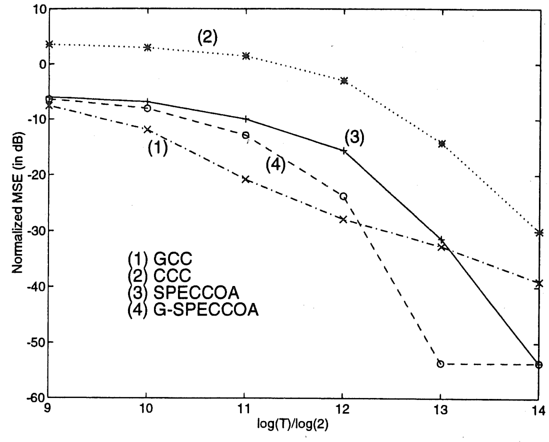

Figure 4: Performance comparison of GCC, SPECCOA, and G-SPECCOA as a function of collection time from 512 samples to 16,384 samples in a WGN environment, SNR = −10 dB.

the various algorithms. For the collection time of 4096 samples the GCC produced 469 estimates of 10, 487 estimates of 48, 34 of 44, and 10 of 52, whereas SPECCOA produced 1000 estimates of 48. This simulation also illustrates that G-SPECCOA and SPECCOA perform very comparably, significantly outperforming the CCC by 20 dB for a collection size of 2048 samples. The effect may become even more pronounced at longer collection times, however these simulation results can not show this because both SPECCOA and G-SPECCOA reach the minimum attainable NMSE for this simulation which does not include time interpolation. Furthermore, at such long collects, the reliability of the results is inadequate to draw sound conclusions since there are an insufficient number of errors for statistical significance in 1000 trials.

In the next simulation, we investigate the case where only WGN is present in the environment. For only WGN and finite collection time, it is not necessarily expected that SPECCOA or G-SPECCOA should outperform GCC. Without interference, the only causes of degradation in the GCC statistic are noise from the environment and finite-time averaging effects. Therefore, the SPECCOA and G-SPECCOA algorithms no longer have an advantage over GCC since signal-selectivity is not needed. However, in the ideal case of infinite collect (approached with long collection times) without detrimental effects due to finite-time averaging, the SPECCOA and G-SPECCOA algorithms should once again outperform GCC. In this idealized case, the spectral correlation functions used in the SPECCOA and G-SPECCOA algorithms contain no contributions from either environmental or finite-time averaging noise. Because the environmental noise does not exhibit any spectral correlation at a cycle frequency other than zero, the spectral correlation function at a cycle frequency other than zero should be noise-free [4, 5, 6]. On the other hand, the power spectrum, which can be viewed as the spectral correlation function at a cycle frequency of zero, utilized by the GCC algorithm contains environmental noise in spite of the absence of finite-time averaging noise. As a consequence, the SPECCOA and G-SPECCOA algorithms should be able to outperform GCC in this case of long collection time. This drawback of GCC is absent in the conventional cross-correlation method, but that method has no filtering capability at all.

The simulation results of Figure 4 concur with the above reasoning. For short-to-moderate collection times of up to 4096 samples, GCC outperforms both SPECCOA and G-SPECCOA.[3] However, as the collection time becomes larger (4096 samples), G-SPECCOA surpasses GCC. The figure also suggests that SPECCOA may also surpass GCC for collection times greater than 16,384 samples although the lack for reliability in the simulations at such long collection times precludes a sound conclusion. It is also evident from Figure 4 that both SPECCOA and G-SPECCOA outperform CCC by 10 and 20 dB, respectively, for a collection time of 4096 samples. One final note illustrated by this figure is that G-SPECCOA noticably outperforms SPECCOA over

[3]These results do not agree with those in [9], where SPECCOA is shown to outperform GCC at lower collects. We believe the results reported in [9] to be incorrect.

the entire collection-time range.

V Conclusions

In the presence of interfering signals, we have demonstrated that cyclostationarity-exploiting algorithms, which possess a signal-selectivity property, outperform a common generalized cross-correlation algorithm for TDOA estimation. When interference is co-channel with the SOI, conventional GCC algorithms suffer from ambiguity and resolvability problems rendering them incapable of accurate TDOA estimation. In contrast, algorithms such as SPECCOA, which are based on a cyclostationary model of the SOI, do not share these same ambiguity and resolvability problems and are able to obtain an accurate estimate of the TDOA despite the presence of co-channel interference. In this paper, we have also considered a generalization of the SPECCOA algorithm called G-SPECCOA. Similar to SPECCOA, G-SPECCOA exhibits the desirable signal-selectivity property allowing it considerable tolerance to noise and interference. Computer simulation results show that for cases of co-channel interference, both G-SPECCOA and SPECCOA are able to accurately estimate the TDOA whereas the GCC fails to do so. Moreover, both G-SPECCOA and SPECCOA are shown to outperform CCC. For the case of WGN only, G-SPECCOA and SPECCOA are able to outperform GCC only when the collection time becomes relatively large. Furthermore, both G-SPECCOA and SPECCOA outperform CCC for all collection times investigated.

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