Blind-Adaptive Linear Frequency-Shift Filtering *

Grace K. Yeung
Mission Research Corporation
Monterey, California

William A. Gardner
University of California, Davis
Davis, California

Abstract

The technique of FREquency-SHift (FRESH) filtering exploits the spectral redundancy property of cyclostationarity that is exhibited by many communications signals for applications such as co-channel interference suppression, signal cancellation, and signal separation. In cases where prior knowledge of the signal of interest is not available for the realization of optimum FRESH filters and training signals are not available for adaptation, blind adaptation is necessary. This paper briefly reviews the theory of optimum FRESH filtering, reviews two recently proposed algorithms, and presents simulation results for blind-adaptive linear FRESH filtering.

1 Introduction

The theory and applications of optimum FREquency-SHift (FRESH) filtering have received some attention recently in regard to the problem of separating signals that overlap both in time and in frequency. By exploiting the spectral redundancy property of cyclostationarity inherent in most radio signals, FRESH filtering provides a better solution to the problem of co-channel interference removal than does linear time-invariant (LTI) filtering. In addition, the use of FRESH filtering becomes especially attractive when only single-antenna reception is available or when directions of arrival of the signals involved are not sufficiently different thereby prohibiting the use of spatial filtering. Performance evaluation of optimum FRESH filtering reveals that a substantial degree of signal separability is possible even with 100% temporally and spectrally overlapping signals [1]. However, prior statistical knowledge of the signal of interest is required for the realization of optimum FRESH filters. When this knowledge is not available, training signals are needed for adaptive implementations of FRESH filters. However, in practice, training signals may not always be available. Therefore, methods for blind adaptation need to be sought.

In this paper, two algorithms proposed by the second author of this paper for blind-adaptive linear FRESH filtering are presented. These two algorithms, namely the Estimated-Crosscorrelation Least Squares (ECLS) algorithm and the Constant-Modulus Least Squares (CMLS) algorithm, are simulated using MATLAB. The ECLS method is based on replacing the unknown crosscorrelation between the received data and the signal to be extracted with an estimate obtained directly from the received data. This is possible due to the signal-selectivity property of cyclostationary signals. The CMLS method is based on the concept of property restoral, specifically restoral of the constant-modulus property of the signal to be extracted.

A review of optimum FRESH filtering is given in Section 2. Formulation of the two blind-adaptive FRESH filtering algorithms and their computer simulation results are presented in Sections 3 and 4, respectively. Both simulated data and data recorded from cellular radio transmitters in a laboratory were used in the computer simulations.

2 Review of Optimum FRESH Filtering

The basic concept of FRESH filtering is that of using spectral components in some bands to estimate, enhance, or cancel those in other bands by exploiting the spectral redundancy exhibited by cyclostationary signals. Hence, in addition to the complex scaling operation that LTI filters apply to individual spectral components of the received signal, the implementation of FRESH filters involves frequency-shifting operations, resulting in linear periodic (or polyperiodic) time-variant filters. Specifically, it is well known that while optimum filters for stationary signals are time-invariant, those for cyclostationary signals with a single period (or multiple incommensurate periods) are periodically (or polyperiodically) time-variant. More elaborate studies of the theory of optimum FRESH filtering are available in [1]-[4].

Consider the received signal

$$r(t) = s(t) + i(t), \tag{1}$$

where $s(t)$ is the cyclostationary signal of interest

*This work was supported in part by the National Science Foundation under grant MIP-91-12800.

(SOI) and $i(t)$ consists of both signals not of interest (SNOIs) and noise. The general expression for the optimum linear-conjugate-linear (LCL) FRESH filtering of a complex-valued received signal $r(t)$ to produce an estimate $\hat{s}(t)$ of $s(t)$ is given by

$$\hat{s}(t) = \int_{-\infty}^{\infty} h_1(t,\tau) r(\tau)\, d\tau + \int_{-\infty}^{\infty} h_2(t,\tau) r^*(\tau)\, d\tau, \quad (2)$$

where $h_1(t,\tau)$ and $h_2(t,\tau)$ are periodically or polyperiodically time-varying linear filters with Fourier series representations

$$h_1(t,\tau) = \sum_{p=0}^{L-1} h_p(t-\tau) exp(i2\pi\alpha_p\tau) \quad (3)$$

and

$$h_2(t,\tau) = \sum_{q=0}^{M-1} g_q(t-\tau) exp(i2\pi\beta_q\tau). \quad (4)$$

The inclusion of the conjugate signal in (2) is essential for optimum time-variant filtering if $r(t)$ and $s(t)$ are analytic-signal or complex-envelope representations of two real-valued signals [2]. The frequency-shift parameters α_p and β_q can be any of the cycle frequencies and conjugate cycle frequencies of the SOI and SNOIs and their linear combinations with integer coefficients. A cycle frequency η is one for which the frequency-shifted version $x(t)e^{i2\pi\eta t}$ is correlated with $x(t+\tau)$ for some τ. A conjugate cycle frequency ν is one for which the frequency-shifted version $x(t)e^{-i2\pi\nu t}$ is correlated with $x^*(t+\tau)$ for some τ. The values L and M correspond to the number of linear paths and the number of conjugate-linear paths in the filter, respectively, and can, in principle, be infinite. The important problem of selecting appropriate frequency-shift parameters is addressed in [1] and [2]. Using (3) and (4) in (2), we obtain

$$\hat{s}(t) = \sum_{p=0}^{L-1} h_p(t) \otimes [r(t) exp(i2\pi\alpha_p t)] + \sum_{q=0}^{M-1} g_q(t) \otimes [r^*(t) exp(i2\pi\beta_q t)], \quad (5)$$

which is equivalent to a multivariate (dimension $L+M$) optimum LTI filtering problem in which the input signals are frequency-shifted versions of $r(t)$ and $r^*(t)$.

For the purpose of analyzing the implementation of optimum FRESH filters in discrete time, assume $r(t)$ is a low pass signal with bandwidth less than unity, and let the variable t be the integer-valued time index corresponding to unity sampling increment. Then, for a finite-impulse-response (FIR) FRESH filter with weight vector $\mathbf{w}$, (5) can be expressed as

$$\hat{s}(t) = \mathbf{w}^H \mathbf{y}(t), \quad (6)$$

where the elements of $\mathbf{w}$ correspond to the coefficients of $h_p(t)$ and $g_q(t)$, and $\mathbf{y}(t)$ is defined as

$$\mathbf{y}(t) \triangleq [p_{\alpha_0}(t-t_1)\cdots p_{\alpha_0}(t-t_m)\cdots p_{\alpha_{L-1}}(t-t_1)\cdots p_{\alpha_{L-1}}(t-t_m) \; p_{-\beta_0}(t-t_1)^*\cdots p_{-\beta_0}(t-t_m)^*\cdots p_{-\beta_{M-1}}(t-t_1)^*\cdots p_{-\beta_{M-1}}(t-t_m)^*]^T, \quad (7)$$

where $p_\mu(t) \triangleq r(t)e^{i2\pi\mu t}$, the t_k are appropriately chosen delays (e.g. $t_k = k - m/2$), and m is the length of each of the LTI filters $h_p(t)$ and $g_q(t)$.

The weight vector $\mathbf{w}$ that minimizes the average squared error (over N time samples)

$$\left\langle |\hat{s}(t) - s(t)|^2 \right\rangle_N \quad (8)$$

is given by

$$\mathbf{w} = \hat{\mathbf{R}}_{\mathbf{yy}}^{-1} \hat{\mathbf{R}}_{\mathbf{y}s}, \quad (9)$$

where

$$\hat{\mathbf{R}}_{\mathbf{yy}} = \left\langle \mathbf{y}(t)\mathbf{y}^H(t) \right\rangle_N \quad (10)$$

and

$$\hat{\mathbf{R}}_{\mathbf{y}s} = \left\langle \mathbf{y}(t)s^*(t) \right\rangle_N. \quad (11)$$

(The notation $\langle\cdot\rangle_N$ represents time averaging over N samples, the $\hat{}$ symbol represents estimates obtained from finite-time averaging, and superscript H denotes conjugate transposition.)

The least squares (LS) solution given in (9) indicates that statistical knowledge of $s(t)$ is required for implementation of the optimum LS FRESH filter. When this knowledge is not available and when training signals cannot be provided, blind-adaptive methods must be sought.

3 Blind-Adaptive FRESH Filtering

In this section, two approaches to performing blind-adaptive linear LS FRESH filtering are presented. These two methods are based on modifying the LS FRESH approach outlined in the previous section such that, in many cases, a good signal estimate can be obtained using only information from the received signal $r(t)$.

3.1 The ECLS Method

For the Estimated-Crosscorrelation Least Squares method, equation (9) is modified as

$$\mathbf{w} = \hat{\mathbf{R}}_{\mathbf{yy}}^{-1} \hat{\mathbf{R}}, \quad (12)$$

where $\hat{\mathbf{R}}$ is an estimate of the crosscorrelation $\hat{\mathbf{R}}_{\mathbf{y}s}$ derived solely from the received signal $r(t)$. Application of this method requires that the modulation type of the SOI be known. Observe, for example, that the element in $\hat{\mathbf{R}}_{\mathbf{y}s}$ corresponding to the i^{th} frequency shift α_i is given by

$$\left\langle p_{\alpha_i}(t-t_k)s^*(t)\right\rangle_N . \tag{13}$$

Due to the signal-selectivity property of cyclostationary signals, we have (for large N)

$$\left\langle p_{\alpha_i}(t-t_k)s^*(t)\right\rangle_N \cong \left\langle p_{\alpha_i}(t-t_k)r^*(t)\right\rangle_N \tag{14}$$

if α_i is a cycle frequency of $s(t)$ but not $i(t)$ and

$$\left\langle p_{-\beta_i}(t-t_k)^*s^*(t)\right\rangle_N \cong \left\langle p_{-\beta_i}(t-t_k)^*r^*(t)\right\rangle_N \tag{15}$$

if β_i is a conjugate cycle frequency of $s(t)$ but not $i(t)$. For all other α_i and β_i, except those that correspond to cycle frequencies of both $s(t)$ and $i(t)$, the left-hand-sides of (14) and (15) approach 0 as $N \rightarrow \infty$.

For the special case of $\alpha_i = 0$, which is necessarily a cycle frequency of both $s(t)$ and $i(t)$, the corresponding m elements in $\hat{\mathbf{R}}_{\mathbf{y}s}$,

$$\left\langle p_0(t-t_k)s^*(t)\right\rangle_N, \quad k=1,\cdots,m \tag{16}$$

approach, as $N \rightarrow \infty$, the limit autocorrelation of $s(t)$ at $\tau = -t_k$, $R_s(-t_k)$, because $s(t)$ and $i(t)$ are statistically independent. Therefore, an estimate of (16) can be obtained using an estimate of $R_s(\tau)$, or its Fourier transform $S_s(f)$, the power spectral density (PSD) of $s(t)$. An expression for $S_s(f)$ in terms of $S_s^{\alpha_i}(f)$, the spectral correlation function for $s(t)$, or $S_{ss^*}^{\beta_i}(f)$, the conjugate spectral correlation function for $s(t)$, can be derived from the mathematical model of $s(t)$ for some signals (like AM, BPSK, PAM, digital QAM, and possibly others) [3]. Furthermore, the signal-selectivity property results in

$$S_s^{\alpha_i}(f) = S_r^{\alpha_i}(f) \tag{17}$$

and

$$S_{ss^*}^{\beta_i}(f) = S_{rr^*}^{\beta_i}(f) \tag{18}$$

if α_i and β_i are cycle frequencies of $s(t)$ but not $i(t)$. Therefore, an estimate of $S_s(f)$, and hence (16), can be obtained using estimates of the right-hand-side of (17) or (18). The rest of $\hat{\mathbf{R}}$ is determined according to (14)-(15) and the discussion that follows (15). As a result, all elements of $\hat{\mathbf{R}}_{\mathbf{y}s}$ can be estimated solely from the received waveform $r(t)$, provided that all the non-zero frequency shifts are cycle frequencies or conjugate cycle frequencies of either only $s(t)$ or only $i(t)$.

For example, for a complex BPSK SOI with carrier frequency f_0, we have

$$S_s(f) = |S_{ss^*}^{2f_0}(f-f_0)|. \tag{19}$$

Using (18) in (19) with $\beta_i = 2f_0$, the first m elements of $\hat{\mathbf{R}}$ (corresponding to $\alpha_0 = 0$) can be approximated using

$$\hat{R}_s(\tau) \cong \mathcal{F}^{-1}\{|\hat{S}_{rr^*}^{2f_0}(f-f_0)|\} \tag{20}$$

and the rest of $\hat{\mathbf{R}}$ can be estimated as discussed previously. Using the weight vector given by (12) in (6), we have then obtained an estimate of $s(t)$ solely from the received signal $r(t)$.

3.2 The CMLS Method

An alternative to the ECLS method is the Constant-Modulus Least Squares method (based on the constant modulus algorithm originally proposed for time-invariant filtering by Treichler and Agee [5]) obtained by replacing the training signal $s(t)$ in $\hat{\mathbf{R}}_{\mathbf{y}s}$ with the modulus-normalized version,

$$\tilde{s}(t) = \frac{\hat{s}(t)}{|\hat{s}(t)|}, \tag{21}$$

of the estimate

$$\hat{s}(t) = \mathbf{w}^H\mathbf{y}(t), \tag{22}$$

where

$$\mathbf{w} = \hat{\mathbf{R}}_{\mathbf{yy}}^{-1}\hat{\mathbf{R}}_{\mathbf{y}\tilde{s}}. \tag{23}$$

The sequence of equations (21),(23), and (22) are iterated until the minimum mean-squared error (MMSE)

$$\min_A \left\langle |s(t) - A\hat{s}(t)|^2\right\rangle_N \tag{24}$$

is approached. The correct scaling factor A needs to be determined because the resulting signal estimate $\hat{s}(t)$ has unity modulus. The CMLS approach is applicable, when the signal-to-interference-ratio (SIR) is greater than unity, to FM, FSK, BPSK, QPSK, and digital QAM signals with constant modulus.

4 Computer Simulations

Here, performances of the ECLS and CMLS algorithms for co-channel interference removal are illustrated by computer simulations. In the first example, we have two 80% spectrally overlapping complex BPSK signals with 100% excess-bandwidth, equal baud-rates ($f_{b_1} = f_{b_2} = 1/16$), and equal average powers (SIR= 0dB). White Gaussian noise is added to obtain a 20dB SNR, and the number of data points N is 4096. The SOI is centered at zero frequency and the carrier offset, f_{os}, of the interferer is determined from

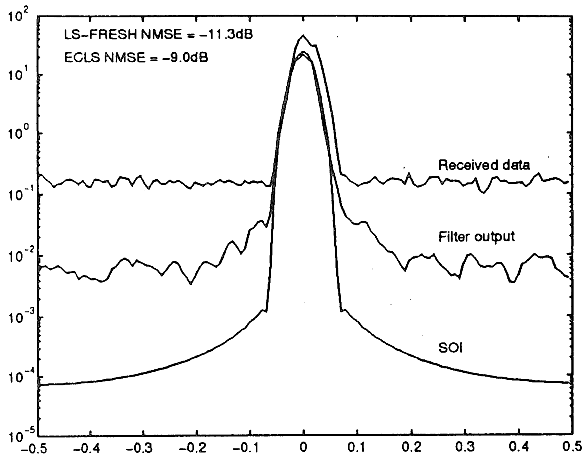


Figure 1: The ECLS method: PSDs of received data, filter output, and SOI in log scale vs. normalized frequency

f_{b_1}, f_{b_2}	SIR	f-shifts (L);(CL)	NMSE(dB): ECLS; LS-FRESH
1/16, 1/16	3	$0, 2f_{os}$; $0, 2f_{os}, \pm f_{b_1}$	−10.3; −12.5
1/16, 1/16	0	$0, 2f_{os}$; $0, 2f_{os}, \pm f_{b_1}$	−9.0; −11.3
1/8, 1/10	3	$0, 2f_{os}, \pm f_{b_1}$; $0, 2f_{os}, \pm f_{b_1}$	−11.2; −13.9
1/8, 1/10	0	$0, 2f_{os}, \pm f_{b_1}$; $0, 2f_{os}, \pm f_{b_1}$	−9.7; −12.2

Table 1: Simulation parameters and NMSE performance results of the ECLS method.

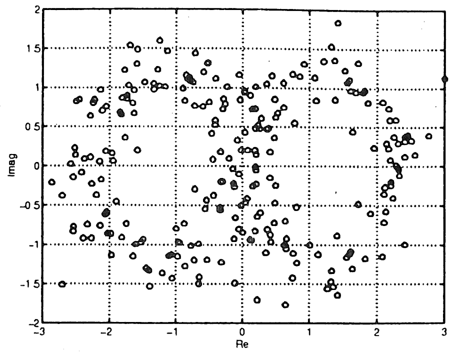
the specified percentage overlap. The ECLS method, with filter length $m = 20$ and four frequency-shift paths ($\alpha_1 = 2f_{os}$, $\beta_{1,2,3} = 2f_{os}, \pm f_{b_1}$) in addition to the zero-frequency-shift paths $\alpha_0 = 0$ and $\beta_0 = 0$, is used to extract the SOI. The result is shown in Figure 1, where the normalized MSE (NMSE) obtained using LS FRESH filtering with a perfect training signal is also shown for comparison. Table 1 shows simulation parameters and NMSEs for this simulation and several other cases, all with $m = 20$ and 80% overlapping signals. We see in these simulations that the ECLS approach yields NMSEs that are within 2 to 3dB of those for the corresponding LS FRESH filters.

Table 2 shows bit-error-rates (BERs) and simulation parameters for example one and several other cases (all with $m = 20$) in which the percentage overlap of signal spectra is much higher. Constellation diagrams of the received data and the ECLS filter output are shown in Figures 2-5 for the four cases in Table 2. In these cases of high percentage spectral overlap of co-channel signals, the use of the ECLS algorithm has enabled reduction in BERs as compared to the case with no filtering (designated BER $r(t)$ in Table 2). Therefore, preceding a decision feedback equalizer or a decision-directed equalizer by the ECLS blind-adaptive FRESH filter could enable proper operation of these conventional methods in cases where the signal of interest is severely corrupted by co-channel interference.

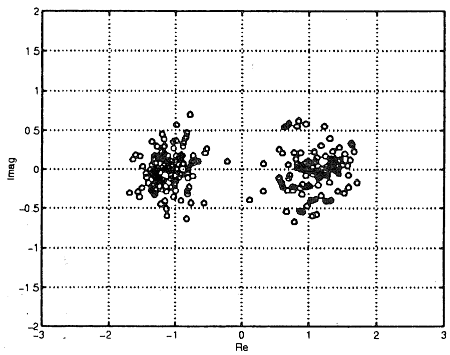
f_{b_1}; f_{b_2}	N; overlap	f-shifts (L);(CL)	BER $r(t)$;ECLS
1/16; 1/16	4096; 80%	$0, 2f_{os}$; $0, 2f_{os}, \pm f_{b_1}$	0.0941; 0
1/16; 1/16	16384; 99.9%	$0, 2f_{os}$; $0, 2f_{os}, \pm f_{b_1}$	0.0968; 0.0313
1/16; 1/16	65536; 99.9%	$0, 2f_{os}$; $0, \pm f_{b_1}, 2f_{os}, 2f_{os} \pm f_{b_2}, 4f_{os}$	0.0906; 0.0215
1/8; 1/10	4096; 99.9%	$0, 2f_{os}, \pm f_{b_1}, \pm f_{b_2}$; $0, \pm f_{b_1}, 2f_{os}, 2f_{os} \pm f_{b_2}$	0.0646; 0.0118

Table 2: Simulation parameters and BER performance results of the ECLS method.

In the second and third examples, the CMLS approach is applied to cellular radio data digitally recorded from commercially available transmitters in a laboratory. In the first scenario, the digital data portion (used for call control information) of the cellular radio data is used. This portion consists of a CPFSK signal which has constant modulus. The SOI and interferer are two independent complex-valued data segments of the same modulation type and with the same baud-rate f_b. The SOI has carrier frequency equal to zero and the interferer has a carrier offset, f_{os}, that results in 80% overlap of the signal spectra. The SIR is 3dB, SNR is 20dB, and $N = 1024$. The CMLS method, with $m = 20$ and four frequency-shift paths ($\alpha_{1,2,3,4} = \pm f_b, \pm 2f_b$) in addition to the $\alpha_0 = 0$ path, is used to extract the SOI. No conjugate-linear paths were employed in this case because the signals involved do not exhibit very strong conjugate spectral coherence. Figure 6 shows the result together with the NMSE obtained using LS FRESH filtering with a perfect training signal for comparison. Also shown are the BERs of the received data and the filter output. The use of the CMLS FRESH filter has reduced the BER from 0.2088 to 0 in this scenario.

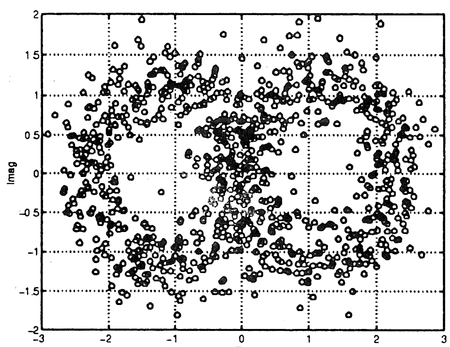


(a) Constellation of $r(t)$

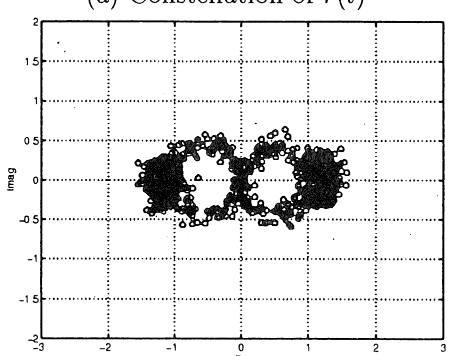


(b) Constellation of $\hat{s}(t)$

Figure 2: The ECLS method on BPSK signal – Table 2 case 1.

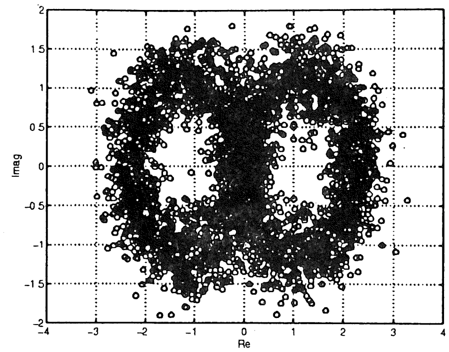


(a) Constellation of $r(t)$

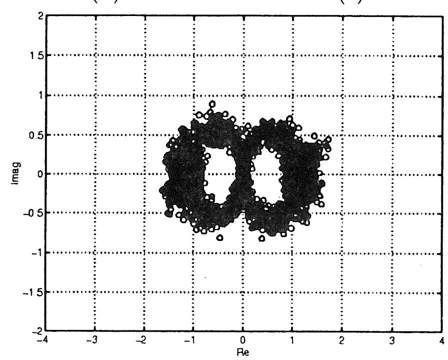


(b) Constellation of $\hat{s}(t)$

Figure 3: The ECLS method on BPSK signal – Table 2 case 2.

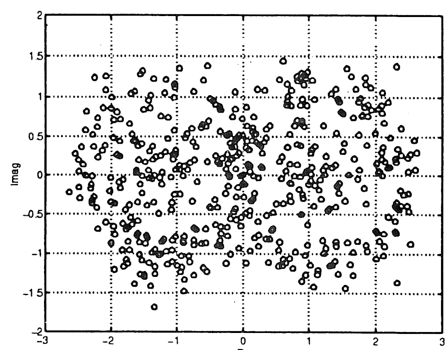


(a) Constellation of $r(t)$

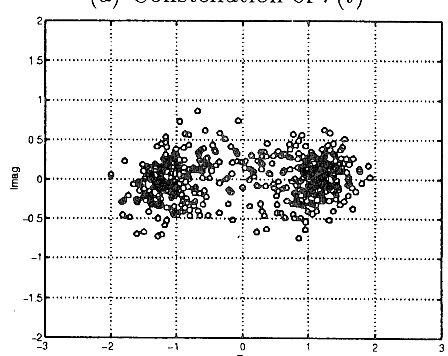


(b) Constellation of $\hat{s}(t)$

Figure 4: The ECLS method on BPSK signal – Table 2 case 3.



(a) Constellation of $r(t)$



(b) Constellation of $\hat{s}(t)$

Figure 5: The ECLS method on BPSK signal – Table 2 case 4.

In the second scenario, the voice portion of the cellular radio data is used. This portion consists of an FM signal with a supervisory audio tone (SAT) in the baseband. The SOI and interferer are two independent complex-valued data segments of the same modulation type with SAT frequencies $f_{sat_1} = 6023$Hz and $f_{sat_2} = 5972$Hz respectively. The interferer with carrier offset f_{os} overlaps the SOI centered at zero frequency by 80%. The SIR is 3dB, SNR is 20dB, and $N = 1024$. The CMLS method is used with $m = 20$ and eleven frequency-shift paths ($\alpha_{0,1,2,3,4} = 0, \pm f_{sat_1}, \pm f_{sat_2}, \beta_{0,1,2,3,4,5} = 0, \pm f_{sat_1}, 2f_{os}, 2f_{os} \pm f_{sat_2}$). Results are shown in Figures 7 and 8.

In both scenarios, the CMLS algorithm is able to converge, within fifteen iterations, to within 2 to 3dB of the LS NMSE. Improvements in the BER performance as shown in Figure 6 and in the quality of the time waveform as shown in Figure 8 are substantial. With much lower SIRs, the algorithm may experience the well-known capture problem which can prevent the algorithm from converging to the global minimum of MSE. With higher percentage spectral overlap, the MSE performance degrades, and larger number of filter taps and longer data collect time are required for extracting the SOI with the same level of performance, resulting in lower convergence rates.

An advantage of the CMLS method is that it can clean up channel distortion when it converges properly. On the other hand, the ECLS method does not experience the convergence problem and can therefore mitigate much more severe interference than that which the CMLS method can accommodate. Moreover, the ECLS method is not restricted to constant-modulus signals. To possibly gain the advantages of both methods, the CMLS algorithm can be used following the ECLS algorithm.

5 Conclusion

In this paper, the usefulness of two relatively new blind-adaptive linear FRESH filtering algorithms are demonstrated via computer simulations. The set of frequency-shift parameters used in the FRESH filters can be determined using conventional cyclic spectral analysis of the received data. The ECLS method requires knowledge of the modulation type of the SOI in order to obtain a mathematical model used for estimating the crosscorrelation and the CMLS method is applicable only to constant-modulus signals. These two methods provide means for mitigating severe co-channel interference in commercial and military communications, shipboard combat systems, navigational environments etc., without prior detailed knowledge of the SOI. In cases of moderate co-channel interference, such as 80% overlap of equal-powered signals, perfect extraction of the SOI is possible. In cases of high percentage spectral overlap (99% or higher), these new methods can be used to precede decision feedback or decision-directed equalizers at the receiver for further reduction in BER.

Linear FRESH filtering can be generalized to nonlinear FRESH filtering for signals exhibiting higher-order cyclostationarity. Examples of such signals are bandwidth-efficient digital communication signals such as duobinary PAM, CPFSK, and filtered digital QAM, as well as some low-probability-of-intercept signals. A special type of nonlinear FRESH filtering called linear-plus-cubic (LPC) FRESH filtering is useful for signals with fourth-order cyclostationarity. The most recent development in LPC FRESH filtering is given in [6] and [7].

References

[1] W. A. Gardner, "Cyclic Wiener Filtering: Theory and Method," *IEEE Trans. on Communications*, Vol. 41, No. 1, January 1993, pp. 151–163.

[2] W. A. Brown, "On the theory of cyclostationary signals," Ph. D. dissertation, Dep. Elec. Eng. Comput. Sci., Univ. California, Davis, CA, 1987.

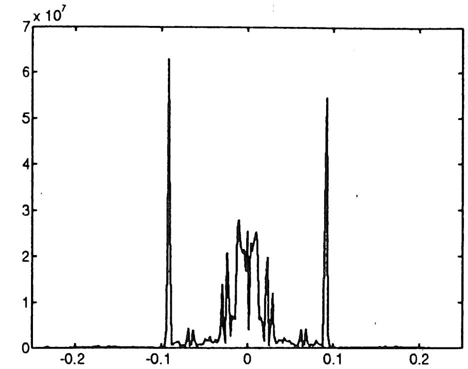
[3] W. A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory.* Englewood Cliffs, NJ: Prentice-Hall, 1987.

[4] W. A. Gardner and W. A. Brown, "Frequency-shift filtering theory for adaptive co-channel interference removal," *Proc. Twenty-Third Annu. Asilomar Conf. Signals, Sys., Comput.,* Pacific Grove, CA, Oct. 30–Nov. 1, 1989, pp. 562–567.

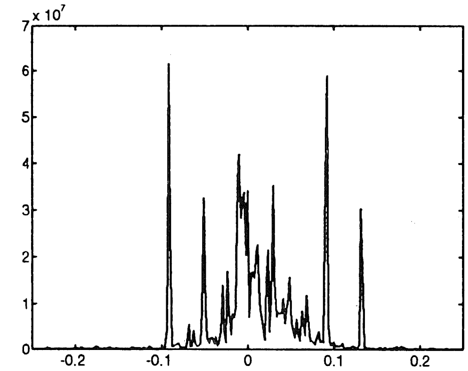
[5] J. R. Treichler and B. G. Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. on Acous., Speech, and Sig. Proc.*, Vol. ASSP-31, No. 2, April 1983.

[6] C. M. Spooner and W. A. Gardner, "Cubic Frequency-Shift Filtering for Cochannel Interference Removal," in *Proceedings of the 1993 International Symposium on Nonlinear Theory and Its Applications*, December 5–9, Honolulu, Hawaii.

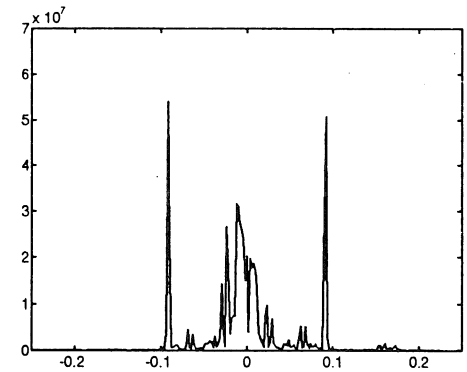
[7] W. A. Gardner, C. M. Spooner and G. K. Yeung, "Frequency-Shift Filtering for Cochannel Signal Separation," in *Proceedings of CRASP Annual Conference: "Cochannel Demodulation" 1994*, June 2, Fort Meade, Maryland.



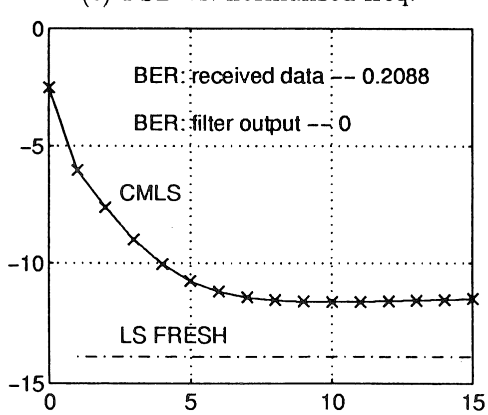
(a) PSD vs. normalized freq.



(b) PSD vs. normalized freq.

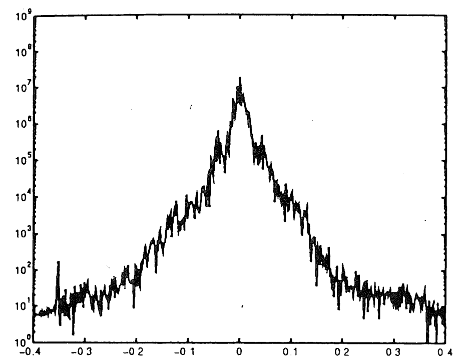


(c) PSD vs. normalized freq.

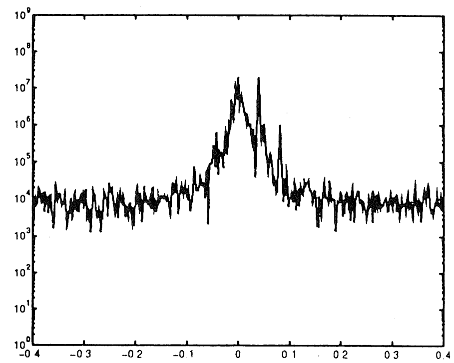


(d) NMSEs (dB) vs. no. of iterations

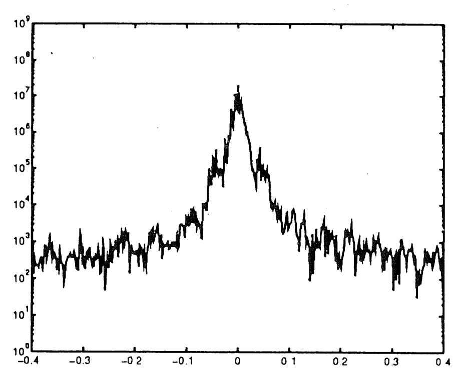
Figure 6: The CMLS method on CPFSK signal: a) PSD of SOI, b) PSD of received data, c) PSD of filter output, d) BERs and NMSEs



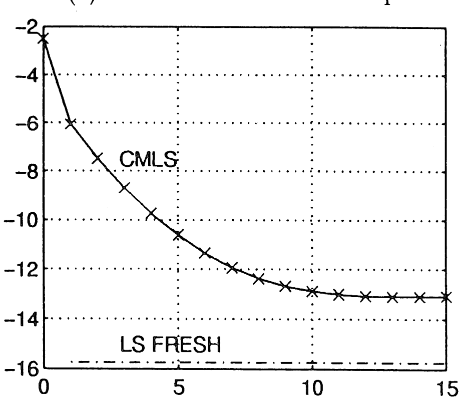
(a) PSD vs. normalized freq.



(b) PSD vs. normalized freq.

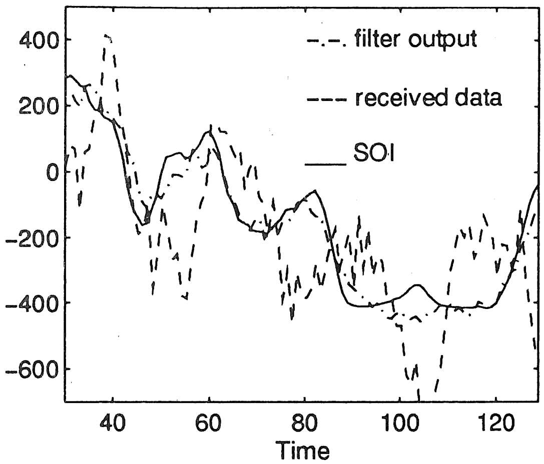


(c) PSD vs. normalized freq.

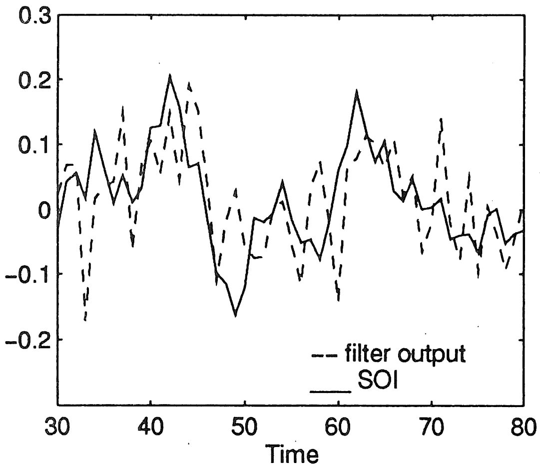


(d) NMSEs (dB) vs. no. of iterations

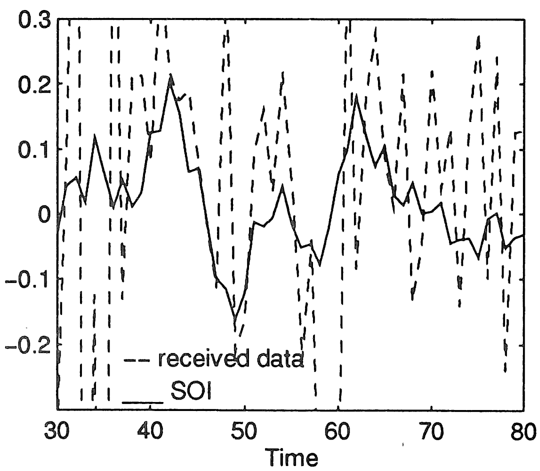
Figure 7: The CMLS method on FM voice signal: a) PSD of SOI, b) PSD of received data, c) PSD of filter output, d) NMSEs



(a) Time waveforms



(b) Demodulated time waveforms



(c) Demodulated time waveforms

Figure 8: The CMLS method on FM voice signal: a) Real time waveforms of $s(t)$, $r(t)$, and $\hat{s}(t)$, b) Demodulated waveforms of $s(t)$ and $\hat{s}(t)$, c) Demodulated waveforms of $s(t)$ and $r(t)$.