Transactions Letters

A New Method of Channel Identification

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Abstract—**A new method of channel identification is proposed. The method exploits the spectral correlation properties of pulse- and carrier-modulated signals to identify channels in the presence of arbitrary noise and nearly arbitrary interference. Although a pilot or training signal is required, no replica of the transmitted pilot/training signal is needed at the receiver. The price paid for this simplicity and the tolerance to extreme channel corruption from noise or interference is that the method is slow. That is, relatively long averaging times are needed for measurement of the spectral correlation of the received signal.**

I. Introduction

A LONG standing problem in data transmission is that of equalizing channel distortion which, in essence, requires identifying the transfer function of the channel. The difficulty of this problem stems from the fact that the input to the unknown channel—the transmitted signal—is not available at the channel output—the receiver—unless special provisions are made for transmission of known training signals or pilot signals. Many techniques for adaptive equalization have been developed, especially for digital data. After an initial phase of equalization has been performed, the problem of fine tuning and tracking has many viable solutions. A particularly successful example for digital data is the decision-directed technique (cf. [1]). The purpose of this paper is to present a new technique for start-up or periodic update or continuous channel identification that is believed to be different from all previously proposed methods and that can use analog as well as digital modulation. The new technique can be based on either 1) initial or periodic replacement of the message signal with a random training signal, or 2) superposition of a random pilot signal on top of the message signal; but the technique is novel in that it does not require full knowledge at the receiver of the particular training or pilot signal that is transmitted. It requires only knowledge of the transmitted pulse shape and transmission of the training or pilot data at a substantially reduced rate, relative to the maximum rate for the channel. (When the message signal is replaced with the training signal, this can be reinterpreted as transmission of the zero symbol repeatedly between isolated random data samples, or it can be reinterpreted as the transmission of isolated pulses with random amplitudes; however, the nonoverlapping channel responses to these isolated pulses cannot be superimposed and averaged to remove channel noise and interference—and thereby identify the channel by deconvolving this pulse response with the known transmitted pulse—since their amplitudes fluctuate randomly with zero mean from one pulse to the next.) Furthermore, the new method does not require pulse synchronization at the receiver.

Although the new technique has the disadvantage of requiring a substantially longer time to accomplish channel identification and, therefore, equalization, than that required by the fastest known methods [1] (and this is unacceptable in many applications), it can be useful when the speed of identification is not crucial and/or there is severe noise or interference present.

The new method exploits the fact that spectral components of PAM signals that are separated in frequency by the pulse rate are completely correlated and the complex correlation value depends on the magnitudes and phases of the spectral components. The concept of spectral correlation is briefly reviewed in Section II and the spectral correlation function for PAM and PAM on AM signals is presented. In Section III, it is shown how the channel transfer function magnitude and phase can be determined from spectral correlation measurements on the received signal, and this is applied in Section IV to PAM signals for baseband channel identification and to PAM on AM signals for passband channel identification. In Section V, a simple FFT-based method for measurement of spectral correlation is presented and some implementation issues are discussed. The method is further illustrated for nonpulsed signals in Section VI.

II. Spectral Correlation

The spectral correlation characteristics of a random signal $s(t)$ can be determined as follows. We pass the signal $s(t)$ through two positive-frequency bandpass filters with center frequencies $f + \alpha/2$ and $f - \alpha/2$ and bandwidths both equal to Δ to obtain the two bandpass signals $s_+(t)$ and $s_-(t)$, respectively. We then downconvert these signals to baseband by multiplication with $\exp(-i2\pi[f + \alpha/2]t)$ and $\exp(-i2\pi[f - \alpha/2]t)$, respectively. Finally, we crosscorrelate these two baseband signals by multiplying the first by the complex conjugate of the second and then time averaging the product over an interval of length T.

For signals that persist indefinitely, we can obtain an idealized measure of the spectral correlation density for spectral components at the two frequencies $f + \alpha/2$ and $f - \alpha/2$ by

Paper approved by the Editor for Modulation Theory and Nonlinear Channels of the IEEE Communications Society. Manuscript received March 27, 1989; revised July 3, 1990. This work was supported by the National Science Foundation under Grant MIP-88-12902.

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IEEE Log Number 9144885.

0090-6778/91/0600-0813$01.00 © 1991 IEEE

making the midband gains of the two filters equal to $1/\sqrt{\Delta}$ and taking the limit as the averaging time T approaches infinity and then taking the limit as the bandwidth Δ approaches zero. When the separation α between the two frequencies is zero, this yields the spectral density of average power $S_s(f)$, which by the Wiener–Khinchin relation is equal to the Fourier transform of the idealized autocorrelation $R_s(\tau)$ of the signal $s(t)$,

$$S_s(f) = \int_{-\infty}^{\infty} R_s(\tau) \exp(-i2\pi f\tau)\, d\tau$$
$$R_s(\tau) = \langle s(t+\tau/2)s^*(t-\tau/2)\rangle$$

where $\langle\cdot\rangle$ denotes the limit of the average over an interval in t of length T as T approaches infinity. For $\alpha \neq 0$, there is the following generalization of the Wiener–Khinchin relation [2]:

$$S_s^\alpha(f) = \int_{-\infty}^{\infty} R_s^\alpha(\tau) \exp(-i2\pi f\tau)\, d\tau$$
$$R_s^\alpha(\tau) = \langle s(t+\tau/2)s^*(t-\tau/2)\exp(-i2\pi\alpha t)\rangle$$

where $S_s^\alpha(f)$ is the spectral correlation density and $R_s^\alpha(\tau)$ is called the *cyclic autocorrelation.* It can be seen that $R_s^\alpha(\tau)$ is nonzero for specific values of α and τ only if the lag-product waveform $s(t+\tau/2)s^*(t-\tau/2)$ contains a finite-strength additive sine-wave component (a spectral line) with frequency α. In this case, the signal $s(t)$ is said to *exhibit cyclostationarity* with *cycle frequency* α [2].

For example, all periodically pulsed signals (with bandwidth exceeding half the pulse rate) exhibit cyclostationarity with α equal to the pulse rate for a range of values of τ [2]. It follows from the generalization of the Wiener–Khinchin relation that all pulsed signals exhibit nonzero spectral correlation density for a range of spectral components separated in frequency by α.

In contrast to this example, signals and noises $n(t)$ that are stationary do not exhibit cyclostationarity and, therefore, their spectral correlation functions $S_n^\alpha(f)$ are identically zero for all $\alpha \neq 0$.

Let us consider three examples. The first example is the *real* PAM signal

$$s(t) = \sum_{n=-\infty}^{\infty} a_n q(t - nT_0 - t_0)$$

where $\{a_n\}$ is a stationary random sequence of pulse-modulating parameters (either analog or digital). It is shown in [2] that the spectral correlation function for this PAM signal is given by

$$S_s^\alpha(f) = \frac{1}{T_0} Q(f+\alpha/2)Q^*(f-\alpha/2) \cdot \tilde{S}_a(f+\alpha/2)\exp(-i2\pi\alpha t_0) \quad (1)$$

for α equal to all integer multiples of the pulse rate $1/T_0$, and $S_s^\alpha(f)$ is zero otherwise. In this expression, $Q(f)$ is the pulse transform

$$Q(f) = \int_{-\infty}^{\infty} q(t)\exp(-i2\pi ft)\, dt$$

$Q^*(f)$ is its conjugate, and $\tilde{S}_a(f)$ is the power spectral density of the stationary random sequence a_n

$$\tilde{S}_a(f) = \sum_{k=-\infty}^{\infty} \tilde{R}_a(k)\exp(-i2\pi kT_0 f)$$
$$\tilde{R}_a(k) = \langle a_{k+n}a_n\rangle$$

where $\langle\cdot\rangle$ here denotes infinite descrete-time average over n.

If the data sequence $\{a_n\}$ is independent and identically distributed, then $\tilde{S}_a(f+\alpha/2) \equiv \langle a^2\rangle$ for all f. As an example, if $s(t)$ has excess bandwidth $\eta 100\%$, then

$$Q(f) \neq 0 \quad \text{for} \quad |f| \leq (\eta+1)/2T_0$$
$$Q(f) = 0 \quad \text{for} \quad |f| > (\eta+1)/2T_0$$

and, therefore,

$$S_s^\alpha(f) \neq 0 \quad \text{for} \quad |f| \leq (\eta+1-|k|)/2T_0$$
$$S_s^\alpha(f) = 0 \quad \text{for} \quad |f| > (\eta+1-|k|)/2T_0$$

for $\alpha = k/T_0$. Thus, the spectral correlation function is nonzero only for $|k| \leq (\eta+1)$.

The second example is the PAM on AM signal (e.g., ASK or BPSK)

$$s(t) = \sum_{n=-\infty}^{\infty} a_n q(t - nT_0 - t_0)\cos(2\pi f_0 t + \phi_0)$$

with independent identically distributed data $\{a_n\}$. If $2f_0T_0 \neq$ integer, then the spectral correlation function is given by [2]

$$S_s^\alpha(f) = \frac{1}{4T_0}\langle a^2\rangle Q(f - f_0 + \alpha/2)Q^*(f + f_0 - \alpha/2) \cdot \exp(-i2\pi[\alpha - 2f_0]t_0 - 2\phi_0) \quad (2)$$

for $\alpha = 2f_0 + k/T_0$ for all integers k.

The third example is the stacked carrier AM signal

$$s(t) = \sum_{k=-N}^{N} a(t)\cos[2\pi(f_0 + k/2T_0)(t - t_0)]$$

where $a(t)$ is a stationary process and $1/2T_0$ is any desired frequency increment. The spectral correlation function for this signal is given by [2]

$$S_s^\alpha(f) = \frac{1}{4}\sum_{j=-(N-|k|)}^{N-|k|} S_a(f + j/2T_0)\exp(-i4\pi\nu t_0) \quad (3)$$

where $\alpha = 2f_0 + k/T_0$ for $-N \leq k \leq N$.

III. Derivation of the Channel Identification Formula

We consider the situation where the transmitted signal $x(t)$ consists of a message signal $m(t)$ on which a pilot signal $p(t)$ is superimposed

$$x(t) = p(t) + m(t).$$

(If a pilot training signal $p(t)$ is used *in place of* the message $m(t)$ during start-up or periodic update identification, then we just let $m(t) = 0$.) The received signal $r(t)$ consists of the distorted transmitted signal plus additive noise and possibly interfering signals $n(t)$

$$r(t) = h(t) \otimes x(t) + n(t)$$

where $h(t)$ is the impulse-response function for the channel and $\otimes$ denotes convolution.

Since the spectral components at frequencies $f \pm \alpha/2$ passing through a real channel are scaled by the channel transfer function $H(f \pm \alpha/2)$ at these two frequencies, then the spectral correlation function at the channel output is given by [2]

$$S_r^\alpha(f) = H(f+\alpha/2)H^*(f-\alpha/2) \left[S_p^\alpha(f) + S_m^\alpha(f)\right] + S_n^\alpha(f). \tag{4}$$

As long as the message $m(t)$ and noise and interference $n(t)$ do not exhibit spectral correlation at the same frequency separation α as for the pilot/training signal $p(t)$, then $S_m^\alpha(f) \equiv S_n^\alpha(f) \equiv 0$ and this equation reduces to

$$S_r^\alpha(f) = H(f+\alpha/2)H^*(f-\alpha/2)S_p^\alpha(f),$$

which can be solved for $H(f)$. That is, since for a real channel [real $h(t)$] we have $H(-\nu) = H^*(\nu)$, then

$$H(\nu) = \left[\frac{S_r^{2\nu}(0)}{S_p^{2\nu}(0)}\right]^{1/2}. \tag{5}$$

It follows from (5) that the channel transfer function $H(\nu)$ can be identified at frequencies ν for which the spectral correlation function $S_p^{2\nu}(f)$ of the transmitted pilot/training signal evaluated at $f = 0$ is nonzero (assuming that $S_m^{2\nu}(0) = S_n^{2\nu}(0) = 0$). This requires knowledge of the statistical parameter $S_p^{2\nu}(0)$ of the transmitted pilot/training signal and measurement of the corresponding statistical parameter $S_r^{2\nu}(0)$ of the received signal.

The result (5) should be contrasted with the conventional result (which follows from (4) with $\alpha = 0$)

$$|H(\nu)| = \left[\frac{S_r(\nu) - S_n(\nu) - S_m(\nu)}{S_p(\nu)}\right]^{1/2} \tag{6}$$

based on the power spectral densities of the transmitted and received signals. From (6), only the magnitude of the transfer function can be identified, and this requires that there be no channel noise or interference or message or that the corresponding power spectral densities $S_n(\nu)$ and $S_m(\nu)$ be known. The alternative conventional result

$$H(\nu) = \frac{S_{rp}(\nu) - S_{rn}(\nu) - S_{rm}(\nu)}{S_p(\nu)} \tag{7}$$

can be used to identify both the magnitude and phase of the transfer function, provided only that the channel noise and interference and message are orthogonal to the pilot/training signal ($S_{rm}(\nu) \equiv S_{rn}(\nu) \equiv 0$). However, measurement of the cross-spectral density $S_{rp}(\nu)$ requires access to the pilot/training signal at the receiver. This requirement is circumvented with (5).

IV. Channel Identification Using Pulsed Signals

If the pilot signal $p(t)$ is PAM, then it follows from (1) that

$$S_p^{2\nu}(0) = \frac{\langle a^2\rangle}{T_0}Q^2(\nu)\exp(-i4\pi\nu t_0)$$

for $\nu = k/2T_0$. Therefore, (5) yields

$$H(\nu)\exp(-i2\pi\nu) = \frac{\left[S_r^{2\nu}(0)\right]^{1/2}}{(\langle a^2\rangle/T_0)^{1/2}|Q(\nu)|}, \quad \nu = k/2T_0.$$

Thus, if the data rate $1/T_0$ for the pilot/training signal is $1/N$ times the (positive-frequency) channel bandwidth B, then we can identify the channel magnitude and phase functions for a baseband channel (to within the linear term $4\pi\nu t_0$ corresponding to the unknown pulse timing) at discrete frequencies $\nu = \alpha/2$ separated by $B/2N$ by measuring the spectral correlation function $S_r^\alpha(0)$ for the received signal and normalizing its square root by the product of the pulse transform and magnitude $|Q(\alpha/2)|$ $(\langle a^2\rangle T_0)^{1/2}$. If the pilot is superimposed on (rather than being a replacement for) the message signal, and the data rate for the message is B, then the assumption $S_m^{2\nu}(f) \equiv 0$ holds for $\nu = kB/2N$ for all k except integer multiples of N. Thus, we can identify the channel in this case at all the discrete frequency points separated by $B/2N$ except those at both end points, $\nu = 0$ and $\nu = B$, and the midpoint $\nu = B/2$. For N sufficiently large, these missing points should present no problem.

For a passband channel, if the pilot signal is PAM on AM, then it follows from (2) and (5) that

$$H(f_0 + k/2T_0)\exp(-i2\pi[kt_0/2T_0 - \phi_0]) = \frac{\left[S_r^{2f_0+k/T_0}(0)\right]^{1/2}}{2(\langle a^2\rangle/T_0)^{1/2}Q(k/2T_0)}. \tag{8}$$

Therefore, except for a linear phase term, the channel can be identified at discrete frequencies centered about f_0, which can be taken to be the center frequency of the channel.

V. Implementation

The spectral correlation function used to identify the channel can be computed by simply frequency-smoothing the conjugate product of shifted FFT's of the *real* data

$$\hat{S}_r^\alpha(f) = \frac{1}{MT}\sum_{m=1}^{M} X_T(f + m/T + \alpha/2) \cdot X_T^*(f + m/T - \alpha/2),$$

where

$$X_T(f) \triangleq \sum_{k=0}^{K-1} r(kT/K)\exp(-i2\pi kfT/K).$$

(For $\alpha = 0$, this reduces to the conventional frequency-smoothed periodogram.) Thus, the measurement required for channel identification is

$$\hat{S}_r^{2\nu}(0) = \frac{1}{MT}\sum_{m=1}^{M} X_T(\nu + m/T)X_T(\nu - m/T).$$

The amount M of smoothing determines the reliability of the estimate. Since the spectral resolution width of the estimate is M/T [2] and this need not be much smaller than the frequency sampling increment from Section IV, $1/2T_0 = B/2N$, then we can use $M/T = B/4N$, which results in $M = BT/4N$ for baseband channel identification. The variance of the estimate is inversely proportional to M [2]; thus, the data segment length T must be much larger than $4N/B$. For $M = 30$ and $N = 30$, this requires about 7000 Nyquist-rate samples, which represents 1 s for a voice channel. Of course, when strong interference is present, more data will be needed in order for that inference to adequately decorrelate in the measurement $\hat{S}_r^{2\nu}(0)$ (as reflected in the assumption in Section III that $S_m^{\alpha}(f) \equiv S_n^{\alpha}(f) \equiv 0$) [2]. For example, if the pilot $p(t)$ is superimposed on the message $m(t)$ and the amplitude levels of $p(t)$ and $m(t)$ are the same, then the power spectral density level of $p(t)$ is a factor of N smaller than that of $m(t)$. Since the variance of the estimate is proportional to the square of the power spectral density [2], then T must be a factor of N^2 larger than that needed when $p(t)$ is transmitted alone. Thus, whereas 1 s is needed for start-up before message transmission on a voice channel, periodic updating during message transmission would require an integration time of about 15 min! However, if the amplitude level of the relatively sparse pulses of $p(t)$ were increased by a factor of $\sqrt{N} \simeq 5.5$ to bring the power spectral density level of $p(t)$ up to that of $m(t)$, then the integration time would be reduced to about 30 s. But this would increase the required dynamic range of the system.

VI. Channel Identification Using Nonpulsed Signals

The general approach to channel identification based on (5) is not limited to pulsed signals such as the PAM and PAM on AM signals considered in Section IV. For example, it follows from (3) and (5) that for a stacked carrier AM signal, we have

$$H(f_0 + k/2T_0)\exp(-i4\pi[f_0 + k/2T_0]t_0) = \frac{4S_r^{2f_0+k/T_0}(0)}{\sum_{j=-(N-|k|)}^{N-|k|} S_a(j/2T_0)}.$$

Thus, except for a linear phase term, either a baseband or passband channel can be identified at frequencies $\nu = f_0 + k/2T_0$ for $-N \le k \le N$, given knowledge of only the power spectral density $S_a(j/2T_0)$ of the amplitude $a(t)$.

However, whereas the low-rate PAM (or PAM on AM) pilot/training signal can be superimposed on a high-rate PAM message, this stacked carrier signal must be transmitted alone since a superimposed message could not in general be recovered.

VII. Comparison to Channel Identification Using Periodic Signals

Both methods presented in Sections IV and V identify the channel at discrete frequencies across the passband. This same result can be accomplished by transmitting periodic test signals. For example, if the amplitude modulation is removed from the pulses in the PAM signal, then it is simply a periodic pulse train with spectral lines at the harmonics of the pulse rate. Similarly, if the amplitude modulation is removed from the sine-wave carriers in the stacked carrier AM signal, then it is simply a sum of sine waves spread across the passband.

Although the periodic-test-signal method is simpler and requires less averaging time, it does require the transmission of spectral lines. In contrast, the new methods do not require transmission of spectral lines, but rather they regenerate spectral lines at the receiver. That is, it can be seen from the discussion in Section II that measurement of spectral correlation inherently involves regeneration of spectral lines by multiplying the received signal by a delayed version of itself, and then measuring the magnitudes and phases of these regenerated spectral lines.

VIII. Conclusion

A new approach to channel identification is proposed. This approach is based on a novel concept and is therefore theoretically unique. It applies to both baseband and passband channels and can utilize either pulsed or nonpulsed pilot/training signals, and the modulation of the pulsed signals can be either analog or digital. The training signal can be sent in place of a message signal during start-up or periodic up-date, or—if pulsed—it can be continuously superimposed on a pulsed message signal. None of these various methods require pulse synchronization or storage of a known training signal at the receiver. They require only knowledge of the transmitted pulse shape for pulsed signals, or knowledge of the power spectral density of a modulating signal at the transmitter for a nonpulsed signal.

Although it is often said, in the literature on the application of higher order statistics to system identification, that second-order autostatistics contain no phase information, this is not true for cyclostationary signals as demonstrated with the new approach proposed in this paper.

Although many competing techniques based on the use of training signals do not actually identify the channel—they just equalize it directly—some methods for minimizing decision errors for digital data (which is the ultimate goal) do indeed require channel identification. The new approach proposed in this paper can identify the channel even in the presence of severe interference or low SNR.

The primary drawback of the new approach is that it is slow. That is, long averaging times can be required to accomplish the required decorrelation when the pilot signal is superimposed on the message signal or when interfering signals are present, and, when pulsed signals are used, the low pulse rate required of the pilot signal can further exacerbate this drawback. A numerical example is given to illustrate this. This drawback severely limits the practical application of this approach.

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