On "The Optimal Linear Receiving Filter for Digital Transmission Over Nonlinear Channels"

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Abstract —**A recent paper shows that the matched-filter/tapped-delay-line structure is optimum not only for linear pulse-modulated signals and linear channel distortion, but also for nonlinear finite-alphabet pulse-modulation and some nonlinear channel distortion. This has important practical applications. Therefore, its connection with other work reported in the literature is brought to light in this note.**

Index Terms —**Optimum receivers, matched filters, nonlinear channels.**

The optimal linear receiving filter for digital transmission derived in the recent paper [1] is closely related to that derived in [2]. The structure of the filter derived in [1]—a parallel bank of matched filters, each followed by a tapped delay line—is identical to that derived in [2]. However, this structure is shown in [2] to be a special case of a more general structure that can be used for MMSE data-symbol estimation, MMSE signal-waveform estimation, or MMSE estimation of the *a posteriori* probabilities of the data symbols. Also, the solution presented in [1] is not as explicit as that presented in [2], which is expressed directly in terms of symbol correlation, pulse shape, and noise spectrum. The important practical ramifications of less than full dimensionality of the signal set, which is discussed briefly in [1], is treated at length in [3]. In addition, the derivation in [2] accommodates unlimited transmitted-signal pulse-duration and channel memory, whereas that in [1] is restricted to finite duration pulses (infinite excess bandwidth) and finite channel memory.

On the other hand, it is explained in [1] that the signal model adopted (in both [1] and [2]) can be used to model some nonlinear channels, which were not explicitly considered in [2], by expanding the symbol alphabet and reinterpreting the signal pulses. This has important practical applications.

Other work related to [1] includes [4], where the related signal-waveform estimation problem is studied and it is shown that the optimum waveform estimator functions like a regenerative repeater; [5], where an analogous receiver structure is derived for optical digital data transmission (which can be interpreted in terms of a random nonlinear channel); [6], where a novel interpretation delay of the matched-filter tapped-line structure as a means for exploiting the inherent frequency diversity in pulse-modulated signals that results from the spectral correlation that is characteristic of cyclostationary processes, is given, and the value of this for suppression of co-channel interference and distortion due to frequency-selective fading is explained, and where the role of the fractionally spaced equalizer for implementing this is clarified; and [7] and [8], where the functions $G_x^{(n)}(f)$ that arise in the receiving filter design equation [1, (2.8)] are shown to be spectral correlation density functions and are explicitly calculated for many types of communication signals.

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Linear Codes with Covering Radius 2 and Other New Covering Codes

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Abstract —**This work gives infinite families of linear binary codes with covering radius $R = 2$ and minimum distance $d = 3$ and $d = 4$. Using the constructed codes with $d = 3$, $R = 2$, families of covering codes with $R > 2$ are obtained. The parameters of many constructed codes with $R \geq 2$ are better than the parameters of known codes. The parity check matrices of constructed codes with $d = 4$, $R = 2$ are equivalent to complete caps in projective geometry.**

I. Introduction

Covering codes are being extensively studied, see, e.g., [1], [2], [4]–[7], [10]–[12], and [17].

We consider linear binary covering codes.

Let an $[n,k,d]R$ code be a linear binary code of length n, dimension k, minimum distance d and covering radius R, Denote by $t[n,k]$ the smallest covering radius of any linear binary code of length n and dimension k. Let r be the number of check symbols of a code. Let $\mu_d[n,R]$ denote the density of the covering of binary n-dimensional space by spheres with radius R, whose centers correspond to the $[n,k,d]R$ codewords (cf. [5]). Let

$$\bar{\mu}_d[R] = \liminf_{n \to \infty} \mu_d[n,R]. \tag{1}$$

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