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Interference-Tolerant Time-Difference-of-Arrival Estimation for Modulated Signals

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Interference-Tolerant Time-Difference-of-Arrival Estimation for Modulated Signals

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Abstract—**A new method for estimation of the difference in the times of arrival of a wavefront at two separate sensors is introduced. This new method, called SPECCORR, exploits the spectral correlation property that essentially all modulated signals exhibit to obtain estimates that are highly tolerant to severely corruptive noise and interference. This tolerance of the SPECCORR method is explained theoretically and demonstrated with simulations.**

I. Introduction

The difference in the times of arrival of a wavefront at two separate sensors, such as a pair of radio antennas, can be used to estimate the direction of arrival of the wavefront. If the time-difference-of-arrival (TDOA) is measured with a pair of antennas at two different locations (e.g., at two different times from a moving platform), then the distance to the source of the wavefront can also be estimated. A standard approach to estimating TDOA is to use as an estimate the lag value at which the cross-correlation function for the signals from the two sensors reaches its maximum value.

There are also a variety of what are called *generalized cross-correlation* methods. These various methods have been designed to reduce the effects of measurement noise and interfering signals, both of which corrupt the measurements of the signal of interest. Although these methods can be effective for noise that is uncorrelated from one sensor to the other, and for isolated narrow-band interference, they can fail when the measurements are severely corrupted by interfering signals that are comparable to (or greater than) the signal of interest in power spectral density level and bandwidth. This includes one (or more) broad-band interferers as well as multiple closely spaced narrow-band interferers. The purpose of this paper is to introduce an entirely new approach that is highly tolerant to severe corruption by noise and interference. This new approach is applicable as long as the signal of interest exhibits the property of *cyclostationarity*, that is, as long as the lag product of the signal exhibits spectral lines. Most modulated signals used in communications, radar, and telemetry do, in fact, exhibit cyclostationarity [1]. The improvements in performance obtained with the new approach can be attributed to the exploitation of the *spectral self-coherence* property that is characteristic of signals that exhibit cyclostationarity.

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II. Background

A. TDOA Estimation

The standard cross-correlation approach to TDOA estimation is based on the facts that 1) the time-average cross correlation

$$R_{yz}(\tau) \triangleq \langle y(t+\tau) z(t) \rangle \tag{1}$$

of the two signals from a single wavefront

$$y(t) = x(t - \tau_o) \quad \text{and} \quad z(t) = x(t) \tag{2}$$

received by a pair of sensors is equal to the translated time-average autocorrelation

$$R_{yz}(\tau) = R_x(\tau - \tau_o),$$

and 2) the autocorrelation

$$R_x(\tau) \triangleq \langle x(t+\tau) x(t) \rangle \tag{3}$$

peaks (reaches its absolute maximum) at $\tau = 0$ for all signals $x(t)$ (and only at $\tau = 0$ for nonperiodic signals) [1]. Consequently, the cross correlation $R_{yz}(\tau)$ peaks at the TDOA τ_o. This result is exact if $\langle \cdot \rangle$ represents an infinitely long time average. When the averaging time is finite as it is in practice, then the location of the peak is only an estimate of the TDOA.

The cross-correlation approach to TDOA estimation can also be justified by the fact that the value of τ that maximizes $R_{yz}(\tau)$ is equal to the value that minimizes the mean squared error

$$\langle [y(t+\tau) - z(t)]^2 \rangle = R_y(0) + R_z(0) - 2R_{yz}(\tau) \tag{4}$$

for finite as well as infinitely long time averages.

One approach to optimizing the TDOA estimate is to reinterpret the problem as that of system identification.

That is, if we interpret $z(t)$ and $y(t)$ in (2) as the excitation and response of a system consisting of a pure delay, then the transfer function of this system is exp $(-i2\pi f \tau_o)$. It is well known that the transfer function model that best fits a system, in the sense that the time-averaged squared error between the system output and model output (when both are subjected to the same excitation) is minimum, is given by the Wiener filter formula [1]

$$H(f) = \frac{S_{yz}(f)}{S_z(f)}, \tag{5}$$

where $S_{yz}(f)$ and $S_z(f)$ are the cross-spectral density and spectral density obtained by Fourier transformation of $R_{yz}(\tau)$ and $R_z(\tau)$. Thus, the TDOA estimate can be taken to be the value of τ at which the model impulse response

$$h(\tau) = \int_{-\infty}^{\infty} \frac{S_{yz}(f)}{S_z(f)} e^{i2\pi f\tau} \, df \tag{6}$$

peaks.[1] Use of (2) in (6) yields

$$h(\tau) = \delta(\tau - \tau_o), \tag{7}$$

which does indeed peak at the correct value. Of course, when only finite-length records of $y(t)$ and $z(t)$ are available, the ideal spectral densities in (6) must be replaced with estimates, in which case the value of τ at which the peak occurs is only an estimate of the TDOA.

The TDOA estimate provided by an estimated version of (6) can be particularly useful when the signals $y(t)$ and $z(t)$ are corrupted by noise and interference [2]. This is especially so for narrow-band interference, because the peaks in the denominator $S_z(f)$ deemphasize those parts of the numerator that are corrupted most by such interference [assumed to be common to both $y(t)$ and $z(t)$]. In contrast to this, the standard method, which is based on seeking the peak of an estimated version of

$$R_{yz}(\tau) = \int_{-\infty}^{\infty} S_{yz}(f) e^{i2\pi f\tau} \, df, \tag{8}$$

does not provide such deemphasis.

Based on this type of reasoning, the whole class of TDOA estimators of the general form

$$\max_{\tau} g(\tau), \tag{9a}$$

where

$$g(\tau) = \int_{-\infty}^{\infty} S_{yz}(f) W(f) \, df, \tag{9b}$$

in which $W(f)$ is a weighting function intended to reduce the effects of measurement noise and interference, has been proposed and studied in depth [3]–[5]. In most applications, it is required that both $S_{yz}(f)$ and $W(f)$ be obtained directly from the measurements $y(t)$ and $z(t)$. For example, the cross-periodogram (possibly smoothed)

$$S_{yz_T}(f) \triangleq \frac{1}{T} Y_T(f) Z_T^*(f), \tag{10}$$

where

$$Y_T(f) \triangleq \int_0^T y(t) e^{-i2\pi ft} \, dt$$

$$Z_T(f) \triangleq \int_0^T z(t) e^{-i2\pi ft} \, dt, \tag{11}$$

is typically used for the cross-spectral density estimate, and for the case in which $W(f) = 1/S_z(f)$, the smoothed periodogram

$$S_{z_T}(f)_{\Delta f} \triangleq \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{z_T}(\nu) \, d\nu, \tag{12}$$

where

$$S_{z_T}(\nu) \triangleq \frac{1}{T} \left| Z_T(f) \right|^2, \tag{13}$$

is typically used.

An alternative to the method based on (5) can be obtained by identifying the z-input/y-output system with the model $H(f)$ given by (5) and the y-input/z-output inverse system with model given by

$$H'(f) = \frac{S_{zy}(f)}{S_y(f)}, \tag{14}$$

and then forming the geometric mean of $H(f)$ and the reciprocal of $H'(f)$,

$$\begin{aligned} \left\{H(f)[1/H'(f)]\right\}^{1/2} &= \left[\frac{S_{yz}(f)}{S_{yz}^*(f)} \frac{S_y(f)}{S_z(f)}\right]^{1/2} \\ &= \frac{S_{yz}(f)}{\left|S_{yz}(f)\right|} \left[\frac{S_y(f)}{S_z(f)}\right]^{1/2}, \end{aligned} \tag{15}$$

where the fact that $S_{yz}(f) = S_{zy}^*(f)$ has been used. If $S_y(f) = S_z(f)$, which will usually be the case, then by inverse Fourier transformation of (15), we obtain (9) with $W(f) = 1/|S_{yz}(f)|$, which yields

$$g(\tau) = \int_{-\infty}^{\infty} \arg\left\{S_{yz}(f)\right\} e^{i2\pi f\tau} \, d\tau, \tag{16}$$

in which arg $\{\cdot\}$ denotes the phase of the complex quantity within the braces.[2]

[1]This method is sometimes referred to as the ROTH method [3].

[2]This method is sometimes referred to as the PHAT method [3].

If $S_y(f) \neq S_z(f)$ because of differing interferences or measurement noises for $y(t)$ and $z(t)$, then a reasonable alternative to the weight $W(f) = 1/S_z(f)$ is the geometric mean[3] [6]

$$W(f) = [S_z(f) S_y(f)]^{-1/2} \quad (17)$$

This yields the standard cross correlation of the whitened versions of $y(t)$ and $z(t)$ obtained by filtering with transfer functions of $1/\sqrt{S_y(f)}$ and $1/\sqrt{S_z(f)}$, respectively.

In the case where no weighting is used, $W(f) \equiv 1$, (9) reduces to the standard cross-correlation method (8). Consequently, all methods for which $W(f) \not\equiv 1$ are referred to as *generalized cross-correlation methods*.

Although the generalized cross-correlation methods are intuitively attractive, all such methods have been found to perform poorly when the measurements are severely corrupted by interfering signals, that is, signals that are comparable to (or greater than) the signal of interest in power spectral density level and bandwidth. One approach to obtaining improved methods for source location in severely corruptive environments is to increase the number of sensors from two to obtain a sensor array with enhanced directional reception capability (cf. [7]–[10]).

The purpose of this paper is to introduce an entirely new type of cross-correlation method for two-sensor reception that is inherently tolerant to both noise and interference and can, in principle, provide arbitrarily accurate TDOA estimates when sufficiently long measurement records are available. However, the method only applies to signals of interest that exhibit cyclostationarity, which most modulated signals do. Also, the attainable accuracy is, in practice, limited by the coherence time of the cyclostationarity, such as the coherence time of regenerated spectral lines from sine wave carriers and pulse trains, which can be quite long at the transmitter but can be substantially shortened by the transmission channel, for example, by time-varying Doppler effects.[4]

B. Cyclostationarity

A signal $x(t)$ is said to *exhibit cyclostationarity* if the sinusoidally weighted time-average autocorrelation (defined with a symmetric lag of $\pm\tau/2$ for mathematical convenience)

$$R_x^\alpha(\tau) \triangleq \langle x(t+\tau/2)x(t-\tau/2)e^{-i2\pi\alpha t}\rangle \quad (18)$$

is not identically zero for some sine wave frequency α [1], [11], [16]. The function $R_x^\alpha(\tau)$ is called the *cyclic autocorrelation* and the parameter α is called the *cycle frequency*.[5] It can be shown that the Fourier transform

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau)e^{-i2\pi f\tau}\,d\tau \quad (19)$$

is the density of spectral correlation for the pair of frequency components of $x(t)$ with frequencies $f+\alpha/2$ and $f-\alpha/2$. That is,

$$S_x^\alpha(f) = \lim_{T\to\infty}\frac{1}{T}\langle X_T(t, f+\alpha/2)X_T^*(t, f-\alpha/2)\rangle, \quad (20)$$

where

$$X_T(t,f) \triangleq \int_{t-T/2}^{t+T/2} x(u)e^{-i2\pi f u}\,du, \quad (21)$$

and where $\langle\cdot\rangle$ denotes the average over all time t [1], [11], [16]. For $\alpha = 0$, $R_x^\alpha(\tau)$ and $S_x^\alpha(f)$ reduce to the conventional autocorrelation (3) and spectral density of time-averaged power.

If $x(t)$ exhibits cyclostationarity, then $y(t)$ and $z(t)$ in (2) exhibit joint cyclostationarity,

$$\begin{aligned} R_{yz}^\alpha(\tau) &\triangleq \langle y(t+\tau/2)z(t-\tau/2)e^{-i2\pi\alpha t}\rangle \\ &= R_x^\alpha(\tau-\tau_o)e^{-i\pi\alpha\tau_o}, \end{aligned} \quad (22)$$

and, therefore, the density of spectral correlation

$$S_{yz}^\alpha(f) \triangleq \lim_{T\to\infty}\frac{1}{T}\langle Y_T(t, f+\alpha/2)Z_T^*(t, f-\alpha/2)\rangle \quad (23)$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} R_{yz}^\alpha(\tau)e^{-i2\pi f\tau}\,d\tau \\ &= S_x^\alpha(f)e^{-i2\pi(f+\alpha/2)\tau_o} \end{aligned} \quad (24)$$

is not identically zero. For $\alpha = 0$, $R_{yz}^\alpha(\tau)$ and $S_{yz}^\alpha(f)$ reduce to the conventional cross-correlation and cross-spectral density (which is the density of spectral correlation for the pair of frequency components of $y(t)$ and $z(t)$ with frequency f).

It can be shown [1], [11], [16] that if $y(t)$ and $z(t)$ are related by the convolution

$$y(t) = \int_{-\infty}^{\infty} h(t-u)z(u)\,du, \quad (25)$$

then

$$S_{yz}^\alpha(f) = H(f+\alpha/2)S_z^\alpha(f). \quad (26)$$

Therefore, if $z(t)$ exhibits cyclostationarity with cycle frequency α, then $S_z^\alpha(f) \not\equiv 0$, and (26) yields

$$H(f) = \frac{S_{yz}^\alpha(f-\alpha/2)}{S_z^\alpha(f-\alpha/2)} \quad (27)$$

for all f for which the denominator is nonzero. Observe that for $\alpha = 0$, (27) reduces to the Wiener filter formula

[3]This method is sometimes referred to as the SCOT method [3].

[4]The effective coherence time can be increased by using spectral-line tracking techniques such as phase-lock loops.

[5]Although (18) looks quite similar to the *Woodward radar ambiguity function*, which can be applied to any signal over a finite interval, the average over *all* time in (18) will be zero unless the lag product of the signal exhibits a spectral line with frequency α. (See the Appendix and [16] for more discussion.)

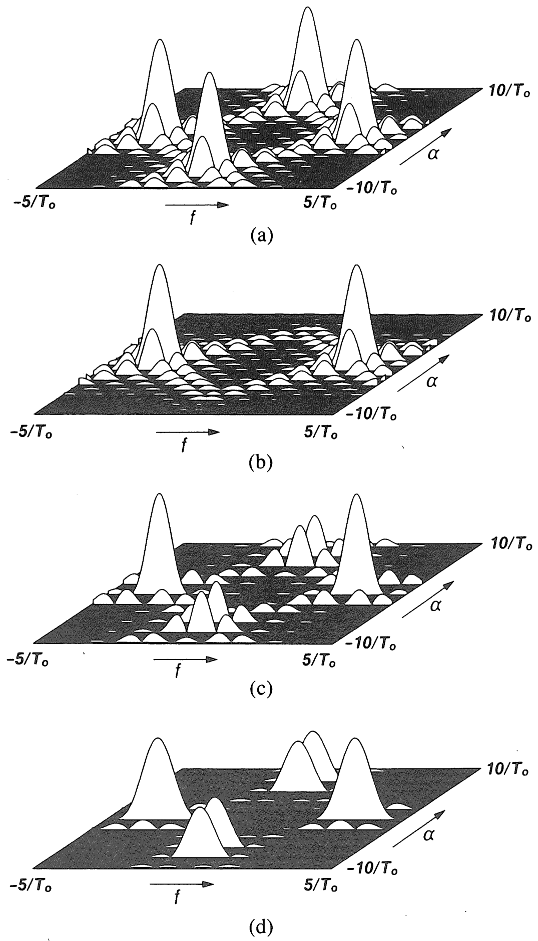


Fig. 1. Graphs of calculated spectral correlation magnitudes. (a) BPSK. (b) QPSK. (c) SQPSK. (d) MSK.

(5).

Essentially all modulated signals exhibit cyclostationarity. This includes analog amplitude, phase, and frequency modulation, digital amplitude-, phase-, and frequency-shift keying, analog and digital pulse-amplitude, pulse-position, and pulse-width modulation, etc. The spectral correlation characteristics of a wide variety of modulation types are derived in [1], [12], [13], and [16]. As an example, the magnitude of the spectral correlation function for four types of phase-shift keyed signals (BPSK, QPSK, and SQPSK with full-duty-cycle rectangular envelopes, and MSK which as a full-duty-cycle half-cosine envelope) are graphed as the heights of surfaces above the plane with coordinates f and α, in Fig. 1.

III. The SPECCORR Method

On the basis of the formula (27) and by analogy with the TDOA estimation method based on (6) and (10)–(13), the following new SPECCORR (SPECtral CORrelation Ratio) method that exploits the spectral correlation property of the signal of interest, which is assumed to exhibit cyclostationarity with cycle frequency α, was recently proposed in [16]:

$$\max_{\tau} g_\alpha(\tau), \tag{28a}$$

where

$$g_\alpha(\tau) \triangleq \left| \int_{-\infty}^{\infty} \frac{S_{yz_T}^\alpha(f)_{\Delta f}}{S_{z_T}^\alpha(f)_{\Delta f}} e^{i2\pi f\tau}\, df \right| \tag{28b}$$

and

$$S_{yz_T}^\alpha(f)_{\Delta f} \triangleq \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{yz_T}^\alpha(\nu)\, d\nu \tag{29a}$$

$$S_{yz_T}^\alpha(f) \triangleq \frac{1}{T} Y_T(f+\alpha/2) Z_T^*(f-\alpha/2) \tag{29b}$$

$$S_{z_T}^\alpha(f)_{\Delta f} \triangleq \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{z_T}^\alpha(\nu)\, d\nu \tag{30a}$$

$$S_{z_T}^\alpha(f) \triangleq \frac{1}{T} Z_T(f+\alpha/2) Z_T^*(f-\alpha/2). \tag{30b}$$

To see that this method can be expected to be immune to noise and interference, we consider the corrupted measurements

$$y(t) = x(t-\tau_o) + n(t) \tag{31}$$

$$z(t) = x(t) + m(t), \tag{32}$$

where $n(t)$ and $m(t)$ each consist of noise and interference that is assumed to be statistically independent of the signal of interest $x(t)$. It can be shown that

$$S_{yz}^\alpha(f) = S_x^\alpha(f) e^{-i2\pi(f+\alpha/2)\tau_o} + S_{nm}^\alpha(f) \tag{33}$$

and

$$S_z^\alpha(f) = S_x^\alpha(f) + S_m^\alpha(f). \tag{34}$$

Regardless of the extent of spectral overlap of $x(t)$, $n(t)$, and $m(t)$, we have

$$S_{nm}^\alpha(f) \equiv 0 \quad \text{and} \quad S_m^\alpha(f) \equiv 0 \tag{35}$$

as long as $n(t)$ and $m(t)$ do not exhibit cyclostationarity with the particular cycle frequency α [e.g., the baud rate of a phase-shift keyed signal $x(t)$]. In contrast to this, the conventional generalized cross-correlation method is based on (28)–(30) with $\alpha = 0$, and (35) does not hold for $\alpha = 0$ unless $n(t)$ and $m(t)$ are statistically independent of each other (which cannot be if $n(t)$ and $m(t)$ contain interference due to the same interfering signal wavefronts impinging on both sensors). Thus, it is the noise and interference immunity of spectral correlation measurements that render the SPECCORR method immune to such corruption. Of course, we have complete immunity only for infinite averaging time T. However, for finite but sufficiently large T, we do have substantial tolerance to such corruption because then we have

$$\left| S_{nm_T}^\alpha(f)_{\Delta f} \right| \ll \left| S_{x_T}^\alpha(f)_{\Delta f} \right| \quad \text{and} \quad \left| S_{m_T}^\alpha(f)_{\Delta f} \right| \ll \left| S_{x_T}^\alpha(f)_{\Delta f} \right|, \tag{36}$$

assuming that the smoothing product is large, $T\Delta f \gg 1$. The effects of the parameters T and Δf on the bias and variance of spectral correlation estimates are described in detail in [14] and [16].

In practice, the denominator can be quite small (theoretically zero) at some values of f (even though $n(t)$ and $m(t)$ may contain white noise). To avoid numerical problems, the denominator can be modified by replacement with

$$Q[S_{z_T}^{\alpha}(f)_{\Delta f}],$$

where

$$Q[s] \triangleq \begin{cases} s, & |s| \geq s_o, \\ s_o, & |s| < s_o, \end{cases} \tag{37}$$

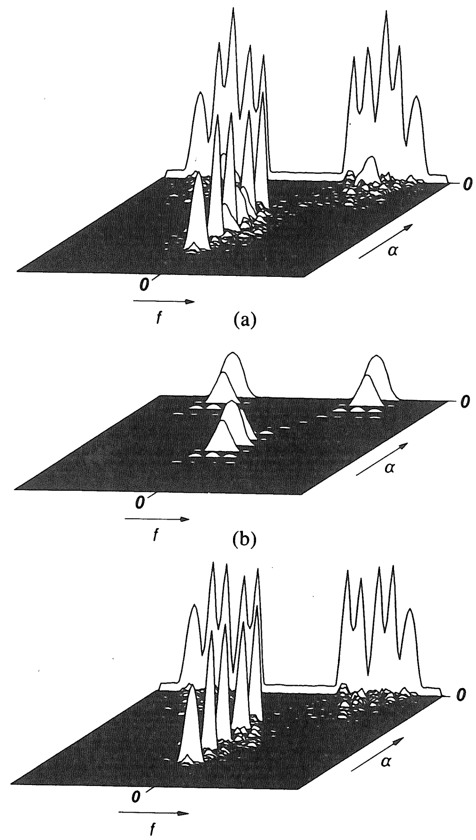
for some appropriate threshold level s_o.

As shown in the next section, the integration time T required to obtain a sufficient level of noise and interference suppression can be quite long. Although segmenting the data and averaging the TDOA estimates obtained from each segment can help in accommodating long integration times in conventional cross-correlation methods [17], this technique is not likely to be useful for the SPECCORR method because, as explained in [17], the effective integration time is much less than the actual integration time when the signal of interest is severely corrupted (e.g., when SNR is low).

IV. Simulations

To demonstrate the tolerance to noise and interference exhibited by the SPECCORR method, we consider a BPSK signal of interest corrupted by interferences for four different cases ranging from broad-band to narrow-band and including multiple and single interferers. In all cases, uncorrelated broad-band noises are also added to the two received signals. The BPSK signal has carrier frequency of $f_o = 0.25/T_s$ and baud rate of $\alpha_o = 0.0625/T_s$. It has full-duty-cycle half-cosine envelope, which results in an approximate bandwidth of $B_o = 0.1875/T_s$. The TDOA for the signal of interest is $\tau_o = 48T_s$. The length of the segment of data processed to obtain the TDOA estimates is 2048 baud intervals. T_s is the time-sampling increment.

Case 1—Multiple AM Interferences: Five AM signals are used as interferers with carrier frequencies of $f_1 = 0.156/T_s, f_2 = 0.203/T_s, f_3 = 0.266/T_s, f_4 = 0.313/T_s, f_5 = 0.375/T_s$, and bandwidths of $B_1 = 0.04/T_s$, $B_2 = 0.05/T_s$, $B_3 = 0.045/T_s$, $B_4 = 0.04/T_s$, $B_5 = 0.08/T_s$, and corresponding TDOA's of $\tau_1 = 28T_s$, $\tau_2 = 68T_s$, $\tau_3 = 78T_s$, $\tau_4 = 38T_s$, $\tau_5 = 58T_s$. The signal-to-interference ratio (SIR) of each AM signal is 0 dB and the SNR is 0 dB. Thus, the total SINR is -8 dB. The magnitude of the measured spectral correlation function for this highly corrupted signal is shown in Fig. 2(a), and that for the uncorrupted signal is shown in Fig. 2(b). (Since $S_z^{-\alpha}(f) = S_z^{\alpha}(f)^*$, only the half plane corresponding to $\alpha < 0$ is shown.) These measurements were obtained using the frequency-smoothed cyclic periodogram (30), except that both time t and frequency f were discretized with sampling increments of $T_s = T/N$ and $F_s = 1/T$, where $N =$ 32 768. The smoothing product is $T\Delta f = 1024$.



(c)

Fig. 2. Graphs of measured (simulated) spectral correlation magnitudes. (a) BPSK signal plus five AM interferers plus white noise. (b) BPSK signal alone. (c) Five AM interferers plus noise alone.

The surface along $\alpha = 0$ is the power spectral density function, from which it is easily seen [by comparing Fig. 2(a) and (b)] that the signal of interest is completely masked. However, for $\alpha \neq 0$, we see that the signal of interest is easily distinguished. Its features at $\alpha = -2f_o$, $\alpha = -2f_o \pm \alpha_o$, and $\alpha = -\alpha_o$ are clearly visible. Thus, the phase information associated with any of these cycle frequencies is easily extracted.

The function $g_\alpha(\tau)$, specified by (28b) with $T\Delta f = 200$, was computed and graphed for $\alpha = \alpha_o$ (with a threshold s_o for the denominator that was exceeded 95 percent of the time). The result is shown in Fig. 3(a). There is an unmistakable peak at the correct value of TDOA, $\tau_o = 48T_s$. But a smaller spurious peak occurs at $\tau \cong 35T_s$. To eliminate the ambiguity caused by the presence of spurious peaks, the processing time T could be increased. Also, it can be helpful to band limit the integrand, $S_{yz_T}^{\alpha_o}(f)_{\Delta f}/S_{z_T}^{\alpha_o}(f)_{\Delta f}$, in (28b) in order to reduce measurement noise outside the band of the signal of interest (when the location of this band is known) before inverse Fourier transformation. Fig. 3(b) shows $g_{\alpha_o}(\tau)$ when the integrand is weighted by a raised-cosine window centered at f_o with width of $3\alpha_o$, and indeed only one peak (at the right value of TDOA) exists, but it occurs within an oscillatory burst. In contrast to these promising results for SPECCORR, we see from the graph for $\alpha = 0$ [which corresponds to the conventional generalized cross-corre-

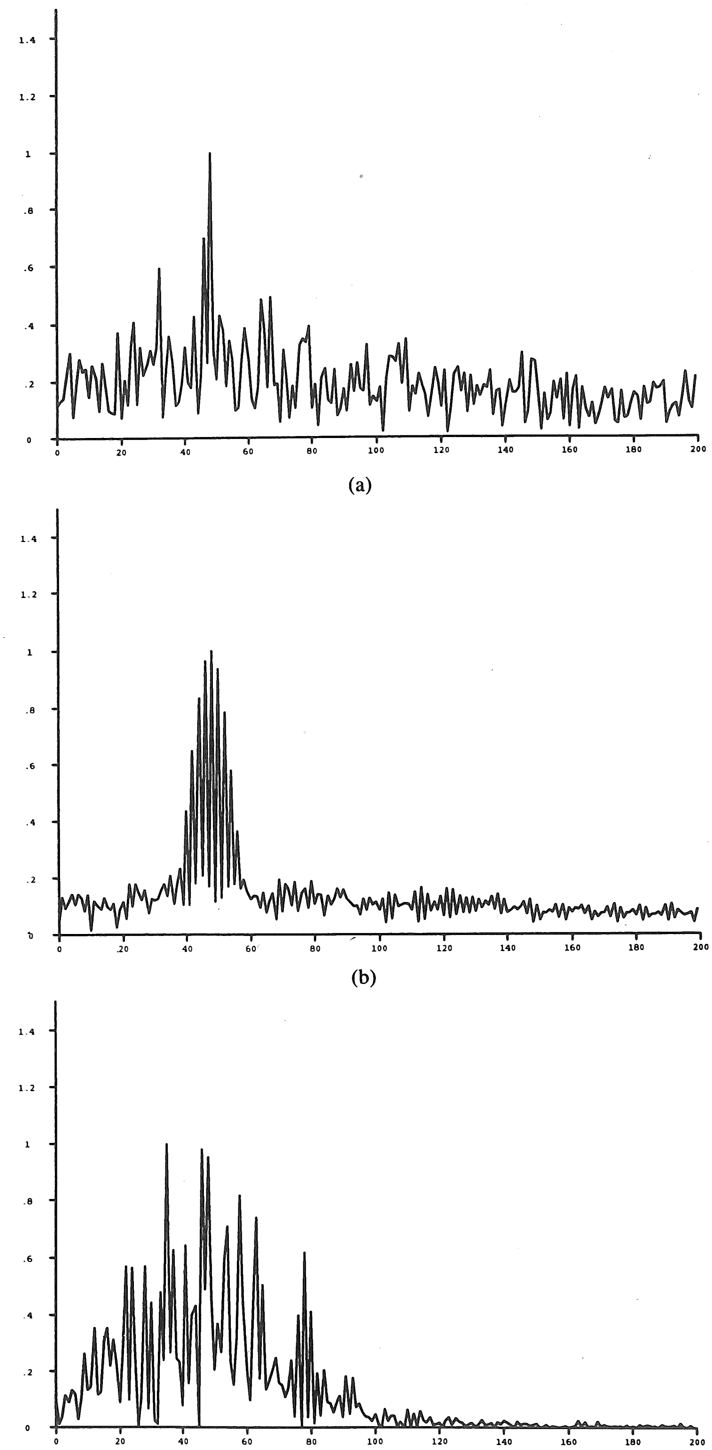


Fig. 3. (a) Graph of generalized cyclic cross-correlation function for SPECCORR method (28)–(30) with α = baud rate for case 1. (b) Graph of generalized cyclic cross-correlation function for SPECCORR method (28)–(30) with α = baud rate for case 1, but with the integrand in (28b) band limited. (c) Graph of generalized cross-correlation function for conventional MMSE-system-identification method (28)–(30) with $\alpha = 0$ for case 1.

lation method based on (6)] in Fig. 3(c) that the peak of interest is only one of many peaks any one of which might be taken as the TDOA estimate. Although the conventional generalized cross-correlation method is somewhat immune to narrow-band interference (see case 4), we see that it fails in this case where multiple moderately narrow-band interferences are present.

Case 2—Wide-Band Interference: The interfering sig-

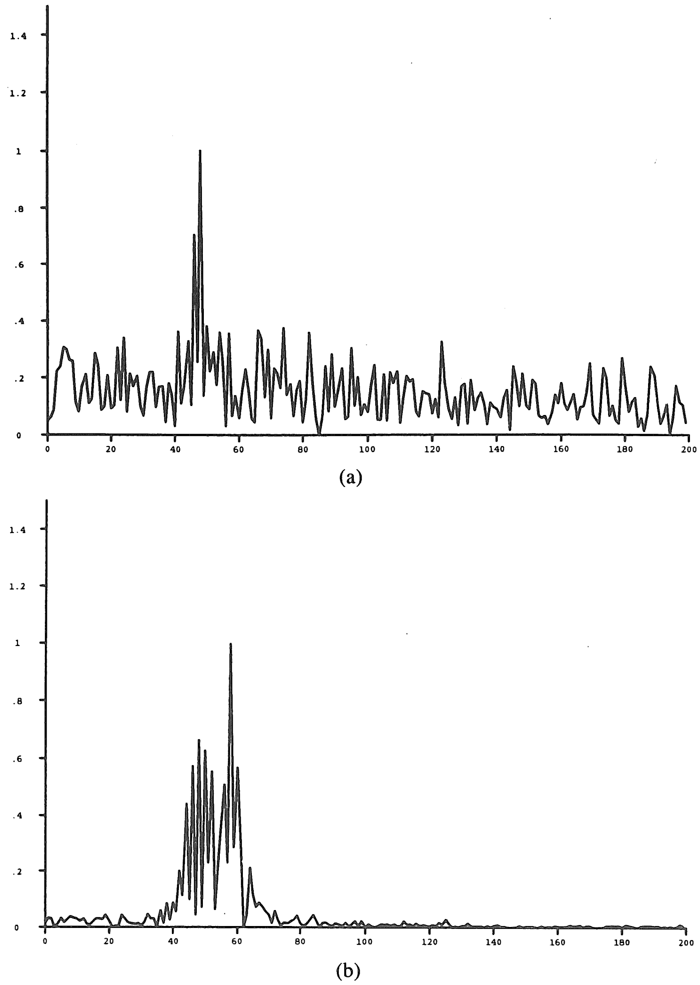


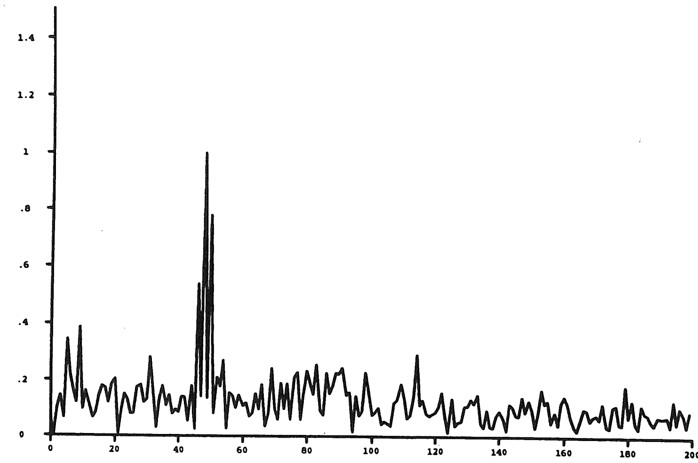
Fig. 4. (a) Graph of generalized cyclic cross-correlation function for SPECCORR method (28)–(30) with α = baud rate for case 2. (b) Graph of generalized cross-correlation function for conventional MMSE-system-identification method (28)–(30) with $\alpha = 0$ for case 2.

nal for this case is a BPSK signal like the signal of interest but with carrier frequency of $f_1 = 0.21875/T_s$, baud rate of $\alpha_1 = 0.10/T_s$, and TDOA of $\tau_1 = 58T_s$. It also has full-duty-cycle half-cosine envelope which yields a bandwidth of $B_1 = 0.3/T_s$. Note that even though the interferer is the same type of signal as the signal of interest, it does not exhibit spectral correlation at $\alpha = \alpha_o$ which is the cycle frequency being exploited by the SPECCORR method. The SIR and SNR are both 0 dB and the total SINR is -3 dB. The function $g_\alpha(\tau)$ for $\alpha = \alpha_o$ is shown in Fig. 4(a). It is clear that the dominant peak occurs at the correct TDOA corresponding to the signal of interest. The conventional generalized cross-correlation method is particularly inferior when a wide-band interference, which spectrally masks the signal of interest, is present. This is a result of the facts that i) the phase of S_{yz} contains a linear term with slope τ_1, which is the TDOA of the interferer, over a wider band than the term with slope τ_o, and ii) the denominator S_z, instead of deemphasizing the interference in S_{yz}, might well deemphasize the signal of interest itself [see (6)]. As a result, $g_o(\tau)$ shown in Fig. 4(b) displays a strong peak at the TDOA $\tau = \tau_1$ of the interferer and a rather weak one at the desired TDOA $\tau = \tau_o$.

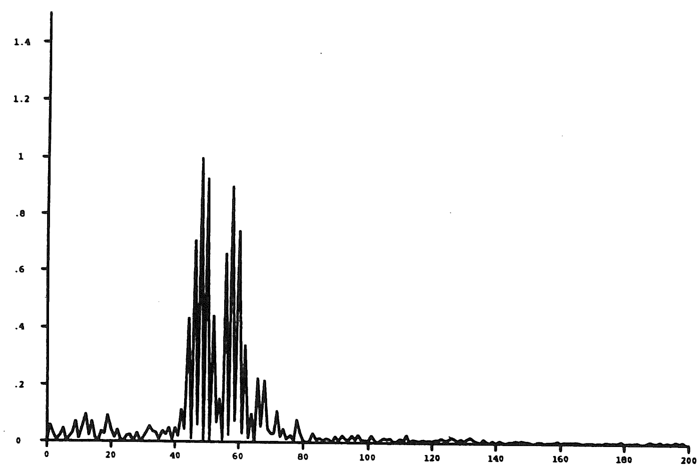
When the bandwidth of the interferer is increased, the performance of the SPECCORR method remains essentially the same, but the performance of the conventional method is degraded.

Case 3—Co-band Interference: The interference for this case is an AM signal which has a TDOA $\tau_1 = 58T_s$ and the same carrier frequency and bandwidth as that of the BPSK signal of interest. Also, the SIR and SNR are both 0 dB and the combined SINR is -3 dB. The functions $g_\alpha(\tau)$ for $\alpha = \alpha_o$ and $\alpha = 0$ are shown in Fig. 5(a) and (b), respectively. It is clear that the conventional generalized cross correlation fails to combat the interference, and either of the two peaks shown in Fig. 5(b) might be taken as the TDOA estimate. On the other hand, the SPECCORR method yields a distinct peak at the correct TDOA as shown in Fig. 5(a).

Case 4—Narrow-Band Interference: In this last case, a BPSK signal with carrier frequency of $f_1 = 0.2/T_s$, baud rate of $\alpha_1 = 0.025/T_s$, bandwidth of $B_1 = 0.075/T_s$, and TDOA of $\tau_1 = 58T_s$ is employed as the interferer. Again, SIR and SNR are both 0 dB to yield a total SINR of -3 dB. The functions $g_\alpha(\tau)$ for $\alpha = \alpha_o$ and $\alpha = 0$ are shown in Fig. 6(a) and (b). Since B_1 is small compared to B_o,



(a)



(b)

Fig. 5. (a) Graph of generalized cyclic cross-correlation function for SPECCORR method (28)–(30) with α = baud rate for case 3. (b) Graph of generalized cross-correlation function for conventional MMSE-system-identification method (28)–(30) with $\alpha = 0$ for case 3.

the conventional generalized cross-correlation method performs relatively well since it is able to notch out or deemphasize the interference and to thereby render a strong peak corresponding to the correct TDOA τ_o. Nevertheless, since the deemphasis leaves the phase of S_{yz} intact, a small peak due to the interferer is still visible. In contrast, no contribution from the interferer is visible for the SPECCORR method. When the bandwidth of the interferer is decreased, the performance of both methods improves and the superiority of the SPECCORR method eventually becomes negligible. □

In practice, with real rather than simulated signals, the performance of the SPECCORR method will be limited by the coherence time of the cyclostationarity which must exceed the total integration time T (see the Appendix). Since $T = 32\,768T_s$ and $\alpha_o = 0.0625/T_s$, then the coherence time must exceed $2048/\alpha_o$. In other words, the percent bandwidth $\Delta\alpha/\alpha_o$ of the baud-rate sine wave in the lag product [see (18)] must be smaller than 0.05 percent, which is reasonable for applications in which time-variant Doppler (or Doppler-tracking error) is not excessive.

To conclude this discussion of the simulation results, we see that in all four cases, the SPECCORR method consistently outperforms the conventional generalized cross-correlation method, and the TDOA-estimation function $g_{\alpha_o}(\tau)$ is essentially the same for all cases. This is a result of the relatively large value used for the integration time T. In fact, (27) and (33)–(35) reveal that as T grows without bound, $g_{\alpha_o}(\tau)$ will become a Dirac delta function centered at $\tau = \tau_o$ for all four cases (assuming that the coherence time of the baud-rate sine wave in the lag product is infinite and the absolute bandwidth of the signal is infinite).

V. Conclusions

A new method for TDOA estimation has recently been introduced in [16]. This method, called SPECCORR, ex-

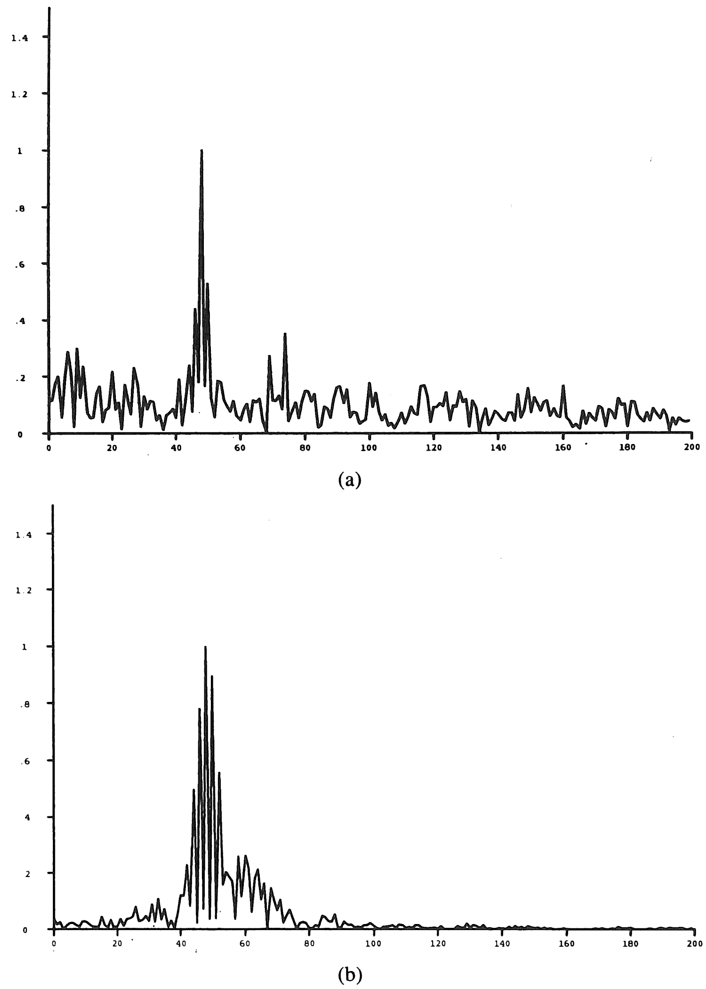


Fig. 6. (a) Graph of generalized cyclic cross-correlation function for SPECCORR method (28)–(30) with α = baud rate for case 4. (b) Graph of generalized cross-correlation function for conventional MMSE-system-identification method (28)–(30) with $\alpha = 0$ for case 4.

ploits the spectral correlation property that essentially all modulated signals exhibit to obtain estimates that are highly tolerant to severely corruptive noise and interference. In all cases considered in this paper, which include wide-band, narrow-band, multiple, and single strong interferers, it is shown that the SPECCORR method outperforms the conventional method. Only for very narrow-band interference do the two methods perform comparably. However, the method does not apply to unmodulated signals or, more generally, signals that do not exhibit cyclostationarity, unless differential Doppler shifts can be exploited (as explained in the Appendix). The method also is limited in performance by the coherence time of cyclostationarity.

In work not reported here, the spectral-self-coherence-exploiting approach has been generalized from two-sensor receivers to sensor arrays for signals with narrow relative bandwidth [16]. A more extensive study of the overall potential and limitations of this new approach to signal source location is currently under way. A competing spectral-self-coherence exploiting algorithm for TDOA estimation, called SPECCOA (SPECtral COherence Alignment), is described in [18], where it is shown to perform comparably to SPECCORR for BPSK signals, but with integration time reduced by a factor of 8 (to 256 band intervals).

Appendix
Relationship to the Ambiguity Function Method

If $x(t)$ does not exhibit cyclostationarity, but the versions of $x(t)$ that occur in the two sensor outputs $y(t)$ and $z(t)$ are Doppler shifted by different amounts due to different relative rates of motion between each sensor and the source of $x(t)$, then the positive-frequency portions (or complex envelopes) of the $x(t)$ components in $y(t)$ and $z(t)$ will be jointly *locally cyclostationary* [16] with cycle frequency equal to the difference in Doppler shifts. The same is true for each interference component in $y(t)$ and $z(t)$. However, the cycle frequency will be different if the differential Doppler shift is different due to the location of the source of interference (and, therefore, the difference in relative rates of motion between the interference source and the two sensors) being different from that of the signal. Consequently, the interference tolerance offered by the SPECCORR method can, in principle, be exploited to some degree even for a stationary signal if differences in differential Doppler shifts between signal and interference are large enough. This is, in fact, an alternative way of viewing the well-known approach of exploiting separation of signal and interferer in ambiguity function measurements due to frequency-difference-of-arrival (FDOA) in order to obtain tolerance to interference for TDOA estimation [15].

However, the separability based on FDOA is not theoretically unlimited as it is for a cyclostationary signal (with infinite coherence time of the cyclostationarity) because Doppler-shifted stationary signals can be only *locally* jointly cyclostationary. This is a result of the fact that the Doppler effect does not shift all positive frequencies by the same amount. The Doppler effect expands or contracts the frequency scale so that frequency f becomes frequency af for all f and some fixed (for time-invariant Doppler) expansion/contraction factor a. Only for narrow relative bandwidth $|f - f_o|/f_o \ll 1$ at a center frequency f_o can this be approximated by a shift in frequency (by the amount $\nu \triangleq (1 - a) f_o$). The actual shifts vary from one end of the band of width B to the other end by the amount $(1 - a)B$. Thus, the range of the differential Doppler shift for a signal $s(t)$ received by a pair of sensors is

$$\Delta\alpha_s = |a_{s_y} - a_{s_z}| B_s. \tag{A1}$$

The nominal differential Doppler shift is

$$\alpha_s = (a_{s_y} - a_{s_z}) f_o, \tag{A2}$$

and this can be considered to be the cycle frequency of joint local cyclostationarity between the signal components $s(t)$ [corresponding to $x(t)$ in (31) and (32)] in $y(t)$ and $z(t)$ only if

$$\Delta\alpha_s \ll B_s, \tag{A3}$$

which is equivalent to

$$|a_{s_y} - a_{s_z}| \ll 1. \tag{A4}$$

Thus, in order to obtain substantial separation between the signal $s(t)$ and the interference $i(t)$ [in $n(t)$ and $m(t)$ in (31) and (32)] by exploiting local cyclostationarity, we require (A4) to be satisfied for the signal, but not for the interference, or if it is satisfied for the interference then the difference $|\alpha_s - \alpha_i|$ in the values of cycle frequencies for the signal and interference must be large enough:

$$|\alpha_s - \alpha_i| > (\Delta\alpha_s + \Delta\alpha_i)/2. \tag{A5}$$

Condition (A5) ensures that the distance along the α axis between the signal and interference features in the cross spectral correlation surface (see Fig. 2(a) which is discussed in Section IV), as well as in the ambiguity surface, exceeds half the sum of their widths in the α direction so that they are indeed distinct features.

As a matter of fact, even modulated signals that are called cyclostationary are only locally cyclostationary because no carrier oscillator or bauding clock is free of phase fluctuations. They all have finite coherence times. Thus, we still require conditions (A3) and (A5) (with $\Delta\alpha$ reinterpreted as the reciprocal of the coherence time) to separate signal from interference on the basis of the cyclostationarity of modulated signals and interferers. But these conditions are easily satisfied in many applications. This is discussed further at the end of Section IV.

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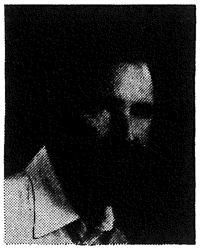

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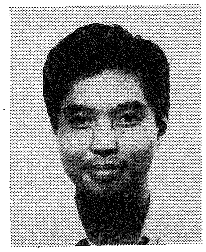

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