L_g and L_s may be regarded as the inductances caused by the lack of calibration in the measurement [6]. The input impedance seen from the gate port of this 10-element circuit with the drain shorted gives the relationship between L_{eff} of 9-element circuit and L_g and L_s. From Miller's theorem [7], L_s may be decomposed into two series impedances at the input (Z_i) and at the output (Z_0). Since $C_d \ll C_{gl}$, L_{eff} of 9-element circuit may be approximated by adding L_g and the imaginary part of the impedance Z_i as

$$L_{eff} = L_g + L_s\left(1 - \frac{g_{ml}\tau}{C_{gl}}\right). \tag{16}$$

From (8), one may find $1/R_0$ is the real part of the drain current excluding the current flowing through C_d when a unit voltage is applied to the drain port with the input shorted. For the 10-element circuit, this current may be accounted by two currents, $1/R_{0l}$ and the additional in-phase current produced by y_{ml} accounting for the frequency dependent term $\omega^2 S_R$ in (15). The additional current is due to the small voltage drop $\omega^2 C_d L_g$ across L_g and $\omega^2 C_s L_s$ across L_s which, in turn, causes the voltage drop $\omega^2(C_s L_s - C_d L_g)$ across C_{gl} ($\omega C_{gl} R_{1l} \ll 1$ is assumed) and it produces the in-phase current with $1/R_{0l}$ as

$$\omega^2 S_R = \omega^2 g_{ml}(C_s L_s - C_d L_g) \tag{17}$$

provided $\omega^2 L_g C_d \ll 1$ and $\omega^2 L_s C_s \ll 1$.

From (16) and (17), one may obtain the correction inductances L_g and L_s. The effect of L_g and L_s may then be subtracted from the measured S parameters and the remaining 8-elements of the 10-element circuit may be obtained from (5) to (12). The newly obtained element values are exactly the same as those of 8-element circuit except R_1 and R_f, which are designated as R_{1l} and R_{fl}. R_{fl} is somewhat changed and R_{1l} is reduced with the same frequency dependence of R_1. Calculated S parameters of this 10-element circuit (circles) closely predict the measured data (dots) over the whole measured frequency range while Minasian's model (triangles) is valid in the lower frequency region.

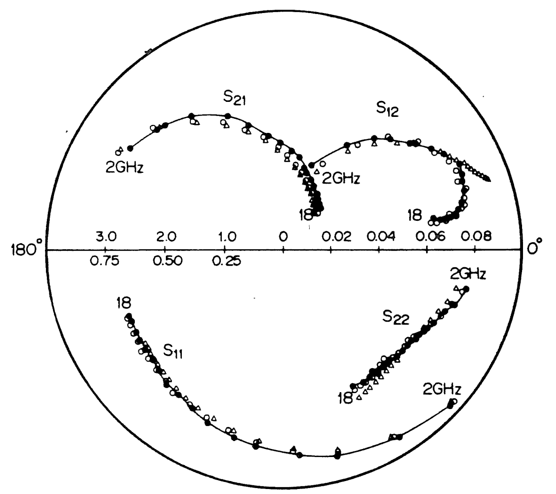

Fig. 4. S-parameters calculated from this 10-element equivalent circuit (○) and from Minasian's (△) with the original measurement data (•). The 10-element circuit values are as follows: R_{1l} = 7.2 ohm, C_{gl} = 0.49 pF, C_d = 39 fF, C_s = 77 fF, R_{fl} = 67 ohm, L_g = 34 pH, L_s = 24 pH, R_{0l} = 460 ohm, g_{ml} = 39 mmho, and τ = 4.0 ps. Minasian's model is as follows: R_1 = 10.2 ohm, C_g = 0.512 pF, C_d = 36 fF, C_s = 72 fF, R_0 = 388 ohm, g_m = 39.6 mmho, and τ = 4.28 ps.

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Simplification of MUSIC and ESPRIT by Exploitation of Cyclostationarity

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It is shown that for the problem of multiple-source location using antenna arrays, the algorithms MUSIC and ESPRIT can be simplified by exploitation of the property called cyclostationarity, which is exhibited by modulated signals. The advantages of the modified algorithms are reduction in the required number of array elements and associated reduction in the SVD computations. The disadvantages are the requirement of either knowing or measuring frequency parameters, such as carrier frequency or baud rate, and the requirement of longer integration time for correlation measurement, as well as the requirement of measuring a different correlation matrix for each signal of interest.

I. Introduction

Eigenstructure methods for direction-of-arrival (DOA) estimation ideally enable the individual DOAs of multiple interfering signals to be determined from estimates of correlation matrices provided that the number m of sensors (or sensor pairs) in a memoryless array exceeds the number of signal sources. The purpose of this letter is to introduce two new techniques, one of which requires only two sensors and the other of which requires only two matched pairs of sensors. This reduction in the required number of sensors is accomplished by exploiting knowledge of frequency parameters of individual sources, such as baud rates or carrier frequencies. One of the techniques presented is a counterpart of the well-known eigenstructure method MUSIC [1], [2], and the other is a counterpart of the more recently proposed eigenstructure method ESPRIT [3], [4]. In MUSIC and ESPRIT, the $m \times m$ correlation matrix of the data from m sensors (two such matrices for ESPRIT) is estimated, and then singular-value-decomposition (SVD) methods are used to estimate and subtract off the additive component of the correlation matrix due to additive noise (assumed to be independent from one sensor to another), and to replace the resultant matrix with an approximant having reduced rank equal to the estimated number of signal sources.[1] Then the direction vectors (in the MUSIC method) or actual DOAs (in the ESPRIT method) are determined from the eigenvalues and eigenvectors (or their orthogonal complement) of the reduced rank matrix. In the two new methods introduced herein, the correlation matrix estimates are replaced with cyclic correlation matrix estimates that reflect the cyclostationarity of the signals, assuming these are bauded and/or carrier modulated signals as they would be in radar and radio communication applications. By selecting the cycle frequency param-

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[1]This rank-reduction step can be circumvented in the ESPRIT method.

eter in these estimates to correspond to the cycle frequency of any one of the signals (e.g., the doubled carrier frequency or the baud rate), the contributions to the cyclic correlation matrix estimate from the other signals (assumed not to possess the same cycle frequency) and any noises (possibly correlated from one sensor to another) converge to zero (ideally) as the integration time used in the estimates grows without bound. As a consequence of this, the estimated $m \times m$ cyclic correlation matrix has approximate rank equal to unity, and the desired direction vector (in the MUSIC-like method) or actual DOA (in the ESPRIT-like method) can be obtained from the largest eigenvalue and corresponding eigenvector (generalized eigenvector for the ESPRIT-like method). Thus there is no need for the number m to be any larger than 2, although any value larger than unity will work.

The disadvantages of these new cyclic correlation (CYCCOR) methods are that they require either prior knowledge or estimation of a cycle frequency for each source of interest, and they require longer integration times in order for signals not of interest and noise to decorrelate in the cyclic correlation estimates. Also, a different cyclic correlation matrix for each signal of interest must be estimated. Methods for the required detection and estimation of cycle frequencies (e.g., baud rates and doubled carrier frequencies) that have been developed for purposes of synchronization in conventional communications and radar systems can be used [7] or more general methods of cyclic spectral analysis for signal detection, classification, and parameter estimation can be applied [8]. Formulas for bias and variance of cyclic correlation and cyclic spectrum estimates provided in [8], [9], can be used to predict the required integration time to be used in the cyclic correlation estimates for specific SNRs and SIRs (signal-to-interference ratios).

In order to be brief, it is assumed in the following that the reader is familiar with the MUSIC and ESPRIT methods described in [1]–[4].

II. Cyclic Correlation

The $m \times m$ CYCCOR matrix for the m vector $\boldsymbol{x}(t)$ of analytic signals of the data from m sensors is defined by

$$\boldsymbol{R}_{xx}^{\alpha}(\tau) \triangleq \langle \boldsymbol{x}(t + \tau/2)\boldsymbol{x}^{\dagger}(t - \tau/2) \exp(-i2\pi\alpha t)\rangle \tag{1}$$

where α is the cycle frequency parameter, τ is the lag parameter, $(\cdot)^{\dagger}$ denotes conjugate transpose, and $\langle \cdot \rangle$ denotes infinite time average. Since the complex sine wave factor can be associated with either data vector in the lag product, we see that the cyclic correlation is the cross correlation between frequency-shifted as well as time-shifted data vectors. For a stationary process, this cross correlation is ideally zero for all $\alpha \neq 0$. But for a cyclostationary process (which essentially all modulated signals are [5], [6]), this cross correlation is nonzero for discrete values of α related to the periodicities of cyclostationarity (which are typically baud rates, doubled carrier frequencies (for real data), spreading code repetition rates, chip rates, hop rates, and their harmonics, sums, and differences).

III. MUSIC-Like CYCCOR Method

If the received data vector of analytic signals for a narrow-band planar sensor array is expressed as

$$\boldsymbol{x}(t) = \boldsymbol{A}s(t) + \boldsymbol{n}(t) \tag{2}$$

where $s(t)$ is any one signal of interest, $\boldsymbol{A}$ is its associated direction vector, and $\boldsymbol{n}(t)$ consists of all other signals and noises present in $\boldsymbol{x}(t)$, and if we choose α to be a cycle frequency of only this one signal[2] $s(t)$, then we have from (2) (see [9])

$$\boldsymbol{R}_{xx}^{\alpha}(\tau) = R_{s}^{\alpha}(\tau)\boldsymbol{A}\boldsymbol{A}^{\dagger}. \tag{3}$$

The matrix (3) is obviously a rank one matrix and the right (or left) eigenvector associated with its one eigenvalue is $\boldsymbol{A}$ (or $\boldsymbol{A}^*$). (Since $R_s^{\alpha}(\tau)$ is, in general, complex, this matrix is neither symmetric nor Hermitian symmetric.) Thus by using a rank-one approximant to an estimate of $\boldsymbol{R}_{xx}^{\alpha}$ obtained from a finite-time average in (1), we can obtain an estimate of the direction vector $\boldsymbol{A}$.

[2]This requires that signals arriving from different directions not share a common cycle frequency. This rules out multipath propagation.

IV. ESPRIT-Like CYCCOR Method

If, corresponding to every sensor represented in (2), there is a matched sensor in the same plane displaced by a distance Δ (in the same direction for every sensor), then the received vector for this matched array can be expressed as

$$\boldsymbol{y}(t) = \boldsymbol{A}\phi s(t) + \boldsymbol{m}(t) \tag{4}$$

where $\boldsymbol{m}(t)$ plays the same roll as $\boldsymbol{n}(t)$ in (2), and ϕ is given by

$$\phi = \exp[i\omega\Delta \sin(\theta)/c] \tag{5}$$

in which ω is the center frequency of the signal $s(t)$, c is the speed of propagation, and θ is the *AOA* of the signal $s(t)$, relative to the displacement direction. As in Section III, we have from (2) and (4) the cyclic cross correlation

$$\boldsymbol{R}_{xy}^{\alpha}(\tau) = R_{s}^{\alpha}(\tau)\phi^*\boldsymbol{A}\boldsymbol{A}^{\dagger}. \tag{6}$$

With the use of (3) and (6) we obtain

$$\boldsymbol{R}_{xx}^{\alpha}(\tau) - \lambda\boldsymbol{R}_{xy}^{\alpha}(\tau) = R_{s}^{\alpha}(\tau)[1 - \lambda\phi^*]\boldsymbol{A}\boldsymbol{A}^{\dagger}. \tag{7}$$

Consequently, $\lambda = 1/\phi^*$ is the generalized eigenvalue of the rank-one pair of matrices $\boldsymbol{R}_{xx}^{\alpha}(\tau)$ and $\boldsymbol{R}_{xy}^{\alpha}(\tau)$. Thus by using the SVD method we can estimate ϕ from estimates of $\boldsymbol{R}_{xx}^{\alpha}(\tau)$ and $\boldsymbol{R}_{xy}^{\alpha}(\tau)$ obtained from finite-time averages. From an estimate of ϕ and knowledge of ω and Δ we can obtain an estimate of the *AOA* using (5).

V. Discussion

It is obvious that for both methods described herein there is no inherent reason for the array size m (which is the dimension of the vector $\boldsymbol{A}$ in (3) and (6)) to be any larger than two, in which case the computational complexity of the SVD required is minimal—in fact, trivial (see Appendix). However, there might be some advantage to usimg $m > 2$ in practice.

The value of the lag parameter τ is important since the required integration time for a given accuracy in the rank-one approximant will usually be minimum when $|R_s^{\alpha}(\tau)|$ is maximum. Unlike the conventional autocorrelation ($\alpha = 0$) which peaks at $\tau = 0$, the cyclic autocorrelation can reach its maximum at $\tau \neq 0$ [9]. For example, for typical PAM signals, the maximum occurs at half the baud interval for α equal to the baud rate; whereas for AM, the maximum (of the cyclic conjugate correlation) occurs at $\tau = 0$ for α equal to twice the carrier frequency for real data or equal to zero for the analytic signal. In some cases it might be advantageous to use in place of the cyclic autocorrelation its Fourier transform, the cyclic spectrum [10].

Although the CYCCOR methods presented here do not require estimation and subtraction of a noise-power matrix, and do not require that the noise be uncorrelated from one sensor to another, they do require a long enough integration time to insure that the noise as well as the interfering-signal components in the cyclic correlation matrix are sufficiently small. This integration time will undoubtedly be longer than that typically required in the MUSIC and ESPRIT methods.[3]

The cyclic correlation defined by (1) is appropriate if a baud rate is to be exploited. However, if a carrier frequency is to be exploited, then the cyclic conjugate correlation, defined by

$$\boldsymbol{R}_{xx^*}^{\alpha}(\tau) \triangleq \langle \boldsymbol{x}(t + \tau/2)\boldsymbol{x}'(t - \tau/2) \exp(-i2\pi\alpha t)\rangle \tag{8}$$

where $(\cdot)'$ denotes transpose (without conjugate), must be used with $\alpha = 0$ and similarly with $\boldsymbol{R}_{xy^*}^{\alpha}(\tau)$ [9]. In this case, the Hermitian matrix $\boldsymbol{A}\boldsymbol{A}^{\dagger}$ in (3), (6), and (7) becomes the symmetric non-Hermitian matrix $\boldsymbol{A}\boldsymbol{A}'$; however, (3), (6), and (7) are still rank-one matrices. Also, $R_s^{\alpha}(\tau)$ in (3), (6), and (7), and ϕ^* in (6) and (7) becomes $R_{ss^*}^{\alpha}(\tau)$ and ϕ, respectively, in this case.

In conclusion, the new CYCCOR methods proposed here have both advantages and disadvantages relative to previous methods. The best method will inevitably depend on the constraints imposed by each particular application. It is hoped that the immunity to noise and interfering signals gained through use of cyclic correlations and longer integration times, the possibility of reducing the number of sensors to only two or four, and the corresponding reduction

[3]Extensive simulations of a related method of AOA estimation based on a generalized cyclic cross correlation function for a two-element array yielded excellent performance but at the expense of a relatively long integration time [10].

in computational complexity will render SVD techniques practically feasible in a broader range of applications.

Appendix

Consider the generalized eigenequation

$$\boldsymbol{R v} - \lambda \boldsymbol{S v} = \boldsymbol{0}$$

where $\boldsymbol{R}$ and $\boldsymbol{S}$ are 2×2 matrices, $\boldsymbol{S}$ is nonsingular, and λ and $\boldsymbol{v}$ are the eigenvalue and right eigenvector. The two values for λ are given by

$$\lambda = \frac{a + d}{2} \pm \left[\frac{a^2 + d^2}{4} - \frac{1}{2}\mathrm{ad} + \mathrm{bc}\right]^{1/2}$$

and the corresponding right eigenvectors (un-normalized) are given by

$$\boldsymbol{v}' = [1 \quad (\lambda - a)/b]$$

where

$$\boldsymbol{S}^{-1}\boldsymbol{R} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

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An On-Line Least-Squares Parameter Estimator with Finite Convergence Time

ROMEO ORTEGA

We present a new on-line parameter estimator with the following features: 1) It reduces to the linear least-squares (LS) estimator after a set of regression vectors that span the full dimension of the parameter space has been processed, and 2) When the regression model is linear in the parameters, the parameter and prediction errors converge to zero in finite time. Both, continuous- and discrete-time versions of the algorithm are given.

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Notation: In this letter we treat parameter estimation for both discrete- and continuous-time systems. When refering to time sequences or continuous-time functions we will denote $(\cdot)_t$ or $(\cdot)(t)$, respectively. The argument is omitted if the relation applies to both cases.

I. Introduction and Problem Formulation

We are concerned with the problem of on-line parameter estimation of regression models of the form

$$y = g(\phi, \theta) \tag{1.1}$$

where $\theta \in \mathbb{R}^n$ is a vector containing the unknown parameters, $\phi: \mathbb{R}_+(Z_+) \to \mathbb{R}^n$, $y: \mathbb{R}_+(Z_+) \to \mathbb{R}$ are time functions (sequences) of measurable signals.

Our problem is to determine a mapping from the data ϕ, y to the model parameters θ to define an on-line estimate $\hat{\theta}$ with suitable properties. A common and natural way to choose the estimate is by minimizing in discrete time

$$\sum_{t=1}^{N} [y_t - g_t(\phi_t, \theta)]^2 + (\theta - \hat{\theta}_{-1})^T P_{-1}^{-1}(\theta - \hat{\theta}_{-1}) \tag{1.2a}$$

or in continuous time

$$\int_0^t \{y(\zeta) - g[\phi(\zeta), \theta]\}^2 \, d\zeta + [\theta - \hat{\theta}(0)]^T P(0)^{-1}[\theta - \hat{\theta}(0)] \tag{1.2b}$$

where $\hat{\theta}_{-1}(\hat{\theta}(0))$ is the initial estimate, and $P_{-1} = P_{-1}^T > 0$ ($P(0) = P(0)^T > 0$) is a measure of confidence in the latter. The procedure is known as the LS solution. In mathematical statistics it is shown that the LS solution has particularly simple statistical properties and enjoys wide popularity in control, signal processing, and prediction theories.

It is well known (e.g., [1], [3]) that when the regression model is linear in the parameters, that is

$$y = \theta^T \phi \tag{1.3}$$

the LS algorithm is given by

$$\hat{\theta}_t = \hat{\theta}_{t-1} + P_t \phi_t e_t \tag{1.4a}$$

$$P_t^{-1} = P_{t-1}^{-1} + \phi_t \phi_t^T \tag{1.5a}$$

$$e_t = y_t - \hat{\theta}_{t-1}^T \phi_t \tag{1.6a}$$

or

$$\dot{\hat{\theta}}(t) = P(t)\,\phi(t)\,e(t) \tag{1.4b}$$

$$\dot{P}(t)^{-1} = \phi(t)\,\phi(t)^T \tag{1.5b}$$

$$e(t) = y(t) - \hat{\theta}(t)^T \phi(t). \tag{1.6b}$$

To establish convergence of the parameter estimates to the true values of the parameters, we study the following equations obtained from (1.3)–(1.6)

$$\tilde{\theta}_t = (I - P_t \phi_t \phi_t^T)\tilde{\theta}_{t-1} \tag{1.7a}$$

or

$$\dot{\tilde{\theta}}(t) = -P(t)\,\phi(t)\,\phi(t)^T\,\tilde{\theta}(t) \tag{1.7b}$$

where

$$\tilde{\theta} = \hat{\theta} - \theta. \tag{1.8}$$

The following facts regarding these equations are well known.

Fact 1.1 [1]: Equation (1.7a) is globally stable if ϕ_t is weakly persistently exciting (PE), i.e., if

$$\rho_1 I \geq \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} \phi_t \phi_t^T \geq \rho_2 I \tag{1.9a}$$

where $\rho_1, \rho_2 > 0$. Furthermore the convergence rate is $1/t$. ∇∇∇

Fact 1.2 [2]: Equation (1.7b) is globally exponentially stable if and only if $\phi(t)$ is PE, i.e., if there exists constants $\rho_1, \rho_2, \delta > 0$ such that