Structurally Constrained Receivers for Signal Detection and Estimation

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Abstract—A general approach to the problem of designing structurally constrained receivers for signal detection and estimation is proposed. The approach is based on the *constrained Bayesian methodology* wherein risk-minimizing inference (or decision) rules are modified (constrained) by replacement of true posterior probabilities with estimated posterior probabilities. The estimators are structurally constrained minimum-mean-squared-error (MMSE) estimators for random posterior probabilities. This methodology is, in essence, an extension and generalization of the well-known linear MMSE estimation methodology. The approach is employed to design linearly constrained coherent receivers for signals in additive and multiplicative noise, and quadratically constrained noncoherent receivers for signals in additive noise. An analysis of these receivers shows that they are very similar to those that are optimum for additive Gaussian noise. The methodology provides a unified theory of receiver design based on the constrained MMSE criterion. This unification yields new insight into this old approach, clarifying both strengths and weaknesses of the approach.

I. INTRODUCTION

A. Purpose

OPTIMUM receivers often must be approximated by more practical structures before they can be implemented. This has led to the popular design alternative of imposing structural constraints before optimization. Structural constraints are also often imposed to facilitate mathematical optimization. For example, the time-correlation receiver, shown in Fig. 1, for detecting the presence of a signal in corrupted observations, $\{Y(t); t \epsilon T\}$, is a widely used structure. One can optimize this receiver by solving for the correlator function $\phi(\cdot)$ and threshold level γ that minimize the probability of detection error. Aside from intuitive appeal, this structure is attractive since it is, in fact, optimum when $\{Y(t)\}$ is composed of a deterministic signal plus Gaussian noise.[1] In this case, the optimum $\phi(\cdot)$ and γ are easily determined [1]. However, when $\{Y(t)\}$ is not appropriately modeled as signal plus Gaussian noise, then solving for the optimum $\phi(\cdot)$ and γ can be very difficult. Furthermore, the optimization requires specification of the probability density function (pdf) for the random variable r, under the two hypotheses of signal present and signal absent, for all candidate functions $\phi(\cdot)$. Often this probabilistic modeling information is not available in practice. Thus alternative criteria of optimality are frequently used to

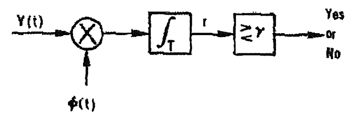

Fig. 1. Correlation receiver.

circumvent this probablistic modeling complication. A popular example of such an alternative is the maximum signal-to-noise ratio (SNR) criterion. This requires specification only of correlation functions, and interestingly leads to an optimum correlator function $\phi(\cdot)$ that is the same as that which minimizes probability of error for additive Gaussian noise [1].

A serious drawback of the maximum SNR structured approach is that it does not appear to generalize to multiple-signal detection (multiple-alternative hypothesis testing). This leads us to the following important point: out of the vast amount of work on structurally constrained receivers during the last two decades, a general approach or methodology, that is not significantly more complicated in application than the maximum SNR approach, seems not to have emerged. As discussed in the preceding example, the *direct* approach of attempting to optimize by seeking the particular receiver, in a structurally constrained class, that minimizes a Bayes risk is unsuccessful in general: the mathematical optimization can be as difficult, if not more so, than the unconstrained problem, and the required specification of a probabilitistic model can be just as prohibitive as it is for the unconstrained problem.

With the preceding as motivation, we propose a general approach to the problem of designing structurally constrained receivers for signal detection and estimation. The methodology that we propose overcomes, to a large degree, both the previously mentioned drawbacks of the direct approach. The price paid for the simplicity of this new approach is suboptimal receiver performance. That is, the receivers derived with this methodology do not, in general, minimize risk subject to structural constraints. However, the methodology is not entirely ad hoc. Rather, it is based on an obvious combination of a simple ad hoc procedure and a genuine structurally constrained risk-minimizing procedure. Furthermore, as demonstrated herein, the methodology yields receivers that are optimum for some classes of problems. The proposed methodology is, in essence, an extension and generalization of the linear minimum-mean-squared-error (MMSE) estimation methodology.

B. Constrained Bayesian Methodology

The methodology that we propose for designing structurally constrained receivers for signal detection is very straightforward. It derives from the fact that all Bayes-risk minimizing

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[1] More generally, this structure is optimum for a class of noise processes with spherical symmetry [19].

signal detectors (hypothesis testers) are most simply described in terms of the posterior probabilities of the hypotheses being tested. If $\{H_i\}_1{}^M$ are M mutually exclusive and exhaustive hypotheses, and C_{ij} is the cost of deciding H_i is true when in fact H_j is true, then the Bayes-risk incurred in deciding H_i is true, given observations $\{Y(t)\}$, hereafter denoted by Y, is

$$R_i = \sum_{j=1}^{M} C_{ij} P[H_j/Y] \tag{1}$$

where $P[H_j/Y]$ is the *posterior* probability of H_j (the probability of H_j, given observations Y). Hence, any risk-minimizing detection rule can be implemented very simply, once the posteriors have been computed from the observations Y. For example, if $C_{ij} = 1 - \delta_{ij}$, where δ_{ij} is the Kronecker delta, then the risk is the probability of detection error and the optimum rule simply announces the hypothesis with the largest posterior. Clearly, the structural complexity of any minimum-risk receiver can be attributed to the computation of the posteriors from the observations; i.e., the implementation of the functionals $\{P[H_i/\cdot]\}_1{}^M$ that map Y into posterior probabilities. Thus, an obvious way to impose a structural constraint on the design of a receiver is to constrain the structure of these functionals. Specifically, the designer might attempt to solve for the specific functional within a prescribed class of functionals that minimizes some appropriate distance between the resultant estimated posterior, denoted by $\tilde{P}[H_i/Y]$, and the true posterior.

The specific prescription of a class of functionals and a distance measure determine the success or failure of any such approach. The success of such an approach should be measured by the tractability of the ensuing optimization problem, and the performance of the resultant receivers. The prescription that we propose guarantees tractability, but not performance. However, preliminary studies reported herein indicate that the performance of receivers designed with the proposed methodology is comparable to the performance of receivers designed with well-known techniques provided that both approaches employ comparable amounts of information about the probabilistic model.

The proposed methodology, to be called the *constrained Bayesian methodology*, is based on the prescription of mean-squared difference as a distance measure, and linear spaces as constraint classes. As a result, optimization requires nothing more than specification of correlations and conditional means of prescribed functionals of the observables, and solution of linear equations.

Summarizing, the structurally constrained receivers that we propose employ *linear-space-constrained, minimum-mean-squared-error (L-MMSE) estimates of posterior probabilities* as true probabilities. Thus, the probability estimates are obtained through a structurally constrained risk-minimizing procedure, and they are then used in an ad hoc procedure (as if they were true probabilities).

The fundamental properties of L-MMSE estimators for posterior probabilities are summarized in Appendix A. As stated there, the L-MMSE estimate of the posterior $P[H_i/Y]$ is identical to the L-MMSE estimate, denoted by $\tilde{\delta}(H_i)/Y$, of the random indicator $\delta(H_i)$:

$$\delta(H_i) \triangleq \begin{cases} 1, & \text{if } H_i \text{ is true} \\ 0, & \text{if } H_i \text{ is false} \end{cases} \tag{2}$$

$$\tilde{P}[H_i/Y] \equiv \tilde{\delta}(H_i)/Y. \tag{3}$$

Thus, if we interpret the vector of random indicators $\delta\{(H_i)\}_1{}^M$ as a vector of signal parameters, then the structurally constrained receiver that announces (as true) the hypothesis with the largest estimated posterior is identical to the receiver that announces (as detected) the signal with parameter value that is closest (in Euclidean norm) to the estimated parameter value $\{\tilde{\delta}(H_i)/Y\}_1{}^M$. Hence, the proposed methodology (for the special case $C_{ij} = 1 - \delta_{ij}$) can be interpreted as a formalism for an *estimation theorist's approach* to signal detection.

As stated in Appendix A, the L-MMSE estimate of a discrete random signal or signal parameter x with range $\{X_i\}_1{}^M$ is identical to the mean of the L-MMSE estimated posterior distribution

$$\tilde{X}/Y = \sum_{i=1}^{M} X_i \tilde{P}[X_i/Y]. \tag{4}$$

Thus the alternative estimation theorist's approach, by which the signal (from the set $\{X_i\}_1{}^M$) that is closest to the estimated signal $\tilde{X}/Y$ is announced as the detected signal (cf. [2]), is intimately related to the preceding approach, the difference being that the former employs the mode of the estimated posterior distribution whereas the latter employs the mean.

Observe that the constrained Bayesian methodology, which is based on *optimum* approximations to posterior probabilities $P[H_i/Y]$ (or, equivalently, optimum approximations to the ratio of densities of observables $f_{y/H_i}(Y)/f_y(Y)$), is substantially different from the more conventional approaches based on nonoptimum series approximations to densities of observables $f_{y/H_i}(Y)$ and $f_y(Y)$ (cf. [18]).

C. History

The basic ingredient of the proposed methodology—the L-MMSE estimator for posterior probabilities—is not new. This estimator has appeared in a number of works in the field of pattern recognition since as early as 1962 [3]-[7]. The primary role of this estimator has been that of justification for the use of various empirical procedures related to linear least squares approximations to discriminants for pattern classification [8]. Similarly, the fact that posterior probabilities can be characterized as unconstrained MMSE estimates of random indicators is not new. This fact has been used by a number of investigators for obtaining various characterizations of posterior probabilities (and pdf's), and for establishing links between detection and estimation problems (cf. [9], [10]). However, there has been—to my knowledge—no published work on analytical characterization of the L-MMSE estimator for posterior probabilities, and on analytical evaluation of the obvious methodology based on this estimator.

The results toward this end that are presented in this paper were motivated by the discovery (reported in [11]) of the simple but revealing characterization (4) of the L-MMSE estimator for a discrete random parameter x in terms of the L-MMSE estimators for the posterior probabilities of x. This characterization shows that the L-MMSE estimators for the posterior probabilities of x are more basic than the L-MMSE estimator for x, and it immediately suggests the constrained Bayesian methodology as a natural extension and generalization of the well-known methodology for L-MMSE estimation of random parameters.

Preliminary results and speculations were presented in [12]. Results that complement those presented herein are currently in preparation for publication [17].

D. Scope

Functionals that are constrained to be in linear spaces are by no means limited to linear functionals. In this paper we consider both linear functionals and quadratic functionals. The linearly constrained receivers employ posterior estimates of the form

$$\tilde{P}[H_i/Y] = \phi_0{}^i + \int_T \phi_1{}^i(\tau)\bar{Y}(\tau)\, d\tau, \tag{5}$$

whereas the posterior estimates employed by the quadratically constrained receiver include the additional term

$$\int_T\int_T \phi_2{}^i(t,\tau)\bar{Y}(t)\bar{Y}(\tau)\, dt\, d\tau. \tag{6}$$

In (5) and (6), $\bar{Y}(t)$ is the centered observation[2]

$$\bar{Y}(t) \triangleq Y(t) - E\{y(t)\}, \tag{7}$$

and T is the observation interval.

Another class of structurally constrained receivers within the realm of the constrained Bayesian methodology is that class of receivers that employs posterior estimates of the form

$$\tilde{P}[H_i/Y] = \phi_0{}^i + \int_T \phi_1{}^i(t)G[Y(t)]\, dt, \tag{8}$$

where $G(\cdot)$ is a prescribed memoryless nonlinear transformation such as a limiter or clipper. An analysis of these receivers will be reported in a future paper.

The optimal $\phi_0{}^i$, $\phi_1{}^i(\cdot)$, and $\phi_2{}^i(\cdot, \cdot)$ are those that minimize the mean-squared error

$$E\{(P[H_i/y] - \tilde{P}[H_i/y])^2\}. \tag{9}$$

These can be determined from the necessary and sufficient orthogonality condition (from the Hilbert space orthogonal projection theorem [13]):

$$E\{(P[H_i/y] - \tilde{P}[H_i/y])z\} = 0, \qquad \forall z \in L, \tag{10}$$

where L is the Hilbert space of finite mean square random variables generated by the images of the observables y under all functionals in the constraint space. Condition (10) can be reexpressed as

$$E\{\tilde{P}[H_i/y]\, z\} = P[H_i]E\{z/H_i\}, \qquad \forall z \in L, \tag{11}$$

where $\{P[H_i]\}$ are the prior probabilities of the hypotheses.

Substitution of the forms (5) and (6) into (11) yields the following necessary and sufficient linear equations.

1) Linear receiver:

$$\phi_0{}^i = P[H_i]$$

$$\int_T M_2(t,\tau)\phi_1{}^i(\tau)\, d\tau = P[H_i]M_{1/H_i}(t), \qquad \forall t \in T, \tag{12}$$

where

$$M_{1/H_i}(t) \triangleq E\{\bar{y}(t)/H_i\} \tag{13}$$

$$M_2(t,\tau) \triangleq E\{\bar{y}(t)\bar{y}(\tau)\}. \tag{14}$$

2) Quadratic receiver:

$$\phi_0{}^i + \int_T \phi_1{}^i(\tau)M_1(\tau)\, d\tau + \int_T\int_T \phi_2{}^i(\tau,s)M_2(\tau,s)\, d\tau\, ds = P[H_i] \tag{15}$$

$$\phi_0{}^i M_1(u) + \int_T \phi_1{}^i(\tau)M_2(u,\tau)\, d\tau + \int_T\int_T \phi_2{}^i(\tau,s)M_3(u,\tau,s)\, d\tau\, ds = P[H_i]\, M_{1/H_i}(u), \qquad \forall u \in T \tag{16}$$

$$\phi_0{}^i M_2(u,v) + \int_T \phi_1{}^i(\tau)M_3(u,v,\tau)\, d\tau + \int_T\int_T \phi_2{}^i(\tau,s)M_4(u,v,\tau,s)\, d\tau\, ds = P[H_i]\, M_{2/H_i}(u,v), \qquad \forall u,v \in T \tag{17}$$

where for example

$$M_4(u,v,\tau,s) \triangleq E\{\bar{y}(u)\bar{y}(v)\bar{y}(\tau)\bar{y}(s)\} \tag{18}$$

$$M_{2/H_i}(u,v) \triangleq E\{\bar{y}(u)\bar{y}(v)/H_i\}. \tag{19}$$

From (12)-(14) it is clear that the linearly constrained receiver cannot be expected to perform acceptably in those situations where the conditional means $\{M_1(t)/H_i\}_1{}^M$ do not convey a sufficient amount of information about the underlying distributions. For example, in the problem of detection of a zero mean random signal in additive zero mean random noise, $M_1(t)/H_i \equiv 0$ for both hypotheses. Thus, the linearly constrained Bayesian detector would not perform acceptably. However, it is known that a linear estimator-correlator re-

[2] We employ lower case letters to denote random variables, and capitals to denote samples of random variables.

ceiver [14] does indeed have the potential for acceptable performance for this detection problem (and is in fact optimum for Gaussian distributions). But this receiver is actually quadratic—not linear—since the correlation statistic is a quadratic functional of the observations. Furthermore, this receiver requires specification of conditional correlations $\{M_2(t,\tau)/H_i\}$. Hence, it would be more appropriate to compare the linear estimator-correlator detector with the quadratically constrained Bayesian detector which also requires specification of $\{M_2(t,\tau)/H_i\}$ as evidenced by (15)-(17).

The evaluation of the L-constrained Bayesian methodology for random signal detection will be reported in a future paper. In the present paper, we consider only sure signal detection (coherent and noncoherent). In Section II we present our results on the analysis and evaluation of linearly constrained coherent receivers for signal detection, and in Section III we do the same for quadratically constrained noncoherent receivers for detection. In Section IV we briefly analyze linearly constrained receivers for signal parameter estimation.

II. LINEARLY CONSTRAINED RECEIVERS FOR SIGNAL DETECTION

A. General Case

The linearly constrained receiver computes the M posterior estimates

$$\tilde{P}[H_j/Y] = P[H_j] + \int_T \phi_1^j(t)\bar{Y}(t)\,dt, \qquad j = 1,2,\cdots,M \tag{20}$$

and announces as true the hypothesis with the largest such estimated posterior. The function $\phi_1^j(\cdot)$ that determines the jth estimate is the solution to the linear integral equation

$$\int_T k_y(t,\tau)\phi_1^j(\tau)\,d\tau = P[H_j]E\{\bar{y}(t)/H_j\}, \qquad \forall t \in T, \tag{21}$$

where $k_y(t,\tau)$ is the covariance for y, denoted by $M_2(t,\tau)$ in (14). We can express the solution to (21) in terms of the inverse kernel[3] k_y^{-1}, defined by

$$\int_T k_y^{-1}(t,\sigma)k_y(\sigma,\tau)\,d\sigma = \delta(t-\tau), \qquad \forall t,\tau \in T, \tag{22}$$

thereby obtaining

$$\phi_1^j(\tau) = P[H_j]\int_T k_y^{-1}(\tau,\sigma)E\{\bar{y}(\sigma)/H_j\}\,d\sigma. \tag{23}$$

Substituting this solution into (20) yields

$$\tilde{P}[H_j/Y] = P[H_j]\,[1 + T_j(Y)], \tag{24}$$

[3] The tacit assumption that k_y^{-1} exists is not essential to the methodology since the Hilbert space orthogonal projection theorem guarantees the existence of a unique solution to the MMSE estimation problem. Thus, even if k_y^{-1} does not exist, the right member of (21) must be in the range space of k_y and an inverse to k_y can always be defined on its range space.

where the statistic T_j is defined by

$$T_j(Y) \triangleq \int_T\int_T k_y^{-1}(\tau,\sigma)E\{\bar{y}(\sigma)/H_j\}\bar{Y}(\tau)\,d\sigma\,d\tau. \tag{25}$$

Now, we can express this statistic in several different ways, each yielding a different physical interpretation. Two particularly interesting interpretations are the following.

Interpretation 1) Let $h(t,\tau)$ be a factor of the inverse kernel k_y^{-1}, i.e.,

$$\int_T h(t,\sigma)h(\tau,\sigma)\,d\sigma = k_y^{-1}(t,\tau), \qquad \forall t,\tau \in T, \tag{26}$$

and denote the whitened process obtained by filtering $\bar{y}$ with h by $\bar{z}$:

$$\bar{z}(t) \triangleq \int_T h(t,\tau)\bar{y}(\tau)\,d\tau. \tag{27}$$

Thus,

$$k_z(t,\tau) = \delta(t-\tau). \tag{28}$$

Also, let

$$S_j^h(t) \triangleq E\{\bar{z}(t)/H_j\} \tag{29}$$

be a measure of the signal on which $\bar{Z}(t)/H_j$ depends. Then

$$T_j(Y) = \int_T\left[\int_T h(t,\tau)\bar{Y}(\tau)\,d\tau\right]S_j^h(t)\,dt. \tag{30}$$

Thus, the estimation rule whitens the centered observations and then correlates with the measure of the signal (in the whitened observations) as shown in Fig. 2.

Interpretation 2) Assume $\bar{y}(t)$ consists of the sum of a colored component $\bar{v}(t)$ and an independent white component $w(t)$, with power spectral density N_0. Then k_y^{-1} can be expressed as

$$k_y^{-1}(t,\tau) = \frac{1}{N_0}\,[\delta(t-\tau) - g(t,\tau)] \tag{31}$$

where

$$g(t,\tau) \triangleq \int_T k_y^{-1}(t,\sigma)k_v(\sigma,\tau)\,d\sigma. \tag{32}$$

Furthermore, the quantity $\tilde{\bar{V}}(t)/\bar{Y}$, defined by

$$\tilde{\bar{V}}(t)/\bar{Y} \triangleq \int_T g(t,\tau)\bar{Y}(\tau)\,d\tau, \tag{33}$$

is the linear MMSE estimate of the colored component of $\bar{Y}$, given observations $\bar{Y}$. Let

$$S_j^g(t) \triangleq (1/N_0)E\{\bar{y}(t)/H_j\} \tag{34}$$

be a measure of the signal on which $[\bar{Y}(t) - \tilde{\bar{V}}(t)/\bar{Y}]/H_j$

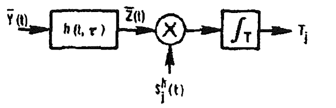


Fig. 2. Realization of statistic T_j (30).

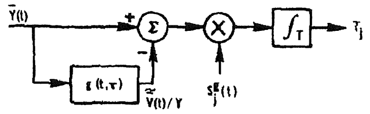


Fig. 3. Realization of statistic T_j (35).

depends. Then

$$T_j(Y) = \int_T S_j^g(t)[\bar{Y}(t) - \tilde{\bar{V}}(t)/\bar{Y}]\, dt. \tag{35}$$

Thus the estimation rule subtracts out the linear MMSE estimate of the colored component of the centered observations, and then correlates with the measure of the signal, as shown in Fig. 3. The similarities between these receivers and those that are optimum for sure signals in additive Gaussian noise [1] are remarkable. The similarities are even stronger for the specific case of sure signals in additive (not necessarily Gaussian) noise discussed in the following text.

B. Sure Signals in Additive and Multiplicative Noise

Consider observations Y of the form

$$Y(t) = Z(t)S_j(t) + N(t), \qquad \forall t \in T \tag{36}$$

under hypothesis H_j. $N(t)$ is a sample of a zero-mean random noise process with autocovariance $k_n(t,\tau)$, and $Z(t)$ is a sample of a random process with nonzero mean denoted by $m_z(t)$ and autocovariance denoted by $k_z(t,\tau)$. $\{S_j(t)\}_1^M$ is interpreted as the exhaustive set of samples (with prior probabilities $P_j = P[H_j]$) of a random signal process $s(t)$ with mean and autocovariance denoted by, respectively,

$$m_s(t) = \sum_{i=1}^{M} P_i S_i(t) \tag{37}$$

$$k_s(t,\tau) = \sum_{i,j=1}^{M} R_{ij} S_i(t) S_j(\tau), \tag{38}$$

where

$$R_{ij} \triangleq P_i \delta_{ij} - P_i P_j. \tag{39}$$

In (39), δ_{ij} is the Kronecker delta. The three processes $n(t)$, $z(t)$, and $s(t)$ are assumed statistically independent. The covariance and conditional means that determine the posterior estimates are easily shown to be

$$k_y(t,\tau) = m_z(t) m_z(\tau) k_s(t,\tau) + k_{n'}(t,\tau) \tag{40}$$

$$E\{\bar{y}(t)/H_i\} = m_z(t)[S_i(t) - m_s(t)] \tag{41}$$

where $k_{n'}$ is defined by

$$k_{n'}(t,\tau) \triangleq k_z(t,\tau)[k_s(t,\tau) + m_s(t) m_s(\tau)] + k_n(t,\tau). \tag{42}$$

Hence the additive and multiplicative noise problem is equivalent (for this methodology) to an additive (only) noise problem with zero-mean noise n' having covariance $k_{n'}(t,\tau)$, and with signals $S_j'(t) = m_z(t)S_j(t)$. In view of this equivalence, we proceed in terms of the additive (only) noise model, and we omit the primes for notational convenience. We then have

$$k_y(t,\tau) = \sum_{i,j=1}^{M} R_{ij} S_i(t) S_j(\tau) + k_n(t,\tau) \tag{43}$$

and

$$E\{\bar{y}(t)/H_j\} = \frac{1}{P_j} \sum_{i=1}^{M} R_{ji} S_i(t). \tag{44}$$

Substituting (43) and (44) into (22) and (23) yields the solution

$$\phi_1^j(\tau) = \sum_{i=1}^{M} W_{ji} \theta_i(\tau), \tag{45}$$

where $\theta_i(\cdot)$ is the solution to the Fredholm equation

$$\int_T k_n(t,\tau) \theta_i(\tau)\, d\tau = S_i(t), \qquad \forall t \in T, \tag{46}$$

and where the matrix W of elements $\{W_{ij}\}$ is given by the formula

$$W = R[I + VR]^{-1}. \tag{47}$$

The elements of the matrix V in (47) are defined by

$$V_{ij} \triangleq \int_T \theta_i(\tau) S_j(\tau)\, d\tau, \tag{48}$$

and I is the identity matrix.

The linear receiver is, therefore, a correlation (or matched-filter) receiver as shown in Fig. 4. The correlators are the same as those employed in the optimum-for-Gaussian-noise receiver [1]. That is, both the linearly constrained receiver and the optimum-for-Gaussian-noise receiver reduce the continuous-parameter sample $\{Y(t)\}$ to the same M correlation statistics $\{\tau_j\}_1^M$,

$$\tau_j(Y) \triangleq \int_T Y(\tau) \theta_j(\tau)\, d\tau. \tag{49}$$

The only differences between the receivers are the values of elements in the linear weighting network W and the biasing vector b used to obtain the final statistics

$$\tilde{P}[H_j/Y] = \sum_{i=1}^{M} W_{ji} \tau_i(Y) + b_j \tag{50}$$

where

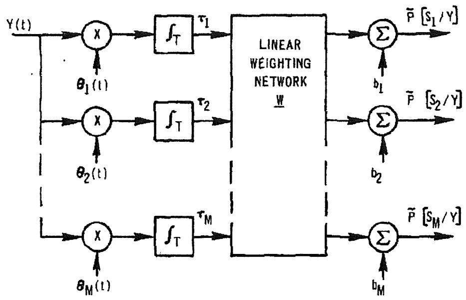


Fig. 4. Linearly constrained coherent receiver for signals in additive and multiplicative noise (50).

$$b \triangleq p - WVp = WR^{-1}p,$$

and p is the vector of priors $\{P_j\}_1{}^M$.

1) Orthogonal signals: Now consider the special case where the noise is white[4] and the signals are mutually orthogonal (e.g., PPM or FSK) with energies $\{E_i\}$. Then, the solution (45) takes the simpler form

$$\phi_1{}^j(\tau) = \frac{1}{N_0} \sum_{i=1}^{M} U_i{}^j S_i(\tau) \tag{51}$$

$$U_i^j \triangleq \alpha_i(\delta_{ij} - \beta_j) \tag{52}$$

$$\alpha_i \triangleq P_i/(1 + E_i P_i/N_0)$$

$$\beta_j \triangleq \alpha_j \bigg/ \sum_{k=1}^{M} \alpha_k. \tag{53}$$

The similarities between this linearly constrained receiver and the optimum-for-Gaussian receiver are very strong. The linearly constrained receiver decides H_j is true if and only if the following jth estimated posterior is largest:

$$\tilde{P}[H_j/Y] = \frac{1}{N_0} \sum_{i=1}^{M} \left[U_i{}^j \int_T S_i(\tau)Y(\tau)\, d\tau - P_i E_i \right] + P_j. \tag{54}$$

The optimum-for-Gaussian receiver decides H_j is true if and only if the following jth statistic is largest [1]:

$$G_j(Y) \triangleq \frac{1}{N_0} \left[\int_T S_j(\tau)Y(\tau)\, d\tau - E_j/2 \right] + \ln(P_j). \tag{55}$$

Or equivalently, the linearly constrained receiver performs $M-1$ tests of the form

$$\sum_{i=1}^{M} L_{jk}(i)[\tau_i{}'(Y) - E_i P_i] \underset{\text{not } H_j}{\overset{\text{not } H_k}{\gtrless}} N_0(P_k - P_j), \tag{56}$$

and the optimum-for-Gaussian receiver performs $M-1$ tests of the form

$$\sum_{i=1}^{M} G_{jk}(i)[\tau_i{}'(Y) - E_i/2] \underset{\text{not } H_j}{\overset{\text{not } H_k}{\gtrless}} N_0[\ln(P_k) - \ln(P_j)] \tag{57}$$

where $\{\tau_j{}'\}$ are the correlation statistics

$$\tau_j{}'(Y) \triangleq \int_T S_j(t)Y(t)\, dt. \tag{58}$$

The coefficient matrices in (56) and (57) are defined by

$$L_{jk}(i) \triangleq (U_i{}^j - U_i{}^k) \tag{59}$$

$$G_{jk}(i) \triangleq \begin{cases} 1, & i = j \\ -1, & i = k \\ 0, & \text{otherwise.} \end{cases} \tag{60}$$

The similarity of these two testing rules, (56) and (57), is evidenced by the properties

$$\sum_{i=1}^{M} L_{jk}(i) = \sum_{i=1}^{M} G_{jk}(i) = 0 \tag{61}$$

$$-1 \leqslant L_{jk}(i) \leqslant 1, \qquad \forall i,j,k$$

$$-1 \leqslant G_{jk}(i) \leqslant 1, \qquad \forall i,j,k. \tag{62}$$

In fact, if all energies $\{E_i\}_1{}^M$ are equal and all priors $\{P_i\}_1{}^M$ are equal, then the receivers are identical! Also, if all priors are equal, the receivers are identical under threshold conditions ($E_i/N_0 \ll 1$).

Furthermore, if we consider binary signal detection ($M = 2$), then (56) and (57) reduce to the following.

a) Linearly constrained rule:

$$[\tau_1{}'(Y) - E_1/2] - [\tau_2{}'(Y) - E_2/2] \underset{H_2}{\overset{H_1}{\gtrless}} \frac{N_0}{2} [(1/P_1) - (1/P_2)] \triangleq \gamma_L. \tag{63}$$

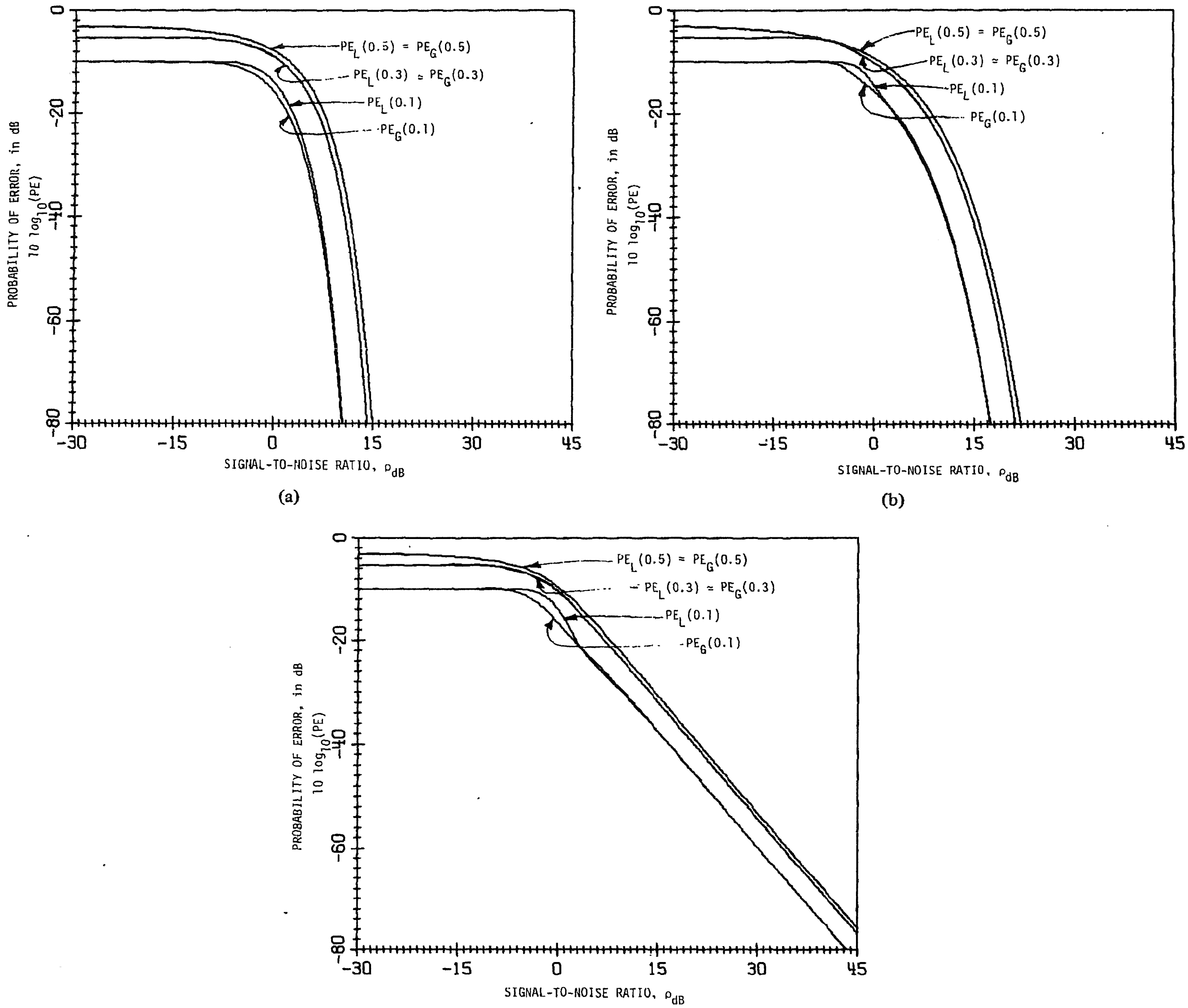
b) Optimum-for-Gaussian rule:

$$[\tau_1{}'(Y) - E_1/2] - [\tau_2{}'(Y) - E_2/2] \underset{H_2}{\overset{H_1}{\gtrless}} \frac{N_0}{2} [\ln(1/P_1) - \ln(1/P_2)] \triangleq \gamma_G. \tag{64}$$

Thus, we see that these two binary receivers differ only in the sensitivity of their thresholds γ to the priors. Both thresholds exhibit odd symmetry about $P_1 = P_2 = \frac{1}{2}$, and approach $+\infty(-\infty)$ as $P_1(P_2)$ approaches 0, as would be expected. However, the linearly constrained receiver is more sensitive to the priors (as would be expected) since $|\gamma_L| \leqslant |\gamma_G|$ for all P_1 and P_2 (with equality if and only if $P_1 = P_2$).

In an effort to determine the relative merits of increased threshold sensitivity, we have evaluated probability-of-error for these two receivers for the three cases where the noise statistic

[4] The assumption that the noise is white does not preclude non-Gaussian noise. For example, the noise could be the sum of Gaussian noise and Poisson impulse noise (with finite rate parameter).



(c)

Fig. 5. Probability of error for linearly constrained binary detector ($PE_L(P_1)$) and for optimum-for-Gaussian-noise binary detector ($PE_G(P_1)$). (a) For Gaussian noise distribution (66). (b) For Laplacian noise distribution (67). (c) For fourth-order Butterworth noise distribution (68).

$$N \triangleq \int_T [S_1(t) - S_2(t)]N(t)\,dt \tag{65}$$

is a sample from: 1) the Gaussian distribution with density function

$$f_n(N) = (\sigma_n\sqrt{2\pi})^{-1} \exp[-(N/\sigma_n)^2/2]; \tag{66}$$

2) the Laplacian distribution

$$f_n(N) = (\sigma_n\sqrt{2})^{-1} \exp(-|N|\sqrt{2}/\sigma_n); \tag{67}$$

or 3) the fourth-order Butterworth distribution

$$f_n(N) = (\sqrt{2}/\pi\sigma_n)[1 + (N/\sigma_n)^4]^{-1}. \tag{68}$$

In all three cases, the difference in probability-of-error for these two receivers is negligible for all values of SNR, and all values of priors that are within an order of magnitude of each other, as shown in Fig. 5. The SNR ρ_{dB} is defined as follows:

$$\rho_{dB} \triangleq 10 \log_{10} (\sigma_s^2/\sigma_n^2) \tag{69}$$

where

$$\sigma_n^2 \triangleq N_0/|T| \tag{70}$$

$$\sigma_s^2 \triangleq \frac{1}{|T|}\int_T [E\{s^2(t)\} - (E\{s(t)\})^2]\,dt = \frac{1}{|T|}\sum_{i=1}^{M} E_iP_i(1-P_i). \tag{71}$$

Similarly, in an effort to determine the relative performance of the linearly constrained receiver for M-ary signaling, we have evaluated the approximate probability of error for the two receivers (56) and (57) for the case where the noise statistics

$$N_i \triangleq \int_T S_i(t)N(t)\,dt, \qquad i = 1,2,\cdots,M \tag{72}$$

are independent samples from the Laplacian distribution (as might occur, for example, with PPM in white impulse noise). The probability of error was approximated in the conventional way using the union bound, which is known to be tight for high SNR's (e.g., $\rho_{dB} \geqslant 10$). Since the two receivers are identical when all priors are equal and all energies are equal, we selected linear distributions of probability and energy ranging over one order of magnitude.

The resultant approximate probability of error is shown in Fig. 6 for the four cases $M = 2^q$, $q = 1, 2, 3, 4$. It can be seen that neither receiver is superior for all alphabet sizes and all levels of SNR. Overall, the performances of the two receivers are comparable. Note that to facilitate comparison of these results with standard results for Gaussian noise [1], we modified our definition of SNR by replacing the signal variance (71) with the mean squared signal (in Fig. 6 only).

2) *Linearly dependent signals:* In contrast to the uniformly good performance of the linearly constrained receiver for orthogonal signals, the performance for linearly dependent signals such as ASK and PSK depends critically on whether the mode or the mean of the estimated posterior distribution is employed. As discussed in Section I-B, the *hypothesis tester's rule* which employs the mode

$$\tilde{P}[H_i/Y] \underset{\text{not } H_j}{\overset{\text{not } H_i}{\gtrless}} \tilde{P}[H_j/Y] \tag{73}$$

can be interpreted as an *estimation theorist's* rule by reexpressing (73) as

$$\|\{\tilde{\delta}(H_k)/Y\}_1^M - \{\delta_{ki}\}_1^M\| \underset{\text{not } H_j}{\overset{\text{not } H_i}{\gtrless}} \|\{\tilde{\delta}(H_k)/Y\}_1^M - \{\delta_{kj}\}_1^M\|, \tag{74}$$

where $\|\cdot\|$ denotes norm in Euclidian M-space, and $\tilde{\delta}(H_k)/Y$ is the L-MMSE estimate of the random indicator $\delta(H_k)$, and satisfies

$$\tilde{\delta}(H_k)/Y \equiv \tilde{P}[H_k/Y]. \tag{75}$$

On the other hand, the following alternative estimation theorist's rule employs the mean

$$|\tilde{X}/Y - X_i| \underset{\text{not } H_j}{\overset{\text{not } H_i}{\gtrless}} |\tilde{X}/Y - X_j|, \tag{76}$$

where X_i is the signal parameter value (e.g., amplitude or phase) that corresponds to the hypothesis H_i, and $\tilde{X}/Y$ is the L-MMSE estimate of the random signal parameter (see Appendix A)

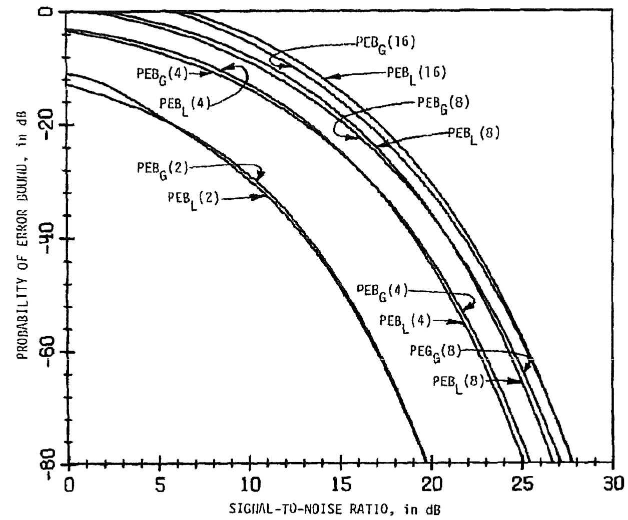


Fig. 6. Union bound on probability of error for linearly constrained M-ary detector ($PEB_L(M)$) and for optimum-for-Gaussian-noise M-ary detector ($PEB_G(M)$), for Laplacian noise distribution (67).

$$\tilde{X}/Y = \sum_{i=1}^{M} X_i \tilde{P}[X_i/Y], \tag{77}$$

where

$$P[H_i/Y] \equiv P[X_i/Y].$$

The performance of these two linearly constrained receivers depends critically on the specific signaling scheme when the signals are linearly dependent. This is amply illustrated by the following two examples. A general comparative analysis will be reported in a future paper [17]. In these examples let $\tilde{X}_{\text{mode}}$ denote the mode of the estimated posterior distribution $\{\tilde{P}[X_i/Y]\}_1^M$, and let $\tilde{X}_{\text{mean}}$ denote the closest of the values $\{X_i\}_1^M$ to the mean $\tilde{X}/Y$ of the estimated posterior distribution.

a) ASK: If $S_j(t) = X_j S(t)$, it can shown that

$$\tilde{X}/Y = m_x N_0/(N_0 + \sigma_x^2 E) + \tau(Y)\sigma_x^2/(N_0 + \sigma_x^2 E), \tag{78}$$

where the statistic $\tau(Y)$ is defined by

$$\tau(Y) \triangleq \int_T S(t)Y(t)\,dt. \tag{79}$$

It follows that, with probability approaching one,

$$\tilde{X}_{\text{mean}} = \begin{cases} X_0, & \text{as } N_0 \to 0 \\ m_x, & \text{as } N_0 \to \infty, \end{cases} \tag{80}$$

where X_0 denotes the correct value of x. Hence, rule (76) performs appropriately. On the other hand, rule (73) does not perform appropriately (for $M > 2$) since it can be shown that

$$\tilde{X}_{\text{mode}} = \begin{cases} \max \{X_i\}_1{}^M, & \text{for } \tau(Y') > 0 \\ \min \{X_i\}_1{}^M, & \text{for } \tau(Y') < 0, \end{cases} \tag{81}$$

where the statistic $\tau(Y')$ is defined by (79) with Y replaced by Y':

$$Y'(t) \triangleq Y(t) - \tilde{S}(t)/Y \tag{82}$$

where $\tilde{S}(t)/Y$ is the zero error-mean, minimum error-variance linear estimate of the random M-ary signal $s(t)$, given observations $Y(t)$. Hence rule (73) always announces either max $\{X_i\}_1{}^M$ or min$\{X_i\}_1{}^M$ as the correct value of x.

b) PSK: If $S_i(t) = \cos(\omega_0 t + X_i)$, where $\omega_0 |T|/2\pi$ is an integer and the mean m_x is zero, then it can be shown that

$$\tilde{X}_{mode} = \text{mode}\left\{-\left|X_i - \tan^{-1}\left[\frac{\sin(X_0) - N_s}{\cos(X_0) + N_c}\right]\right|\right\} \tag{83}$$

where N_c and N_s are defined by

$$N_c \triangleq \frac{2}{|T|} \int_T \cos(\omega_0 t) N(t)\, dt$$

$$N_s \triangleq \frac{2}{|T|} \int_T \sin(\omega_0 t) N(t)\, dt. \tag{84}$$

It follows that $\tilde{X}_{mode} = X_0$ with probability approaching one as N_0 approaches zero. Furthermore, $\tilde{X}_{mode}$ is identical to the maximum likelihood estimate of X, assuming X is an unknown (nonrandom) parameter and $n(t)$ is Gaussian. Hence, rule (73) performs appropriately. On the other hand, rule (76) does not perform appropriately (for $M > 2$) since it can be shown that

$$\tilde{X}/Y = KT(4N_0 + T)^{-1}[\sin(X_0) - N_s], \tag{85}$$

where K is defined by

$$K \triangleq \frac{2}{M} \sum_{i=1}^{M} X_i \sin(X_i). \tag{86}$$

It follows that, as N_0 approaches zero, the probability of announcing the wrong value of x approaches 1 or $1 - 2/M$, depending on the value of $M (M > 2)$, assuming that $\{X_i\}_1{}^M$ are uniformly distributed on $[-\pi, \pi]$.

C. Sure Rate-Signals of Filtered Poisson Processes in Additive and Multiplicative Noise

Consider observations Y of the form

$$Y(t) = \sum_{i=1}^{N_j(t)} Z_i G(t - T_i) + V(t), \qquad \forall t \in [0,T] \tag{87}$$

under hypothesis H_j, where $G(t)$ is a deterministic filter-pulse, and $\{T_i\}$is a sample of the random occurence times of the Poisson counting process $n_j(t)$ with sure rate-signal $S_j(t)$, and $\{Z_i\}$are samples of identically distributed independent multiplicative random noise variables, and $V(t)$ is a sample of an uncorrelated zero-mean additive random noise process. This is a useful model for digital data signals transmitted over fiber optic channels. It can be shown that for linear MMSE estimation, this model is equivalent to a sure signal in additive, uncorrelated, zero-mean noise model [16]. Thus, much of the analysis in Section II-B carries over to this class of signal detection problems. The equivalent model is

$$Y'(t) = S_j'(t) + N'(t), \qquad \forall t \in [0,T] \tag{88}$$

under hypothesis H_j, where

$$S_j'(t) \triangleq m_z \int_0^T S_j(\tau) G(t - \tau)\, d\tau, \tag{89}$$

and the noise has autocovariance

$$k_{n'}(t,\tau) = k_v(t,\tau) + (\sigma_z{}^2 + m_z{}^2) \cdot \sum_{i=1}^{M} P_i \int_0^T S_i(\sigma) G(t - \sigma) G(\tau - \sigma)\, d\sigma. \tag{90}$$

As shown in Section II-B the linear receiver is a correlation (or matched-filter) receiver. For more details, see [16].

III. QUADRATICALLY CONSTRAINED RECEIVERS FOR SIGNAL DETECTION

Since quadratic estimates include linear estimates as a special case (viz., $\phi_2{}^j(\cdot,\cdot) \equiv 0$), then quadratically constrained estimates of posteriors must be at least as accurate as linearly constrained estimates. However, it does not follow that quadratically constrained receivers (as defined in Section I) must perform at least as well as linearly constrained receivers. In fact, it can be shown that, for deterministic signals in additive noise, the relative performance depends on the priors and signal energies. On the other hand there are certain types of signal detection problems for which quadratically constrained receivers always outperform linearly constrained receivers. One such type of signal detection problem that is of considerable practical importance is the noncoherent signal detection problem for which the observations are of the form

$$Y(t) = W_j(t) \cos(\omega_0 t + \Psi) + N(t), \qquad \forall t \in T \tag{91}$$

under hypothesis H_j. $N(t)$ is a sample of a zero-mean random noise process, $W_j(t)$ is a deterministic signal envelope, and Ψ is a sample of a random phase variable. As would be expected, the linearly constrained receiver performs poorly. In fact if ψ is uniformly distributed on $[0, 2\pi]$, then the linearly constrained estimates of the posteriors are just the priors. In contrast to this, the quadratically constrained receiver performs quite well for some noise distributions; as demonstrated in this section, for a certain class of noise distributions, it closely resembles the optimum noncoherent receiver for Gaussian noise.

The quadratically constrained noncoherent receiver computes the M posterior estimates

$$\tilde{P}[H_j/Y] = \phi_0{}^j + \int_T \phi_1{}^j(t)\bar{Y}(t)\,dt + \int_T\int_T \phi_2{}^j(t,\tau)\bar{Y}(t)\bar{Y}(\tau)\,dt\,d\tau, \qquad j = 1,2,\cdots,M \tag{92}$$

and announces as true the hypothesis with the largest estimated posterior. The optimal $\phi_0{}^j$, $\phi_1{}^j(\cdot)$, and $\phi_2{}^j(\cdot,\cdot)$ are the solutions to the three linear integral equations (15)-(17). Nothing very definitive can be said about the form of the functions $\phi_1{}^j(\cdot)$ and $\phi_2{}^j(\cdot,\cdot)$ without some restrictive assumptions about the distributions of the random phase and the noise process. However, if these distributions are such that the functions $\phi_2{}^j(\cdot,\cdot)$ are the kernels of symmetrical, nonnegative definite linear operators on $L^2(T)$, and they factor as

$$\phi_2{}^j(t,\tau) = \int_T \gamma_j(\sigma,t)\,\gamma_j(\sigma,\tau)\,d\sigma, \qquad \forall t,\tau \in T, \tag{93}$$

then the nonlinearity and the memory in the receiver can be separated since the nonlinear term in (92) then factors as

$$\int_T\int_T \phi_2{}^j(t,\tau)\bar{Y}(t)\bar{Y}(\tau)\,dt\,d\tau = \int_T\left[\int_T \gamma_j(t,\sigma)\bar{Y}(\sigma)\,d\sigma\right]^2 dt. \tag{94}$$

This nonlinear term in the estimate can be implemented by the cascade connection of a linear time-varying filter with impulse-response function $\gamma_j(\cdot,\cdot)$, a memoryless square-law device, and an integrator, in that order.

In many cases of practical interest the following assumptions are valid.

Assumption 1) The random phase variable ψ is uniformly distributed on $[0, 2\pi]$.

Assumption 2) The distributions of the random noise process $n(t)$ exhibit even symmetry so that all joint moments of odd order vanish.

With these two assumptions, it follows that the function $\phi_1{}^j(\cdot)$ is identically zero. Hence there is no linear term in the estimate. The simplified equations for $\phi_0{}^j$ and $\phi_2{}^j(\cdot,\cdot)$ are

$$\phi_0{}^j = P[H_j] - \sum_{i=1}^{M} P[H_i] \int_T\int_T \phi_2{}^j(t,\tau)M_2(t,\tau)/H_i\,dt\,d\tau \tag{95}$$

$$\int_T\int_T M_4(u,v,\tau,s)\phi_2{}^j(\tau,s)\,d\tau\,ds = \sum_{i=1}^{M} R_{ij}M_2(u,v)/H_i, \qquad \forall u,v \in T, \tag{96}$$

where

$$R_{ij} \triangleq P[H_i](\delta_{ij} - \phi_0{}^j). \tag{97}$$

Furthermore, with Assumptions 1) and 2), the moments in these equations can be expressed as

$$M_2(t,\tau)/H_j = \tfrac{1}{2} W_j(t)W_j(\tau)\cos[\omega_0(t-\tau)] + k_{n_2}(t,\tau) \tag{98}$$

$$\begin{aligned} M_4(u,v,\tau,s) = {} & k_{s_4}(u,v,\tau,s) + k_{s_2}(u,v)k_{n_2}(\tau,s) \\ & + k_{s_2}(u,\tau)k_{n_2}(v,s) + k_{s_2}(u,s)k_{n_2}(\tau,v) \\ & + k_{s_2}(v,\tau)k_{n_2}(u,s) + k_{s_2}(v,s)k_{n_2}(u,\tau) \\ & + k_{s_2}(\tau,s)k_{n_2}(u,v) + k_{n_4}(u,v,\tau,s). \end{aligned} \tag{99}$$

In these expressions, k_{n_2} and k_{n_4} are the second and fourth joint moments for the noise process $n(t)$, and k_{s_2} and k_{s_4} are given by the formulas

$$k_{s_2}(t,\tau) = \frac{1}{2}\sum_{i=1}^{M} P[H_i]\,W_i(t)W_i(\tau)\cos[\omega_0(t-\tau)], \tag{100}$$

$$\begin{aligned} k_{s_4}(u,v,\tau,s) = {} & \sum_{i=1}^{M} P[H_i]\,W_i(u)W_i(v)W_i(\tau)W_i(s) \\ & \cdot \{\tfrac{1}{4}\cos[\omega_0(u-v)]\cos[\omega_0(\tau-s)] \\ & + \tfrac{1}{8}\cos[\omega_0(u-\tau)]\cos[\omega_0(v-s)] \\ & - \tfrac{1}{8}\sin[\omega_0(u-\tau)]\sin[\omega_0(v-s)]\}. \end{aligned} \tag{101}$$

Without further assumptions about the form of k_{n_4}, nothing more definitive can be said about the form of the solution $\phi_2{}^j$. However, if we make the following assumption, then we can obtain an explicit solution to (96) for $\phi_2{}^j$.

Assumption 3) The fourth joint moment of the noise process is identical in form to the fourth joint moment of a Gaussian process, i.e.,

$$k_{n_4}(u,v,\tau,s) = k_{n_2}(u,v)k_{n_2}(\tau,s) + k_{n_2}(u,\tau)k_{n_2}(v,s) + k_{n_2}(u,s)k_{n_2}(v,\tau). \tag{102}$$

This assumption does not preclude non-Gaussian noise. A discrete-time example is given in Appendix B. (See also [15, addendum B, section 3].) With this assumption, it can be shown that the solution to (96) has the form

$$\phi_2{}^j(t,\tau) = \sum_{i,k=1}^{M} \{a_{ik}{}^j\theta_{s_i}(t)\theta_{s_k}(\tau) + b_{ik}{}^j\theta_{c_i}(t)\theta_{c_k}(\tau) + c_{ik}{}^j[\theta_{s_i}(t)\theta_{c_k}(\tau) + \theta_{c_k}(t)\theta_{s_i}(\tau)]\}, \tag{103}$$

where θ_{s_i} and θ_{c_i} are the solutions to the Fredholm equations:

$$\int_T k_{n_2}(t,\tau)\theta_{s_i}(\tau)\,d\tau = W_i(t)\sin(\omega_0 t), \qquad \forall t \in T$$

$$\int_T k_{n_2}(t,\tau)\theta_{c_i}(\tau)\,d\tau = W_i(t)\cos(\omega_0 t), \qquad \forall t \in T. \tag{104}$$

The coefficients are determined by $3M^2$ simultaneous linear equations.

Thus, with Assumption 3), the quadratically constained receiver is a *generalized quadrature receiver* as shown in Fig. 7. The correlators employed by this receiver are the same as those employed by the optimum-for-Gaussian-noise receiver

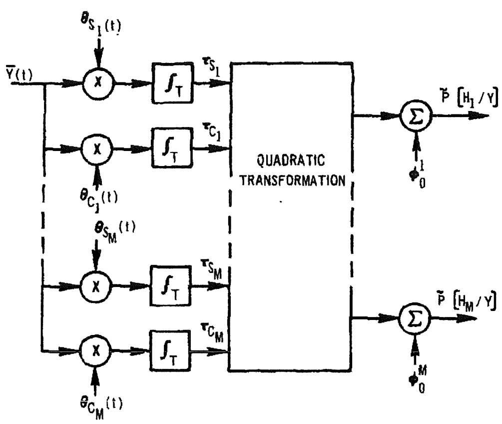


Fig. 7. Quadratically constrained noncoherent receiver for signals in additive noise.

[1]. That is, both receivers reduce the continuous-parameter sample $\{Y(t)\}$ to the same $2M$ correlation statistics $\{\tau_{s_i}, \tau_{c_i}\}_1{}^M$ defined by

$$\tau_{s_i}(Y) \triangleq \int_T Y(t)\theta_{s_i}(t)\, dt$$

$$\tau_{c_i}(Y) \triangleq \int_T Y(t)\theta_{c_i}(t)\, dt. \tag{105}$$

The quadratically constrained receiver then forms its final M statistics as a quadratic combination of these statistics; viz.,

$$\tilde{P}[H_j/Y] = \phi_0{}^j + \sum_{i,k=1}^{M} [a_{ik}{}^j\tau_{s_i}(Y)\tau_{s_k}(Y) + b_{ik}{}^j\tau_{c_i}(Y)\tau_{c_k}(Y) + 2c_{ik}{}^j\tau_{s_i}(Y)\tau_{c_k}(Y)]. \tag{106}$$

The fact that the quadratically constrained receiver is not a generalized quadrature receiver without Assumption 3) is more appropriate than at first might appear. Specifically, it can be shown that for the problem of noncoherent detection of the presence of a single signal, the maximum SNR quadratic receiver does not reduce to the familiar quadrature receiver, that is optimum for Gaussian noise, without Assumption 3).

Now consider the special case where the noise is white and the $2M$ in-phase and quadrature signals,

$$\{W_i(t)\cos(\omega_0 t)\}_1{}^M \text{ and } \{W_i(t)\sin(\omega_0 t)\}_1{}^M,$$

are all mutually orthogonal on T. Also assume that the ith in-phase and quadrature signals have the same energy, denoted by $\frac{1}{2}E_i$. With these assumptions, which are often valid in practice, the quadratic combination of (106) reduces to

$$\tilde{P}[H_j/Y] = \beta_j + \sum_{i=1}^{M} U_i^j[\tau_{s_i}{}^2(Y) + \tau_{c_i}{}^2(Y) - E_i/N_0], \tag{107}$$

where

$$U_i^j \triangleq 4\alpha_i[\delta_{ij} - \beta_j] \tag{108}$$

$$\beta_j \triangleq \gamma_j\alpha_j \Big/ \sum_{k=1}^{M} \gamma_k\alpha_k \tag{109}$$

$$\alpha_i \triangleq \left[\left(\frac{E_i}{N_0}\right)^2 + 8\frac{E_i}{N_0} + \frac{16}{P[H_i]}\right]^{-1} \tag{110}$$

$$\gamma_i \triangleq \frac{P[H_i]E_i}{N_0} + 2. \tag{111}$$

These statistics are very similar to those employed by the optimum-for-Gaussian receiver under the same assumptions of white noise and orthogonal signals; viz.,

$$G_j(Y) \triangleq P[H_j] \exp\{-E_i/4N_0\} \cdot I_0\left\{\frac{1}{N_0}[\tau_{s_i}{}^2(Y) + \tau_{c_i}{}^2(Y)]^{1/2}\right\}, \tag{112}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order [1]. In fact, if all energies $\{E_i\}_1{}^M$ are equal and all priors $\{P[H_i]\}$ are equal, then the two receivers are identical! Also, if all priors are equal, the receivers are identical under threshold conditions ($E_i/N_0 \ll 1$).

Furthermore, if we consider the special problem of detecting the presence of a single signal that is equally likely to be present or absent (i.e., $M = 2$, $W_2(t) \equiv 0$, $P[H_i] = 1/2$), then the two receivers reduce to the following.

Quadratically constrained receiver:

$$[\tau_s{}^2(Y) + \tau_c{}^2(Y)]^{1/2} \underset{H_2}{\overset{H_1}{\gtrless}} N_0\left[\frac{E}{N_0} + \frac{1}{8}\left(\frac{E}{N_0}\right)^2\right]^{1/2} \triangleq \gamma_Q. \tag{113}$$

Optimum-for-Gaussian receiver:

$$[\tau_s{}^2(Y) + \tau_c{}^2(Y)]^{1/2} \underset{H_2}{\overset{H_1}{\gtrless}} N_0 I_0{}^{-1}[\exp(E/4N_0)] \triangleq \gamma_G. \tag{114}$$

It can be shown that $\gamma_Q < \gamma_G$, and that $\gamma_Q \cong \gamma_G$ for $E/N_0 \ll 1$. Thus, the quadratically constrained threshold receiver is identical to the optimum threshold receiver for Gaussian noise.

IV. LINEARLY CONSTRAINED RECEIVERS FOR SIGNAL PARAMETER ESTIMATION

The approach to designing structurally constrained receivers for signal detection that is introduced in Section I extends in an obvious way to problems of signal parameter estimation. A minimum-risk estimate of a random parameter x, given observations Y, is that value of the variable V which minimizes the risk function

$$R(V) = \int C(V,X) f_{x/y}(X/Y)\, dX, \tag{115}$$

where $C(V, X)$ is the cost of estimating the value of x to be

Y, when the true value is X, and where $f_{x/y}(\cdot/Y)$ is the posterior pdf for x. Commonly employed cost functions lead to the following estimation rules.

Minimum-mean-squared-error (MMSE):

$$\hat{X}_{\mathrm{MMSE}} = \operatorname*{mean}_{X} \{f_{x/y}(X/Y)\}. \tag{116}$$

Maximum a posterior probability (MAP):

$$\hat{X}_{\mathrm{MAP}} = \operatorname*{mode}_{X} \{f_{x/y}(X/Y)\}. \tag{117}$$

Maximum likelihood[5] *(ML):*

$$\hat{X}_{ML} = \operatorname*{mode}_{X} \{f_{x/y}(X/Y)/f_x(X)\}$$

$$= \operatorname*{mode}_{X} \{f_{y/x}(Y/X)\}. \tag{118}$$

Paralleling the approach introduced in Section I, an obvious way to impose a structural constraint on the design of a receiver is to constrain the structure of the functionals $\{f_{x/y}(X/\cdot)\}_X$. That is, the L-constrained parameter estimator that we propose employs L-MMSE estimates of posterior densities in place of the true densities in the estimation rules (116)–(118). For example, the linearly constrained MMSE estimate of the posterior pdf is, paralleling (24) and (25),

$$\tilde{f}_{x/y}(X/Y) = f_x(X)[1 + S(X,Y)], \tag{119}$$

$$S(X,Y) \triangleq \int_T \int_T k_y^{-1}(\tau,\sigma) E\{\bar{y}(\sigma)/X\} Y(\tau)\, d\sigma\, d\tau. \tag{120}$$

Furthermore, Interpretations 1) and 2) in Section II with H_i replaced by X apply here to the statistics $S(X, Y)$.

The linearly constrained ML estimate, denoted by $\hat{X}_{\text{L-ML}}$, reduces to

$$\hat{X}_{\text{L-ML}} = \operatorname*{mode}_{X} \{S(X,Y)\}, \tag{121}$$

and the L-MMSE estimate defined by

$$\hat{X}_{\text{L-MMSE}} \triangleq \operatorname*{mean}_{X} \{\tilde{f}_{x/y}(X/Y)\} \tag{122}$$

is identical to the conventional MMSE linear estimate of x, denoted by $\tilde{X}/Y$ (see Appendix A).

In the following, we briefly analyze and compare the newly proposed estimator $\hat{X}_{\text{L-ML}}$ and the well-known estimator $\hat{X}_{\text{L-MMSE}}$ for the specific class of problems where the observations consist of a signal in additive noise:

$$Y(t) = U(t,X) + W(t), \qquad \forall t \in [0,T], \tag{123}$$

where $U(\cdot, X)$ is a deterministic function of its first argument, indexed by X, and where $W(t)$ is a sample of a white (not necessarily Gaussian) noise process with power spectral density N_0. By employing the parallel of (35) for this class of problems, the expression for the statistic $S(X, Y)$ reduces to

$$S(X,Y) = \int_0^T [Y(t) - \tilde{U}(t)/Y]\,[U(t,X) - m_u(t)]\, dt, \tag{124}$$

where $m_u(t)$ is the mean signal, and $\tilde{U}(t)/Y$ is the zero error-mean, minimum error-variance estimate of $U(t, x)$, given observations Y. The mode of $S(\cdot/Y)$ is identical to the mode of the simplified statistics $S'(\cdot/Y)$ obtained from $S(\cdot/Y)$ by eliminating $m_u(t)$. If we denote the error in estimating $U(t, X)$ by $N(t)$,

$$N(t) \triangleq \tilde{U}(t)/Y - U(t,X_0), \tag{125}$$

where X_0 is the true value of X, then S' can be expressed as

$$S'(X,Y) = \int_0^T U(t,X)W(t)\, dt - \int_0^T U(t,X)N(t)\, dt. \tag{126}$$

One might intuitively expect the first time correlation (first term in right member) to be independent of X, and the second time correlation to be minimum near $X = X_0$. Hence the mode of S' would appear to be an appropriate estimator.

In fact, it turns out that for some problems $\hat{X}_{\text{L-ML}}$ is a more appropriate estimator than $\hat{X}_{\text{L-MMSE}}$, whereas the opposite is true for other problems. This is illustrated by the following two examples.

Example 1) Amplitude parameter: If $U(t,X) = XU(t)$, then it can be shown that

$$\hat{X}_{\text{L-MMSE}} = m_x N_0/[N_0 + \sigma_x^2 E] + \tau(Y)\sigma_x^2/[N_0 + \sigma_x^2 E], \tag{127}$$

where m_x and σ_x^2 are the mean and variance of x, respectively, and where

$$\tau(Y) \triangleq \int_0^T Y(t)U(t)\, dt \tag{128}$$

$$E \triangleq \int_0^T U^2(t)\, dt. \tag{129}$$

It follows that for $N_0 \gg \sigma_x^2 E$,

$$\hat{X}_{\text{L-MMSE}} \cong m_x, \tag{130}$$

and for $N_0 \ll \sigma_x^2 E$,

$$\hat{X}_{\text{L-MMSE}} \cong X_0 \triangleq \text{true value of } X. \tag{131}$$

On the other hand, $\hat{X}_{\text{L-ML}}$ does not exist since $S'(X,Y)$ is linear in X. Thus, the conventional estimate $\hat{X}_{\text{L-MMSE}}$ is clearly the more appropriate of the two.

Example 2) Phase-parameter: If $U(t,X) = \cos(\omega_0 t + X)$, where ω_0 is an integer multiple of $2\pi/T$ and X is uniformly distributed on $[-\pi, \pi]$, then it can be shown that

[5] The maximum likelihood (ML) estimate is not a true minimum-risk estimate unless the prior density f_x is uniform.

$$\hat{X}_{\text{L-MMSE}} = -[N_0 + T/4]^{-1} \int_0^T Y(t) \sin(\omega_0 t)\, dt$$
$$= [N_0 + T/4]^{-1} \left[\frac{T}{2} \sin(X_0) - W_s\right] \tag{132}$$

where

$$W_s \triangleq \int_0^T W(t) \sin(\omega_0 t)\, dt. \tag{133}$$

It can also be shown that

$$\hat{X}_{\text{L-ML}} = -\tan^{-1}\left[\frac{\int_0^T Y'(t) \sin(\omega_0 t)\, dt}{\int_0^T Y'(t) \cos(\omega_0 t)\, dt}\right], \tag{134}$$

where

$$Y'(t) \triangleq Y(t) - \tilde{U}(t)/Y. \tag{135}$$

Furthermore, it can be shown that

$$\tilde{U}(t)/Y = \frac{T}{T + 4N_0}\left\{\cos(\omega_0 t + X_0) + \frac{2}{T}[W_c \cos(\omega_0 t) + W_s \sin(\omega_0 t)]\right\}, \tag{136}$$

where

$$W_c \triangleq \int_0^T W(t) \cos(\omega_0 t)\, dt. \tag{137}$$

Substituting (135) and (136) into (134) yields

$$\hat{X}_{\text{L-ML}} = \tan^{-1}\left[\frac{\sin(X_0) - 2W_s/T}{\cos(X_0) + 2W_c/T}\right]. \tag{138}$$

It follows from (132) and (138) that

$$\hat{X}_{\text{L-MMSE}} \cong 2 \sin(X_0) \tag{139}$$

$$\hat{X}_{\text{L-ML}} \cong X_0 \tag{140}$$

for $N_0 T \ll 1$. Furthermore, $\hat{X}_{\text{L-ML}}$ is identical to the true maximum likelihood estimate when $w(t)$ is Gaussian [1]. Thus, the newly proposed estimate $\hat{X}_{\text{L-ML}}$ is clearly superior to the conventional estimate $\hat{X}_{\text{L-MMSE}}$.

V. SUMMARY AND CONCLUSIONS

We have introduced the *constrained Bayesian methodology* for the design of structurally constrained receivers for signal detection and estimation. We have illustrated the use of the methodology by designing linearly constrained receivers for coherent reception of deterministic signals in additive and multiplicative noise, and for signal parameter estimation in additive noise, and by designing quadratically constrained receivers for noncoherent reception of deterministic signals in additive noise. These receivers turn out to be remarkably similar to receivers that are optimum for additive Gaussian noise, under certain restrictive assumptions. This is not surprising since the methodology is, in essence, an extension and generalization of the linear MMSE estimation methodology that has a simple and well known relationship to Gaussian models.

For signal detection in non-Gaussian additive noise environments where the noise pdf's exhibit heavy tails, highly nonlinear receivers that employ, for example, clipping or limiting nonlinearities are known to be superior to linear or mildly nonlinear (e.g., quadratic) receivers. As mentioned in Section I-D, the methodology is currently being evaluated for such applications. Also, as discussed in Section I-D, the methodology is currently being evaluated for application to random signal detection.

It is hoped that the preliminary results presented herein on the evaluation of the proposed methodology will stimulate further investigation of its utility. However, on the basis of the results obtained so far, we can conclude that the methodology at least provides a unified theory of receiver design based on the constrained MMSE criterion. This unification yields new insight into this old approach, clarifying both strengths and weaknesses of the approach. This is further evidenced by the results presented in [17], regarding a general comparative analysis of the conventional decision rule $\tilde{X}_{\text{mean}}$ and the newly proposed rule $\tilde{X}_{\text{mode}}$, defined in Section II-B2.

APPENDIX A

L-MMSE ESTIMATORS FOR POSTERIOR PROBABILITIES

Let y be a set of random variables with samples denoted by Y, and let W and X be related events. Let Ω be the certain event and ϕ the null event. Let L be *any* Hilbert space generated by mean square images of functionals of the random variables y. Let δ be the indicator function for the event X;

$$\delta(X) \triangleq \begin{cases} 1, & \text{if } X \text{ occurs} \\ 0, & \text{if } X \text{ does not occur.} \end{cases} \tag{A1}$$

It can be shown by using the Hilbert space orthogonal projection theorem [13], that the L-MMSE estimate of $\delta(X)$, denoted by $\tilde{\delta}(X)/Y$, is identical to the L-MMSE estimate of the posterior (conditional) probability $P[X/Y]$, denoted by $\tilde{P}[X/Y]$; i.e., the two mean-squared errors

$$E\{[\delta(X/y) - z]^2\} \tag{A2}$$

$$E\{[P[X/y] - z]^2\} \tag{A3}$$

are simultaneously minimized over all z in L by the estimator

$$z = \tilde{\delta}(X/y) = \tilde{P}[X/y]. \tag{A4}$$

This estimator is the orthogonal projection of $\delta(X/y)$ (and of $P[X/y]$) onto L. The linearity of orthogonal projection operators leads directly to the following fundamental properties of unbiased L-MMSE estimators for posterior probabilities:

$$1 \geqslant E\{\tilde{P}[X/y]\} \geqslant 0 \tag{A5}$$

$$\tilde{P}[\Omega/Y] = 1, \tilde{P}[\emptyset/Y] = 0 \tag{A6}$$

$$\tilde{P}[X \cup W/Y] = \tilde{P}[X/Y] + \tilde{P}[W/Y] - \tilde{P}[X \cap W/Y] \tag{A7}$$

$$\left\{\bigcup_i W_i = \Omega\right\} \Rightarrow \left\{\sum_i \tilde{P}[X \cup W_i/Y] = \tilde{P}[X/Y]\right\}. \tag{A8}$$

Similarly, it can be shown that if x is a discrete random variable with range $\{X_i\}$, then the L-MMSE estimate of X denoted by $\tilde{X}/Y$ is identical to the mean of the L-MMSE estimated posterior distribution

$$\tilde{X}/Y = \sum_i X_i \tilde{P}[X_i/Y]. \tag{A9}$$

The analogs of (A5)-(A9) for unbiased L-MMSE estimators for posterior probability distribution functions (denoted by $\tilde{F}_{x/y}(\cdot/\cdot)$) for continuous random variables can also easily be validated:

$$1 \geqslant E\{\tilde{F}_{x/y}(X/y)\} \geqslant 0 \tag{A10}$$

$$\tilde{F}_{x/y}(\infty/Y) = 1, \tilde{F}_{x/y}(-\infty/Y) = 0 \tag{A11}$$

$$\tilde{F}_{x,w/y}(X,\infty/Y) = \tilde{F}_{x/y}(X/Y) \tag{A12}$$

$$\tilde{f}_{x/y}(X/Y) = \frac{d}{dX}\tilde{F}_{x/y}(X/Y) \tag{A13}$$

$$\tilde{X}/Y = \int_{-\infty}^{\infty} X \tilde{f}_{x/y}(X/Y)\, dX \tag{A14}$$

where $\tilde{f}_{x/y}(X/y)$ is the L-MMSE estimator for the posterior pdf $f_{x/y}(X/y)$.

APPENDIX B

GAUSSIAN-LIKE NOISE

Let $\{n_i\}_{-\infty}^{\infty}$ be a sequence of independent identically distributed random variables with finite fourth moment and odd moments all of which are zero. Then $\{n_i\}$ is a discrete-time strict sense white noise process, and has joint second and fourth moments

$$k_n(i,j) = \overline{n^2}\delta_{ij}$$

$$r_n(i,j,p,q) = [(\overline{n^2})]^2[\delta_{ij}\delta_{pq} + \delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp}] + [\overline{n^4} - 3[(\overline{n^2})]^2][\delta_{ij}\delta_{ip}\delta_{iq}]. \tag{B1}$$

Thus, if $\overline{n^4} = 3[(\overline{n^2})]^2$, then r_n is identical in form to the fourth joint moment of white Gaussian noise, although n certainly need not be Gaussian. Now, if n is passed through any stable linear filter, with unit-pulse-response function h_{ij}, then the second and fourth joint moments of the colored output process $\{m_i\}_{-\infty}^{\infty}$ are easily shown to be

$$k_m(i,j) = \overline{n^2}\sum_k h_{ik}h_{jk} \tag{B2}$$

$$r_m(i,j,p,q) = k_m(i,j)k_m(p,q) + k_m(i,p)k_m(j,q) + k_m(i,q)k_m(j,p) \tag{B3}$$

provided that $\overline{n^4} = 3[(\overline{n^2})]^2$. Thus r_m is identical in form to the fourth joint moment of colored Gaussian noise with autocovariance k_m, although m certainly need not be Gaussian. For example, if $\{n_i\}$ is not Gaussian and there exists a finite number K that bounds the memory of the filter

$$h_{ij} = 0, \qquad \forall i,j \ni |i-j| > K, \tag{B4}$$

then $\{m_i\}$ *cannot* be Gaussian.

An example of a non-Gaussian density for which $\overline{n^4} = 3[(\overline{n^2})]^2$ is

$$f_n(N) = [8/3\sqrt{6}\pi\sigma][1 + (N/\sqrt{3}\sigma)^4]^{-2} \tag{B5}$$

where $\sigma^2 \triangleq \overline{n^2}$.

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Convolutional Code Performance in the Rician Fading Channel

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Abstract—**The performance of short constraint length convolutional codes in conjunction with binary phase-shift keyed (BPSK) modulation and Viterbi maximum likelihood decoding on the classical Rician fading channel is examined in detail. Primary interest is in the bit error probability performance as a function of E_b/N_0 parameterized by the fading channel parameters. Fairly general upper bounds on bit error probability performance in the presence of fading are obtained and compared with simulation results in the two extremes of zero channel memory and infinite channel memory. The efficacy of simple block interleaving in combating the memory of the channel is thoroughly explored. Results include the effects of fading on tracking loop performance and the subsequent impact on overall coded system performance. The approach is analytical where possible; otherwise resort is made to digital computer simulation.**

I. INTRODUCTION

THE use of convolutional codes in conjunction with coherent binary phase-shift-keyed (BPSK) modulation has proven to be an effective and efficient means of obtaining error control on the classical additive white Gaussian noise (AWGN) channel. Recent work by Heller and Jacobs [1] has discussed the performance of short constraint length convolutional codes in conjunction with coherent BPSK modulation and Viterbi maximum likelihood decoding on the AWGN channel. Previous work by Jacobs [2] has treated the performance of longer constraint length codes together with sequential decoding. In an increasing number of important applications, however, the AWGN channel provides an entirely inappropriate model of the propagation environment. We will be particularly concerned with those situations where the received signal component is known to undergo fading as the result of an appropriately defined channel scattering mechanism. Examples include HF and tropospheric scatter links [3], the aeronautical channel [4], and the planetary entry channel [5], [6]. In such cases the channel can be adequately modeled by assuming the received signal is a linear combination of a specular and a diffuse scatter component received in the presence of AWGN. Assuming further that the diffuse scatter component can be represented as the product of a complex zero-mean Gaussian process and the original transmitted signal component, we have the classical Rician fading channel.

It is of some interest then to provide a complete characterization of the performance of short constraint length convolutional codes in conjunction with BPSK modulation and Viterbi decoding on the Rician fading channel. The remainder of this paper is devoted to this purpose. Primary interest is in the bit error probability performance as a function of E_b/N_0 parameterized by the fading channel parameters. Fairly general upper bounds on bit error probability performance in the presence of fading are obtained and compared

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