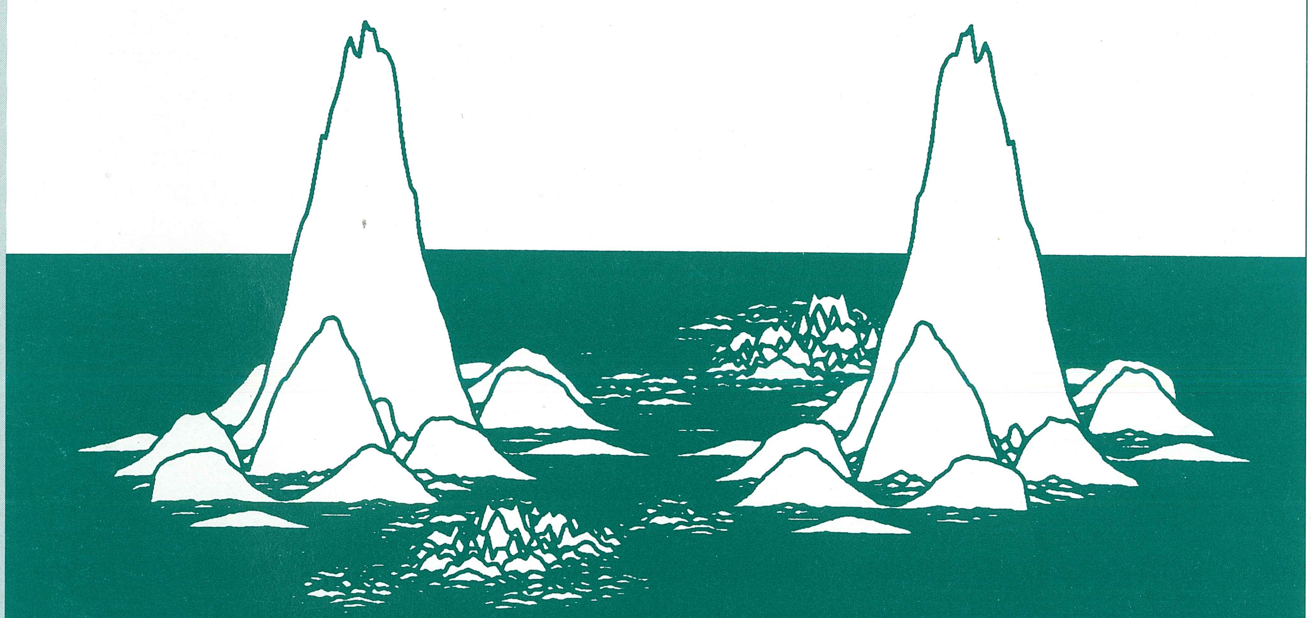
William A. Gardner

STATISTICAL SPECTRAL ANALYSIS

A NONPROBABILISTIC THEORY



PRENTICE HALL INFORMATION AND SYSTEMS SCIENCES SERIES
Thomas Kailath, Series Editor

DO YOUR STUDENTS A FAVOR—BREAK WITH TRADITION—

Why put them on the stochastic process bandwagon

when, instead, you can provide them with a sound philosophical foundation for understanding both theory and method for statistical measurement and analysis of time-series data.

Why make such a radical departure from the orthodox approach? The answer is simple.

The stochastic process, with its hypothetical ensemble of random sample paths and associated probability measure defined on an abstract space, is in many realistic situations irrelevant. It introduces unnecessary abstraction that widens the gap between theory and practice into a chasm that can only be bridged with the magical assumption that all stochastic models of interest are ergodic, which means that abstract expected values are identical to practical time averages. Design methods for controlling resolution, leakage, and reliability do not follow naturally from this approach and are difficult to motivate and conceptualize.

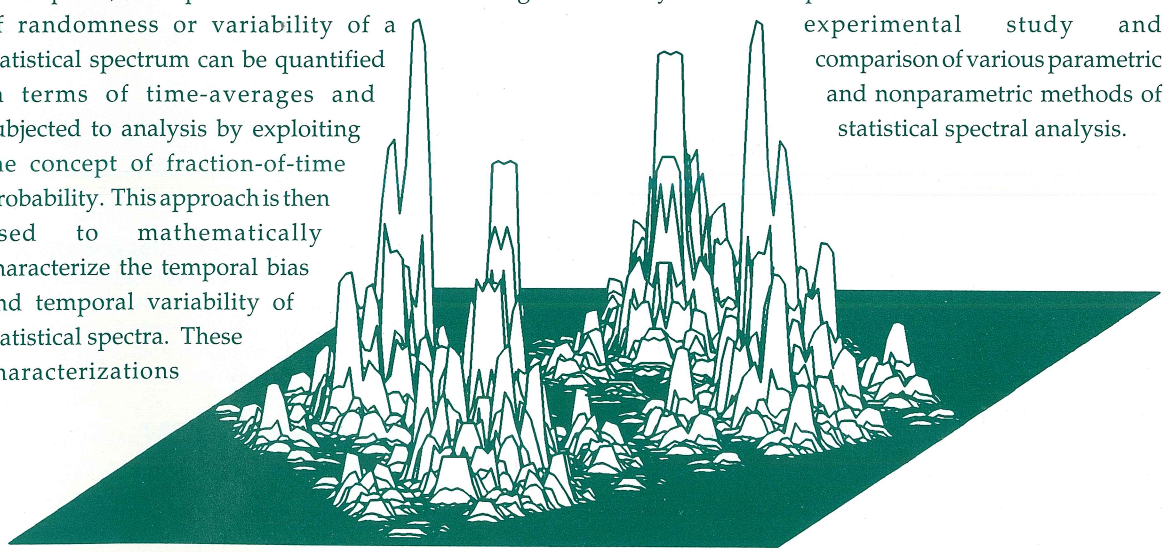
When time-averages are the quantities of interest in research with real data, why not use a theory based directly on time-averages?

Although Professor Gardner recognizes in his previous graduate-level book on stochastic processes (Introduction to Random Processes, Macmillan, 1985, McGraw-Hill, 1990) that stochastic processes have many important applications to real-world problems—this is, in fact, a theme of his book—he also recognizes that it is an unfortunate accident that they have for decades been foisted upon the unsuspecting student as the only way to comprehend and study the problems of statistical measurement and analysis of time-series data. This is not to say that probability is irrelevant to this problem area. Probability, when interpreted as the fraction-of-time of occurrence of an event throughout the life of a time-series, can be defined solely in terms of time averages. And this allows for the development of a complete probabilistic theory of statistical time-series analysis based entirely on time averages—those things which in fact are used in the real world.

In his book *Statistical Spectral Analysis: A Nonprobabilistic Theory* Professor Gardner provides us with the first comprehensive development of theory and method for statistical spectral analysis based entirely on time averages of single time-series. Design methods for controlling resolution, leakage, and reliability follow naturally from this approach and are easy to motivate and conceptualize.

OVERVIEW: PART I

After an introduction to the origins of spectral analysis and its objectives and applications in Chapter 1, the basic elements of empirical spectral analysis are introduced in Chapter 2. The time-variant periodogram for nonstatistical spectral analysis is defined and characterized as the Fourier transform of the time-variant correlogram, and its temporal and spectral resolution properties are described. The effects of linear time-invariant filtering and periodic time sampling are described. Then in Chapter 3, the fundamentals of statistical spectral analysis are introduced. The equivalence between statistical spectra obtained from temporal smoothing and statistical spectra obtained from spectral smoothing is established, and the relationship between these statistical spectra and the abstract limit spectrum is derived. The limit spectrum is characterized as the Fourier transform of the limit autocorrelation, and the effects of linear time-invariant filtering and periodic time-sampling on the limit spectrum are described. Various continuous-time and discrete-time models for time-series are introduced, and their limit spectra are calculated. Chapter 4 presents a wide variety of analog (continuous-time) methods for empirical statistical spectral analysis, and it is shown that all these methods are either exactly or approximately equivalent when a substantial amount of smoothing is done. The spectral leakage phenomenon is explained, and the concept of an effective spectral smoothing window is introduced. Then a general representation for the wide variety of statistical spectra obtained from these methods is introduced and shown to provide a means for a unified study of statistical spectral analysis. In Chapter 5, it is explained that the notion of the degree of randomness or variability of a statistical spectrum can be quantified in terms of time-averages and subjected to analysis by exploiting the concept of fraction-of-time probability. This approach is then used to mathematically characterize the temporal bias and temporal variability of statistical spectra. These characterizations form the basis for an in-depth discussion of design trade-offs involving the resolution, leakage, and reliability properties of a statistical spectrum. The general representation introduced in Chapter 4 is used here to obtain a unified treatment for the wide variety of spectral analysis methods described in Chapter 4. Chapter 6 complements Chapter 4 by presenting a variety of digital (discrete-time) methods for statistical spectral analysis. Chapter 7 generalizes the concept of spectral analysis of a single real-valued time-series to that of cross-spectral analysis of two or more complex-valued time-series. It is established that the cross spectrum, which is a measure of spectral correlation, plays a fundamental role in characterizing the degree to which two or more time-series are related by linear time-invariant transformations. Methods for measurement of statistical cross spectra that are generalizations of the methods described in earlier chapters are presented, and the temporal bias and temporal variability of statistical cross spectra are mathematically characterized in a unified way based on a general representation. In Chapter 8, the application of statistical spectral analysis to time-variant phenomena is studied. Fundamental limitations on temporal and spectral resolution are discussed, and the roles of ensemble averaging and probabilistic models are described. Finally, in Chapter 9, an introduction to the theory of autoregressive modeling of time-series is presented and used as the basis for describing a variety of autoregressive and autoregressive-moving-average parametric methods of statistical spectral analysis. The chapter concludes with an extensive experimental study and comparison of various parametric and nonparametric methods of statistical spectral analysis.



OVERVIEW: PART II

A phenomenon or the time-series it produces is said to exhibit *second-order periodicity* if and only if there exists some quadratic time-invariant transformation of the time-series that gives rise to finite additive periodic components which result in spectral lines. In Part II, a comprehensive theory of statistical spectral analysis of time-series from phenomena that exhibit second-order periodicity that does not rely on probabilistic concepts is developed. It is shown that second-order periodicity in the time-series is characterized by spectral correlation, and that the *degree of spectral coherence* of such a time-series is properly characterized by a spectral correlation coefficient, the *spectral autocoherence function*. A fundamental relationship between superposed epoch analysis (synchronized averaging) of lag products, and spectral correlation, which is based on the *cyclic autocorrelation* and its Fourier transform, the *cyclic spectrum*, is revealed by a synchronized averaging identity. It is shown that the cyclic spectrum is a spectral correlation density function. Relationships to the ambiguity function and the Wigner-Ville distribution are also explained. It is shown that the deterministic theory can be given a probabilistic interpretation in terms of fraction-of-time distributions obtained from synchronized time averages. Several fundamental properties of the cyclic spectrum are derived. These include the effects of time-sampling, modulation, and periodically time-variant filtering, and the spectral correlation properties of Rice's representation for band-pass time-series. The specific spectral correlation properties of various modulation types, including amplitude and quadrature-amplitude modulation, pulse modulation, phase and frequency modulation, and phase- and frequency-shift keying are derived. The basics of cyclic spectrum estimation, including temporal, spectral, and cycle resolution, spectral and cycle leakage, and reliability, are described, and the relationships among a variety of measurement methods are explained. Applications of the cyclic spectrum concept to problems of signal detection, signal extraction, system identification, parameter estimation and synchronization are presented. Finally, an approach to probabilistic analysis of cyclic spectrum estimates based on fraction-of-time distributions obtained from synchronized time averages is introduced. It is emphasized that the fundamental results of the theory of cyclic spectral analysis presented in Part II are generalizations of results from the conventional theory of spectral analysis presented in Part I, in the sense that the latter are included as the special case of the former, for which the cycle frequency is zero (or the period is infinite) or the time-series is purely stationary.

Dr. Gardner is Professor of Electrical Engineering and Computer Science at the University of California, Davis, President of the engineering consulting firm Statistical Signal Processing, Inc., and a Fellow of the IEEE. He is the author of several graduate-level textbooks and professional reference books, and numerous research papers, for which he has received Best-Paper-of-the-Year Awards from the Institute of Electrical and Electronics Engineers and the European Association for Signal Processing.

 ISBN 0–13–844572–9

REVIEWS

One prepublication reviewer of **Statistical Spectral Analysis** *states:*

"The manuscript is well written and the book as a whole is a competent piece of work...it opens up a whole new line of reasoning in the important area of spectral analysis...people who look at and study this book will see a refreshing new approach that might not only help their own research but also help their teaching..."

Another prepublication reviewer adds:

"Professor Gardner's historical preface is well done and is perfectly correct as to what has happened in the development of numerical spectral analysis over the past 40 years and more. The approach that he follows in this book makes a lot of sense...The originality of the manuscript lies mainly in the decision to organize a whole text on a consistent foundation...In my opinion, a good fraction of instructors will be delighted to switch [to this approach]...I would think that half the instructors are dissatisfied with their current book and would like to try an alternative, especially one that has a clear philosophical foundation."

Other quotations from recognized leaders in the field:

"This excellent book is an outstanding contribution that is going to have a tremendous impact. The author has gone very deeply into the subject with much insight...This book can be highly recommended to the engineering profession"

—Enders A. Robinson,
Distinguished Professor
Society of Exploration Geophysicists Medalist
member of National Academy of Engineering

"I admire the scholarship of this book and its radical departure from the stochastic process bandwagon of the past 40 years."

—Professor James Massey,
Fellow of IEEE
Past Chairman, IEEE Information Theory Group
member of National Academy of Engineering

"If we are to go beyond pure mathematical deduction and make advances in the realm of phenomena, theory should start from the data. To do otherwise risks failure to discover that which is not built into the model...Professor Gardner's book demonstrates a consistent approach from data, those things which in fact are given, and shows that analysis need not proceed from assumed probability distributions or random processes. This is a healthy approach and one that can be recommended to any reader."

—Professor Ronald N. Bracewell,
Fellow of IEEE
Fellow of the Royal Astronomical Society
member of International Academy of Astronautics

"It is always hard to go against the established order of things, but I am sure the book will have a considerable impact. It will be a definitive text on spectral analysis."

—Professor Thomas Kailath
Fellow of IEEE
Fellow of Institute of Mathematical Statistics
Past President, IEEE Information Theory Group
member of National Academy of Engineering

"It is important...that until Gardner's...book was published there was no attempt to present the modern spectral analysis of random processes consistently in language that uses only time-averaging rather than averaging over the statistical ensemble of realizations [of a stochastic process]...Professor Gardner's book is a valuable addition to the literature."

—Professor Akiva M. Yaglom
Academy of Sciences of the USSR

CONTENTS

Every chapter concludes with a detailed summary and an extensive set of exercises.