FRESH Filtering of Wideband FM in Interference

For Wide Band Frequency Modulation (WBFM) signals, in order to meet the conditions under which FM is approximately equal to DSB-AM plus a quadrature carrier, we must first pass the signal through a frequency divider which divides the instantaneous frequency of the FM signal by some integer which is large enough to ensure that the phase deviation due to modulation is much smaller than 2π. That is, $s(t) = \sin\left[\int \omega(t)dt\right]$ must be converted to $\tilde{s}(t) = \sin\left[\frac{1}{n}\int \omega(t)dt\right]$ (ignoring any constant additive phase term) for some large enough positive integer n. Here $\omega(t)$ is the instantaneous frequency; that is, it is the time derivative of the time-varying angle of the sinusoid.

If $\omega(t) = 2\pi f_o + b(t)$, where $\int b(t)dt = a(t)$, then

$$\tilde{s}(t) = \sin\left[\frac{1}{n}\int \omega(t)dt\right] = \sin\left[\frac{2\pi f_o}{n}t + \frac{1}{n}a(t)\right]$$

and

$$\sin\left[\frac{2\pi f_o}{n}t + \frac{1}{n}a(t)\right] = \frac{1}{2}\cos\left[\frac{1}{n}a(t)\right]\sin\left[\frac{2\pi f_o}{n}t\right] + \sin\left[\frac{1}{n}a(t)\right]\frac{1}{2}\cos\left[\frac{2\pi f_o}{n}t\right]$$
$$\cong \frac{1}{2}\sin\left[\frac{2\pi f_o}{n}t\right] + \frac{1}{2n}a(t)\cos\left[\frac{2\pi f_o}{n}t\right]$$

where the approximation is close if

$$\frac{1}{n}|a(t)| \ll 2\pi.$$

If we started out with the signal plus corruption $x(t) = s(t) + c(t)$ then, at the output of the frequency divider, we have $y(t) = \tilde{s}(t) + \tilde{c}(t) + e(t)$ where $e(t) = y(t) - \tilde{s}(t) - \tilde{c}(t)$ and $\tilde{c}(t)$ is the frequency divided version of $c(t)$.

Unfortunately, $e(t)$ is not easy to characterize, because a frequency divider is not linear; there are various methods for dividing frequency, one of which is described as follows: At every up-crossing (a zero crossing with a positive slope) of an input FM signal, the divider outputs a fixed signal level and holds that output level until the m-th up-crossing and then switches the polarity of that level and holds it until the 2m-th up-crossing, and switches the polarity again, etc. This will produce a square-wave with fundamental frequency f_o/n where f_o is the frequency of the input FM signal during those up crossings, and $n = 2m$. Alternatively, the polarity of the output square-wave can be switched every n zero crossing (regardless of whether these are up-crossings or down-crossings. The final step for the divider is to pass this square-wave through a bandpass filter that passes the fundamental-frequency component and rejects all higher-order harmonics. Yet another alternative is to select the bandpass filter to pass only the 3^{rd} harmonic (symmetrical square-waves contain only odd-order harmonics). Then n will be only $1/3$ of the n for the fundamental-frequency component.

Sources of distortion to the frequency-divided FM signal in the output of such a frequency divider include any changes in the input FM frequency during the hold period and any changes to the FM signal's zero crossings resulting from additive noise or additive interfering signals or frequency-selective fading. Because of this, one can expect performance, relative to a perfect frequency divider, to degrade as the strengths of any and all these sources of input corruption grow. Like essentially all nonlinear signal processors, a threshold phenomenon can be expected. That is, performance can remain good as sources of corruption grow in strength, until they reach a level at which the divider fails catastrophically. In this regard, the fact that the frequency-divided sine wave at the divider output is attenuated by the factor $1/n$ may exacerbate the impact on the output due to the presence of corruption at the input.

Ignoring the error $e(t)$ in the divider output, as long as $\tilde{c}(t)$ resides in complementary sub-bands on each side of the center frequency f_o/n, then Frequency Shift Filtering (FRESH filtering) can suppress it. But the best suppression requires knowledge of the bands in which $\tilde{c}(t)$ resides. These bands can, however, be determined from knowledge of the bands in which $c(t)$ resides, which may or (unfortunately) may not be known.

In conclusion, if the average power level of $e(t)$ at the output of the frequency divider is low enough, co-channel interference can potentially be suppressed for WBFM signals. For NBFM, for which frequency division is not needed, $e(t) = 0$ and FRESH filtering can indeed suppress co-channel interference.

Although this is of practical interest, as is, it would be more interesting if the conditions under which $e(t)$ has low enough average power to remain below threshold can be determined. This challenge is somewhat related to the problem of threshold characterization for FM demodulators, a topic that has received significant attention in the communications systems literature of the past.

As is, the above concept will be of little practical use until the threshold phenomenon is quantified. This quantification might well depend on the nature of the additive corruption, and this could indicate that the threshold value of the input SCR (signal-to-corruption ratio) must be determined independently for each application. Consequently, experimental determination of the threshold SCR is probably the most pragmatic approach.

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