Cyclic MUSIC Algorithms for Signal-Selective Direction Estimation*

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Abstract

Signal-selective direction finding algorithms that overcome many of the limitations of existing direction finding techniques are presented. The new algorithms automatically classify signals as desired or undesired based on their known spectral correlation properties and estimate only the desired signals' directions of arrival. The signal-selective nature of the new techniques eliminates the need for knowledge of the characteristics of the noise or interference in the environment; furthermore, it enables the new techniques to resolve a number of desired signals not exceeding the number of sensors in the presence of arbitrary noise and a virtually unlimited number of unknown interferers. For example, the interferers can exhibit an arbitrarily high degree of correlation amongst themselves and can arrive from directions arbitrarily close to those of the desired signals.

1 Introduction

Currently popular methods of direction finding (DF) using sensor arrays, such as MUSIC [1] and modified versions of MUSIC [2], suffer from various drawbacks. These include: (1), the requirement that the total number of signals impinging on the array, including both signals of interest (SOIs) and interference, be less than the number of sensors or that the characteristics of interfering signals be known so that their effects can be subtracted; (2), the inability to resolve two signals spaced more closely than the resolution threshold of the array when only one signal is a SOI; and (3), the requirement that the noise characteristics of the sensors and the environment be known or that they be accurately modeled as independent and identically distributed Gaussian random processes [3].

The signal selective DF algorithms presented here effectively circumvent these drawbacks in environments where SOIs exhibit *cyclostationarity (spectral correlation)* [5]. This is accomplished by exploiting the differing spectral correlation characteristics of the different signals.

2 Cyclic MUSIC Algorithms

The fundamental difference between existing techniques and the new techniques presented here is that all existing techniques locate the signal sources using spatial coherence properties (e.g., as measured by the array covariance matrix), whereas the new techniques locate the signals using their spectral coherence properties as well. The theory of spectral correlation is presented in [5] where it is shown that a signal exhibits spectral correlation if it is correlated with a frequency-shifted version of itself, that is, if the *cyclic auto-*

correlation function, defined by

$$R_{ss}^{\alpha}(\tau) \triangleq \left\langle s(t+\tau/2)\left\{s(t-\tau/2)e^{j2\pi\alpha t}\right\}^{*}\right\rangle_{\infty}, \qquad (1)$$

is not identically zero for some *cycle frequency* α and some lag parameter τ. For example, the frequency components in stationary noise are not correlated with each other, so the cyclic autocorrelation of that noise is identically zero for all $\alpha \neq 0$ and is equal to the conventional autocorrelation for $\alpha = 0$. Most communication signals exhibit nonzero spectral correlation at one or more cycle frequencies. For example, a PCM signal with a stationary baseband has nonzero cyclic autocorrelation for α equal to its baud rate.

The spectral correlation concept yields interesting and useful results when applied to analysis of antenna array signals. Consider an array having M sensors which receives $L_{\alpha} < M$ signals $s_i(t)$, $i = 1, \ldots, L_{\alpha}$, that exhibit spectral correlation at a particular α of interest (i.e., they are signals of interest) and an *arbitrary* number of interferers and *arbitrary* noise that do not exhibit spectral correlation at that α. For example, the interferers can exhibit arbitrarily high correlation amongst themselves. However, in the work presented here, the desired signals must not be fully correlated amongst themselves and must be independent of the interference. If the narrowband assumption holds (i.e., if the transit time of the received wave across the array is much less than the period of the carrier), then the received signal can be approximated by

$$\mathbf{x}(t) = \sum_{i=1}^{L_{\alpha}} \mathbf{a}(\theta_i)s_i(t) + \mathbf{i}(t) \qquad (2)$$

$$= \mathbf{A}(\Theta)\,\mathbf{s}(t) + \mathbf{i}(t), \qquad (3)$$

where $\mathbf{a}(\theta_i)$ models the relative gains and phases of the sensors acting on a signal impinging from angle θ_i, and $\mathbf{i}(t)$ contains the interference and noise. The direction vectors $\mathbf{a}(\theta_i)$ are assumed to linearly independent (i.e., $\mathbf{A}(\Theta)$ has full column rank). The cyclic autocorrelation matrix of this received signal is given by

$$\mathbf{R}_{\mathbf{xx}}^{\alpha}(\tau) = \mathbf{A}(\Theta)\,\mathbf{R}_{\mathbf{ss}}^{\alpha}(\tau)\,\mathbf{A}^{H}(\Theta), \qquad (4)$$

where $(\cdot)^H$ denotes conjugate transpose, and where $\mathbf{R}_{\mathbf{ss}}^{\alpha}(\tau)$ is the cyclic autocorrelation matrix of the transmitted signals,

$$\mathbf{R}_{\mathbf{ss}}^{\alpha}(\tau) = \left\langle \mathbf{s}(t+\tau/2)\left\{\mathbf{s}(t-\tau/2)\,e^{j2\pi\alpha t}\right\}^{H}\right\rangle_{\infty}. \qquad (5)$$

If the assumptions made above hold, then $\mathbf{R}_{\mathbf{xx}}^{\alpha}(\tau)$ has rank L_{α}, the number of desired signals. In particular, if only one signal exhibits spectral correlation at the chosen α, then $\mathbf{R}_{\mathbf{xx}}^{\alpha}(\tau)$ is a rank-one matrix given by

$$\mathbf{R}_{\mathbf{xx}}^{\alpha}(\tau) = \mathbf{a}(\theta_1)\mathbf{a}^{H}(\theta_1)R_{s_1 s_1}^{\alpha}(\tau), \text{ for } L_{\alpha} = 1, \qquad (6)$$

with one non-zero eigenvalue whose eigenvector is equal to the direction vector $\mathbf{a}(\theta_1)$ of the signal $s_1(t)$. From this eigenvector the angle θ_1 can be estimated as shown in [6].

In general the cyclic autocorrelation matrix has contributions from $L_\alpha > 1$ signals that exhibit spectral correlation at the chosen α. For $L_\alpha < M$ it has a null space spanned by the columns of $\mathbf{E}_{N,\alpha}$, the eigenvectors corresponding to its zero eigenvalues,

$$\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)\mathbf{E}_{N,\alpha} = 0. \tag{7}$$

Since none of the signals $s_i(t)$ are perfectly correlated with each other, then $\mathbf{R}_{\mathbf{ss}}^\alpha(\tau)$ has full rank equal to L_α. Furthermore since the columns of $\mathbf{A}(\Theta)$ are linearly independent, then (4) and (7) imply that the null space is orthogonal to the direction vectors of the desired signals,

$$\mathbf{E}_{N,\alpha}^H \mathbf{a}(\theta_i) = 0, \quad i = 1, \ldots, L_\alpha. \tag{8}$$

This fact can be used to form a measure of orthogonality $P_{CM}(\theta)$ (also referred to as the spatial spectrum) similar to that used by MUSIC and other algorithms:

$$P_{CM}(\theta) = \frac{\|\mathbf{a}(\theta)\|^2}{\left\|\mathbf{E}_{N,\alpha}^H \mathbf{a}(\theta)\right\|^2}. \tag{9}$$

Thus, the DF algorithm proposed here can be summarized as follows:

1. Choose α to be a cycle frequency of the desired signals;

2. Find the null space, $\mathbf{E}_{N,\alpha}$ of $\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)$, and its rank, Z_α;

3. Determine the number of SOIs, $L_\alpha = M - Z_\alpha$;

4. Search over θ for the L_α highest peaks in $P_{CM}(\theta)$.

This algorithm is referred to as the *Cyclic MUSIC* algorithm since it can be thought of as a modified and simplified MUSIC algorithm that exploits cyclostationarity properties of the desired signals. The function $P_{CM}(\theta)$ is referred to as the Cyclic MUSIC spatial spectrum.

It is interesting to note here that the recently introduced phase-SCORE algorithm for blind adaptive signal extraction [7] yields a null space with the same orthogonality properties as the Cyclic MUSIC algorithm (7)-(9). The phase-SCORE algorithm finds weight vectors to perform signal extraction that satisfy the phase-SCORE eigenvalue equation,

$$\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)\mathbf{w} = \lambda \mathbf{R}_{\mathbf{xx}}\mathbf{w}. \tag{10}$$

Whereas the nonzero eigenvalues' eigenvectors perform extraction, the generalized null space of (10) performs direction finding:

$$\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)\mathbf{E}_{N,\alpha} = 0, \tag{11}$$

$$\mathbf{E}_{N,\alpha}^H \mathbf{a}(\theta_i) = 0, \quad i = 1, \ldots, L_\alpha, \tag{12}$$

$$P_{CM}(\theta) = \frac{\|\mathbf{a}(\theta)\|^2}{\left\|\mathbf{E}_{N,\alpha}^H \mathbf{a}(\theta)\right\|^2}. \tag{13}$$

Note that (11) can be written as

$$\mathbf{R}_{\mathbf{ww}}^\alpha(\tau)\mathbf{E}_{N,\alpha}^{(w)} = 0, \tag{14}$$

where

$$\mathbf{E}_{N,\alpha}^{(w)} = \mathbf{R}_{\mathbf{xx}}^{-1/2}\mathbf{E}_{N,\alpha}, \tag{15}$$

$$\mathbf{w}(t) = \mathbf{R}_{\mathbf{xx}}^{-1/2}\mathbf{x}(t), \tag{16}$$

which can be interpreted as the Cyclic MUSIC algorithm applied to spatially whitened data. This suggests that phase-SCORE–based Cyclic MUSIC (11)-(13) can potentially outperform unwhitened Cyclic MUSIC (7)-(9) when the desired signals are weaker than the interference. Furthermore, the phase-SCORE algorithm (10) simultaneously performs signal extraction [7].

In practice, the use of measured correlation matrices instead of ideal quantities in (7) and (11) reduces the equalities in (7)-(8) and (11)-(12) to approximations. However, as shown in the simulations, these approximations hold sufficiently well after an adequate averaging time. Since the smallest eigenvalues are not identically zero, a statistical test must be applied to determine the rank of $\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)$ if it is unknown. Existing techniques for rank determination such as Minimum Description Length (MDL) [4] apply directly only to the conventional autocorrelation matrix $\mathbf{R}_{\mathbf{xx}}$ (or to the generalized eigenvalue equation $\mathbf{R}_{\mathbf{xx}}\mathbf{w} = \lambda \mathbf{R}_{\mathbf{nn}}\mathbf{w}$, where $\mathbf{R}_{\mathbf{nn}}$ is the autocorrelation of the noise and known interferers), which have positive, real eigenvalues grouped around a positive value (the noise power). The cross-correlation matrices used by the Cyclic MUSIC algorithms do not satisfy these assumptions. This rank determination problem is left as an open research topic. In the proof-of-concept results presented in this paper, the number of desired signals, L_α, is assumed to be known.

The Cyclic MUSIC algorithms circumvent all of the drawbacks mentioned in the introduction because only the desired signals contribute to $\mathbf{R}_{\mathbf{xx}}^\alpha(\tau)$. In particular, the number of signal sources represented by $\mathbf{i}(t)$ is limited only by averaging time and numerical accuracy. Also, regardless of how close the direction of arrival (DOA) of a desired signal is to the DOA of an undesired signal, the algorithms can accurately estimate the direction of the desired signal. Furthermore, no knowledge of the noise covariance or distribution is required.

3 Simulations

3.1 Results

Four simulations supporting the theoretical claims made for the Cyclic MUSIC algorithms are presented. The array consists of four isotropic sensors spaced uniformly on a circle having diameter equal to half of the carrier wavelength. The receiver has a complex bandwidth of 10.24 MHz. Signal powers are given in dB SWNR (signal to white noise ratio). Unless otherwise specified, the noise is additive white Gaussian noise (AWGN) that is uncorrelated from sensor to sensor (i.e., the noise covariance matrix is equal to the identity matrix). Unless specified otherwise, each received signal has a power level of 10 dB SWNR. In the simulations presented here, the SOI is a 4 Mb/s BPSK signal transmitted with Nyquist-shaped pulses using a 100% excess bandwidth. The Cyclic MUSIC algorithms are simulated with $\alpha = 4$ MHz and $\tau = 0$. The MUSIC algorithm is simulated using the identity as the assumed noise covariance matrix. Unless stated otherwise, MUSIC uses exact *a priori* knowledge of the total number of signals in the environment. The averaging time is equal to 20 μs (roughly 800 bauds of the 4 Mb/s BPSK signal).

In the first simulation, the signal selectivity and accuracy of Cyclic MUSIC are tested. The BPSK SOI arrives from 60°, and an FM interferer of comparable bandwidth arrives from

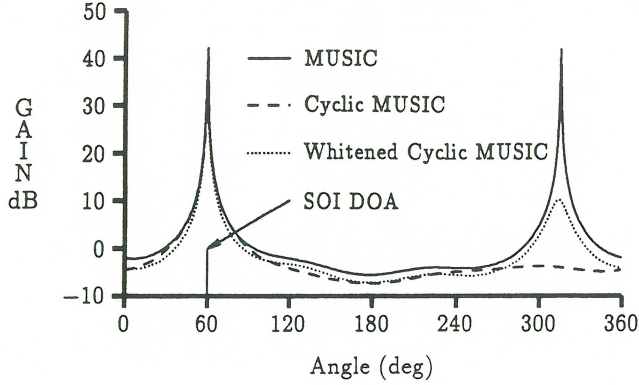−15°. The resulting spatial spectra are shown in Figure 1.

Figure 1: Spatial spectra for environment containing one SOI and one interferer.

In the second simulation, the Cyclic MUSIC algorithms accurately estimate a SOI DOA in the presence of an interferer arriving from nearly the same direction. The 4 Mb/s BPSK SOI arrives from 60°, and a 3 Mb/s 16-QAM interferer with 100% excess bandwidth arrives from 63°. The resulting spatial spectra are shown in Figure 2.
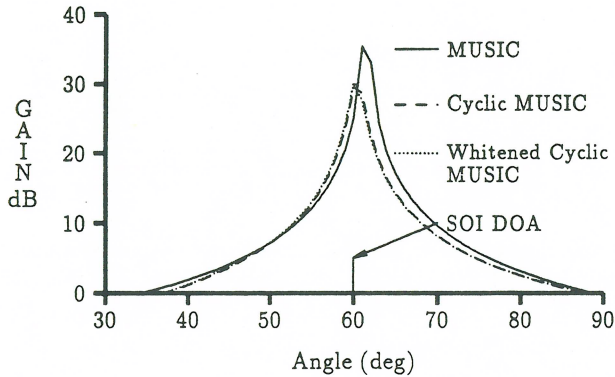

Figure 2: Spatial spectra for environment containing one SOI with 60° DOA and one interferer with 63° DOA.

In the third simulation, the independence of Cyclic MUSIC from the noise characteristics is demonstrated. The BPSK SOI has -5 dB SWNR and arrives from 60°. AWGN is present but is correlated from sensor to sensor such that noises on neighboring elements have a correlation coefficient of 0.5, and noises on diametrically opposite elements have a correlation coefficient of 0.25, yielding an actual noise covariance matrix given by

$$\mathbf{R_{nn}} = \begin{bmatrix} 1 & .5 & .25 & .5 \\ .5 & 1 & .5 & .25 \\ .25 & .5 & 1 & .5 \\ .5 & .25 & .5 & 1 \end{bmatrix}. \quad (17)$$

The resulting spatial spectra are shown in Figure 3.

In the fourth simulation, the performance of Cyclic MUSIC for an overloaded array is illustrated. A total of five signals impinge on the four-element array. Two 4 Mb/s BPSK SOIs arrive from 60° and 150°, and three interferers consisting of FM, TV, and pulsed radar signals arrive from 10°, 20°,

and 70° respectively. MUSIC is simulated under the assumption that three signals impinge on the array, since any greater number would not leave it a noise subspace with which to form its spatial spectrum. The resulting spatial spectra are shown in Figure 4.
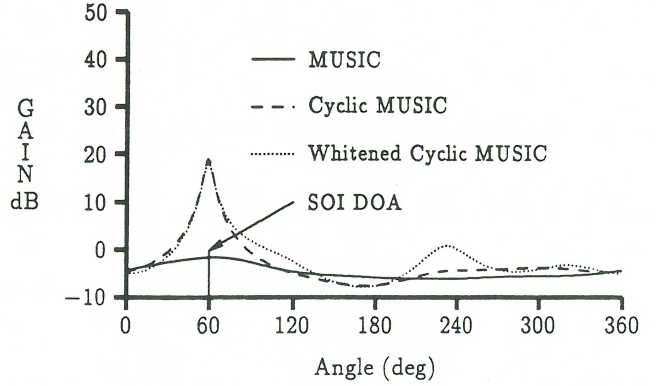

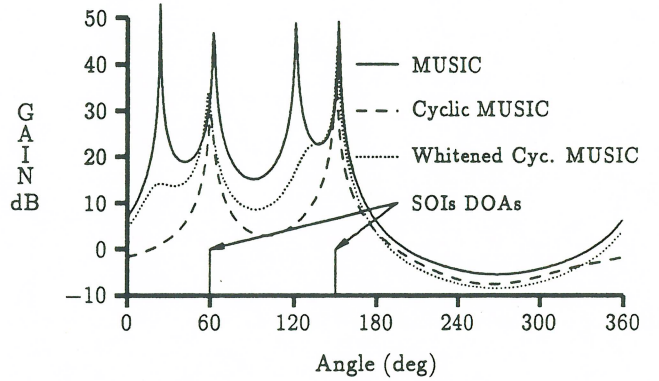Figure 3: Spatial spectra for environment containing one SOI and correlated AWGN.

Figure 4: Spatial spectra for environment containing two SOIs and three interferers.

3.2 Discussion

The first simulation verifies both the accuracy and the signal-selective nature of the Cyclic MUSIC algorithms. Both Cyclic MUSIC and MUSIC estimate the DOA of the BPSK SOI to within 0.2° of the true value of 60°. Due to its signal-selectivity, Cyclic MUSIC ignores the interference, whereas MUSIC also estimates the DOA of the interferer. The bump in the phase-SCORE Cyclic MUSIC spectrum near the interference DOA appears to be due to the pre-whitener acting on the residual *measured* spectral correlation of the interferer; after a sufficiently long averaging time the measured spectral correlation would be zero, and the bump should disappear. Clearly, no post-processing of the output of the Cyclic MUSIC algorithm is necessary to determine which peak is due to the SOI.

The second simulation illustrates the increase in effective resolution afforded by signal selectivity. MUSIC is unable to resolve the two signals, which are only 3° apart. Despite its *a priori* knowledge that two signals impinge on the array, MUSIC yields a peak in the spatial spectrum halfway

between the two true DOAs. In contrast, the Cyclic MUSIC algorithms isolate the signal with cycle frequency equal to the value used in the simulation and estimate its DOA to within 0.2° of the true DOA of 60° while ignoring the interference. Note that if the desired cycle frequency used by the algorithm were changed to 3 MHz, then the Cyclic MUSIC algorithms would estimate the DOA of the 3 Mb/s signal and ignore the 4 Mb/s signal.

The third simulation demonstrates the independence of Cyclic MUSIC from the noise statistics. The correlated noise is not accounted for in MUSIC's assumed noise covariance matrix, causing total failure at this low SNR. In contrast, since the noise is stationary and thus is not spectrally correlated at the α of interest, Cyclic MUSIC ignores it. Consequently, both Cyclic MUSIC algorithms yield a DOA estimate of the SOI within 1.0° of the true DOA of 60°.

The fourth simulation illustrates the performance of Cyclic MUSIC for an overloaded array. With five signals impinging on a four-element array MUSIC does surprisingly well, yielding four DOA estimates, three of which are within 4° of the true DOAs; however, one estimate differs from the true value by 50°, and one signal's DOA is not estimated at all. These errors reduce the significance of the estimates that *are* within 4°. Cyclic MUSIC performs very well here, estimating the two SOI DOAs to within 0.2° and 0.1° of the true values of 60° and 150°, respectively. Phase-SCORE Cyclic MUSIC performs almost as well, yielding errors of 1.5° and 1.0° and a spurious bump at 24°. This slightly degraded performance appears to be due to the inability of the prewhitener to equalize the power of five signals using a four element antenna array.

4 Conclusions

Two signal-selective direction estimation algorithms, referred to as Cyclic MUSIC algorithms, that overcome many limitations of existing techniques by using spectral correlation to select the desired signals and ignore interference have been presented. By estimating DOAs of only the desired signals, the number and characteristics of the interference can be arbitrary and unknown. Also, the signal selectivity enables Cyclic MUSIC to estimate DOAs of the desired signals regardless of how closely spaced they are to the interferers. Furthermore, the noise can be arbitrary and unknown. The price paid for the improved performance of Cyclic MUSIC is the requirement of an averaging time for estimation of the cyclic correlation matrix that exceeds that required for the MUSIC algorithm.

The two algorithms, Cyclic MUSIC and a whitened version of it based on the phase-SCORE algorithm, performed comparably in the simulations. They consistently outperformed MUSIC in environments for which MUSIC's *a priori* knowledge was inaccurate or where its fundamental operating assumptions were violated. Although the whitened Cyclic MUSIC algorithm yielded slightly larger errors than the unwhitened version, its inherent signal extraction ability could outweigh this drawback in some applications.

The similarity of Cyclic MUSIC to MUSIC suggests that spatial smoothing or vector-space MUSIC techniques can be applied to Cyclic MUSIC, extending its applicability to include fully correlated signals of interest. Also, various maximum likelihood methods such as the Alternating Projections algorithm [3] might be modified to benefit from the signal

selectivity of Cyclic MUSIC.

In addition to the fully-correlated sources problem, topics for further investigation include using the inherent signal extraction ability of the phase-SCORE Cyclic MUSIC algorithm to improve performance, and detecting optimally the number of desired signals by analyzing the cyclic autocorrelation matrix.

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