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Robustness of Direction-Finding Methods for Cyclostationary Signals in the Presence of Array Calibration Error*

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Abstract

In this paper it is shown that cyclostationarity-exploiting direction-finding methods can be much less sensitive than conventional direction-finding methods to errors in the array calibration data. In particular, when only a small subset of the signals arriving at the array exhibits the desired cyclostationarity property, it is shown that the signal-selective methods can operate properly in the presence of calibration errors that cause the conventional methods to fail.

I Introduction

The need to estimate the directions of arrival of signals impinging on a sensor array arises in applications such as signals intelligence and commercial communications monitoring in which often very little prior knowledge of the interference and noise characteristics is available. In these applications it has been shown [1, 2, 3, 4, 5, 6, 7] that signal-selective methods that exploit known (or estimable) second-order cyclostationarity properties of the signals of interest can greatly outperform conventional methods in some signal environments. However, in these comparisons it has always been assumed that the array calibration data were known exactly, an assumption that does not hold in many environments in practice due to physical perturbations of the sensors, component drift, and so on.

The effects of erroneous array calibration data on conventional methods have been investigated [8, 9], and techniques for mitigating them (sometimes referred to as self-calibration methods) have been proposed [10, 11, 12]. Primarily, these studies have focused on analyzing and improving the resolving power of conventional direction-finding (DF) methods. The alternative approach taken in this paper is to recognize that the signal-selective DF methods such as Cyclic MUSIC and Cyclic Least Squares inherently mitigate the effects of calibration error in some environments simply by reducing the number of signals that must be simultaneously spatially resolved.

This paper is organized as follows. In Section II the relevant properties of cyclostationary signals are reviewed prior to their use in Section III where the Cyclic MUSIC method for signal-selective DF is summarized. The results of computer simulations are presented in Section IV to demonstrate that Cyclic MUSIC can tolerate much greater calibration error than conventional MUSIC. Conclusions are drawn in Section V.

II Cyclostationarity

In this section the most relevant concepts from the theory of cyclostationarity are reviewed prior to their use in Section III. More detailed treatments can be found in [13, 14, 15].

A vector-valued complex envelope $\boldsymbol{x}(n)$ exhibits cyclostationarity if it is correlated with either a frequency-shifted version of itself (i.e., if it exhibits spectral coherence) for any nonzero frequency shift α or a conjugated and frequency-shifted version of itself for any frequency shift α. Mathematically, this correlation (or spectral coherence) is expressed in terms of the cyclic autocorrelation matrix $\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}}(\tau)$ or the cyclic conjugate correlation matrix $\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}^*}(\tau)$, respectively, where

$$\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}}(\tau) \triangleq \left\langle \boldsymbol{x}(n)\,\boldsymbol{x}(n-\tau)^H\, e^{-j2\pi\alpha n}\right\rangle_\infty \quad (1)$$

$$\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}^*}(\tau) \triangleq \left\langle \boldsymbol{x}(n)\,\boldsymbol{x}(n-\tau)^T\, e^{-j2\pi\alpha n}\right\rangle_\infty, \quad (2)$$

with $\langle f(n)\rangle_N \triangleq \frac{1}{N}\sum_{n=0}^{N-1} f(n)$ and where $(\cdot)^T$ and $(\cdot)^H$ denote the matrix transposition and matrix conjugate transposition operators, respectively. The values of α for which either of these correlation matrices are nonzero are the cycle frequencies of the signals comprising $\boldsymbol{x}(n)$. Since (1) and (2) can be reinter-

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preted as the Fourier coefficients for the matrices of conjugate and non-conjugate lag-product waveforms $\boldsymbol{x}(n)\boldsymbol{x}(n-\tau)^H$ and $\boldsymbol{x}(n)\boldsymbol{x}(n-\tau)^T$, then it can be seen that $\boldsymbol{x}(n)$ exhibits cyclostationarity (or spectral coherence) with cycle frequency α if and only if the lag-product waveforms contain finite-strength additive sine-wave components with frequency α. That is, cyclostationarity means that sine-waves can be generated by multiplying the signal by a delayed and possibly conjugated version of itself, even though the signal itself might not contain any finite-strength additive sine-wave components.

Most digital communication signals exhibit cyclostationarity as a result of the periodic sampling, gating, keying, and mixing operations in the modulator. For example, the cycle frequencies of BPSK are equal to the doubled carrier frequency offset, harmonics of the baud rate, and sums and differences of these. More specifically, if $\boldsymbol{x}(n)$ contains a BPSK signal having carrier offset f_c (relative to the center of the reception band which is downconverted to zero) and baud rate f_b, then $\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}}(\tau)$ is not identically zero for $\alpha = kf_b$ for integers k, and $\boldsymbol{R}^{\alpha}_{\boldsymbol{xx}^*}(\tau)$ is not identically zero for $\alpha = 2f_c + kf_b$ for integers k. The useful values of τ in the correlation matrices are typically between 0 and $1/(2f_b)$. A case of particular interest in this paper is the fact that for a scalar BPSK signal $s(n)$ having carrier frequency offset f_c, the magnitude of the cyclic conjugate correlation $R^{2f_c}_{ss^*}(\tau)$ is maximized at $\tau = 0$ regardless of the pulse shape.

Measurements of these two types of cyclic correlations are useful because they select contributions from only the signal components that exhibit the specified cyclostationarity property and discriminate against all others. This is analogous to the property that measurements of the correlation between a desired signal corrupted by additive interference and noise and an uncorrupted version of the desired signal (e.g., a training signal) select only the contributions from the desired signal and discriminate against all others. The utility of exploiting cyclostationarity to gain signal-selectivity has been demonstrated for many applications, including adaptation of antenna arrays [16, 17], estimation of directions of arrival [1, 2, 3, 4, 5, 6, 7], estimation of time difference of arrival [18], detection [19], and others [13, 15].

III Cyclic MUSIC

In this section the Cyclic MUSIC method for DF is summarized, and it is explained why it can be more robust to calibration error than conventional DF methods.

Under the narrowband assumption (which does not preclude the presence of non-sinusoidal signals as discussed in [20, 7]), the complex envelope $\boldsymbol{x}(n)$ of the M-element sensor array data can be expressed as

$$\boldsymbol{x}(n) = \boldsymbol{A}(\Theta)\,\boldsymbol{s}(n) + \boldsymbol{i}(n) \qquad (3)$$

where the columns of $\boldsymbol{A}(\Theta) = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_L)]$ are the true array response vectors for directions $\Theta = [\theta_1, \cdots, \theta_L]^T$, $\boldsymbol{s}(n) = [s_1(n), \cdots, s_L(n)]^T$ is the corresponding vector of signals, and $\boldsymbol{i}(n)$ denotes the noise.

Given N time samples $\boldsymbol{x}(0), \cdots, \boldsymbol{x}(N-1)$, the Cyclic MUSIC algorithm estimates the directions of arrival of the $L_\alpha \le L$ signals having cycle frequency α as follows:

1. Compute $\hat{\boldsymbol{R}}^{\alpha}_{\boldsymbol{xx}}(\tau)$ or $\hat{\boldsymbol{R}}^{\alpha}_{\boldsymbol{xx}^*}(\tau)$ for an appropriate value of τ using the finite-time-average versions of (1) or (2), respectively.

2. Compute the singular value decomposition $\boldsymbol{U\Sigma V}^H$ of either $\hat{\boldsymbol{R}}^{\alpha}_{\boldsymbol{xx}}(\tau)$ or $\hat{\boldsymbol{R}}^{\alpha}_{\boldsymbol{xx}^*}(\tau)$, and partition $\boldsymbol{U}$ as $[\boldsymbol{U}_s,\ \boldsymbol{U}_n]$ where $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are $M \times L_\alpha$ and $M \times (M - L_\alpha)$ matrices of left singular vectors that correspond to the L_α most dominant and $M - L_\alpha$ least dominant singular values, respectively.

3. Find the L_α highest peaks in $P_s(\theta) = \left\| \boldsymbol{U}_s^H \tilde{\boldsymbol{a}}(\theta) \right\|$ or the $M - L_\alpha$ lowest valleys in $P_n(\theta) = \left\| \boldsymbol{U}_n^H \tilde{\boldsymbol{a}}(\theta) \right\|$, where $\tilde{\boldsymbol{a}}(\theta)$ is the **assumed** array response vector for direction θ.

Several performance advantages of Cyclic MUSIC over conventional DF methods derive from its signal-selectivity: 1) in some applications, the number L_α of signals having cycle frequency α can be less than the number L of all signals, so spatial resolution requirements and computational requirements are reduced; 2) more signals than sensors can be present so long as $L_\alpha < M$; and 3) the spatial characteristics of the noise can be arbitrary. For example, in the scheme [17] for substantially increasing the capacity of a land mobile cellular radio system by using blind adaptive spatial filtering, Cyclic MUSIC could be used to estimate the approximate directions of arrival at the base station of the signals from the mobile users for the purposes of carrier frequency assignment and cell-to-cell hand-off, and the relevant operating parameters would be $L_\alpha = 1$, $L = 48$, and $M \le 64$ (e.g., $M = 8$); in this rapidly changing dense environment, it seems unlikely that any conventional DF method would be able to obtain acceptable accuracy and reliability.

In the presence of array calibration error, an additional benefit of signal selectivity and the associated reduction in spatial resolution requirements becomes apparent. Since it is substantially more difficult to resolve the directions of two closely spaced sources in the presence of calibration error than it is to accurately estimate the direction of one source in the presence of calibration error, it would appear that any reduction in the number of sources that must be spatially resolved should yield a significant increase in performance. This proposition is substantiated by the results of computer simulations in Section IV.

IV Simulation Results

In this section the sensitivity of the MUSIC and Cyclic MUSIC algorithms to perturbations in the positions of the sensors is investigated using computer simulations.

A uniform linear array having 8 sensors nominally separated by half of a wavelength receives 4 spectrally overlapping BPSK signals that all have baud rate equal to 1/4 of the sampling rate and signal-to-noise ratio of 10 dB. Two of the signals have zero carrier offset from the center of the receiver band and arrive from 8 degrees and 2 degrees, respectively. The third and fourth signals have carrier frequencies that are offset from the center of the receiver band by 1/10 and 2/10 of the sampling rate and arrive from 16 degrees and -6 degrees, respectively. Zero-mean complex white Gaussian noise that is uncorrelated from sensor to sensor is also present. The Cyclic MUSIC algorithm is applied using $\hat{\boldsymbol{R}}^{\alpha}_{\boldsymbol{x}\boldsymbol{x}^*}(0)$ with $\alpha = 0$ and $L_\alpha = 2$ to perform DF on the two signals having zero carrier frequency offset ($L_\alpha = 2$), whereas MUSIC must spatially resolve all 4 signals ($L = 4$). By using $L_\alpha = 1$ and $\alpha = 2 \times 1/10$ or $\alpha = 2 \times 2/10$, Cyclic MUSIC could be used to find the directions of the other two signals one at a time. The choice of $\alpha = 0$ and the directions of arrival were made so that Cyclic MUSIC would not be given an unfair advantage (e.g., having to resolve the two most widely separated signals instead of the two most narrowly separated, or having to find the direction of one source instead of two). For both methods, the correlation matrices are estimated from 8192 time samples of received data. At the start of each of the 100 independent trials conducted, the true sensor coordinates $(\tilde{p}_1, \tilde{q}_1), \cdots, (\tilde{p}_M, \tilde{q}_M)$ are perturbed from their assumed positions $(p_1, q_1), \cdots, (p_M, q_M)$ by independent identically distributed Gaussian random variables $(\delta p_1, \delta q_1), \cdots, (\delta p_M, \delta q_M)$ having zero mean and variances $E\left\{(\delta p_m)^2\right\} = E\left\{(\delta q_m)^2\right\} = \beta^2$ wavelength^2 and $E\left\{(\delta p_m)(\delta q_m)\right\} = 0$:

$$(\tilde{p}_m, \tilde{q}_m) = (p_m + \delta p_m, q_m + \delta q_m).$$

As β increases in magnitude, the perturbations become more severe.

The largest bias among the two directions of arrival found by Cyclic MUSIC is plotted versus the perturbation parameter β in Figure 1. The largest bias among the four directions of arrival found by MUSIC is also shown there. From the figure it can be seen that the largest bias incurred by Cyclic MUSIC does not exceed the minimum angular separation of the sources (6 degrees) for $\beta \leq 0.05$, whereas the comparable limit for conventional MUSIC is $\beta \leq 0.005$. The mean-squared errors are not shown because they are dominated by the bias.

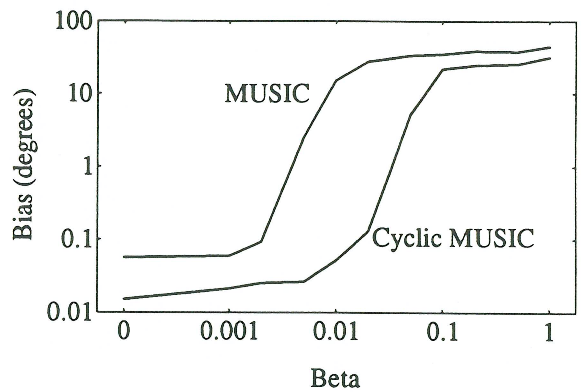

Figure 1: Maximum bias in degrees of direction estimates for MUSIC and Cyclic MUSIC for source directions of 8, 2, 16, and -6 degrees versus the standard deviation β of sensor coordinates.

If the sources are brought closer together (8 deg., 5 deg., 12 deg., and 1 deg. instead of 8 deg., 2 deg., 16 deg., and -6 deg., respectively), then the benefits of signal-selectivity become even more pronounced as shown in Figure 2. MUSIC is unable to resolve the signals even when no calibration error is present ($\beta = 0$), whereas Cyclic MUSIC continues to perform properly for all $\beta \leq 0.01$. The mean-squared errors are not shown because they are again dominated by the bias.

Although results for perturbations in sensor position only are shown here, the results for perturbations in the sensor phases are qualitatively similar: Cyclic MUSIC can tolerate perturbations that are approximately ten times as large as those that MUSIC can tolerate in the two environments considered.

V Conclusions

In this paper it has been shown that Cyclic MUSIC can be more robust to calibration error than MUSIC when only a subset of signals exhibit cyclostationarity with the specified cycle frequency. In particular, Cyclic MUSIC can tolerate perturbations in sensor position or phase that are approximately ten times as large as those that MUSIC can tolerate in

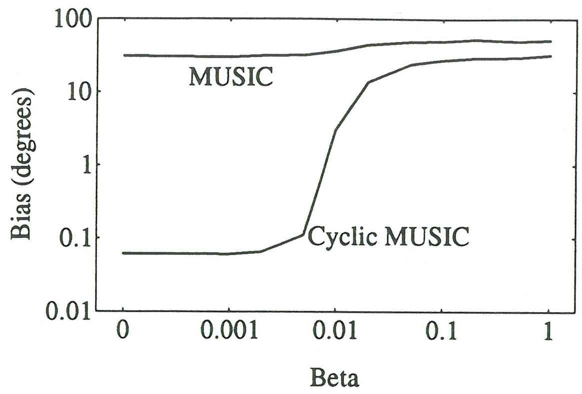

Figure 2: Maximum bias in degrees of direction estimates for MUSIC and Cyclic MUSIC for source directions of 8, 5, 12, and 1 degrees versus the standard deviation β of sensor coordinates.

the two environments considered. These results imply that cyclostationarity-exploiting methods might be more amenable than conventional methods to implementation in practical environments where thermal effects, component drift, and mechanical disturbances exist. A related implication is that self-calibration methods applied to cyclostationarity-exploiting methods might be able to correct larger errors than could be expected for conventional methods.

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