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The SCORE Approach to Blind Adaptive Signal Extraction: An Application of the Theory of Spectral Correlation

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Abstract

A new approach to blind adaptation of antenna arrays is presented that has the capability to extract PCM, AM and FDM-FM communication signals-of-interest (SOIs) from co-channel interference environments using only known spectral correlation properties of those SOIs, i.e., *without* using knowledge of the antenna array manifold, SOI waveform or SOI direction-of-arrival to train the array. The class of *self-coherence restoral (SCORE)* objective functions are introduced, and flexible algorithms for adapting antenna arrays to optimize these objective functions are developed. Using the *theory of spectral correlation*, it is shown via analysis and simulation that these algorithms will maximize the SOI SINR at the output of any antenna array when a single SOI with self-coherence at a known value of frequency separation and an *arbitrary* number of interferers without self-coherence at that frequency separation are impinging on the array. Algorithm modifications are also introduced to allow the SCORE processors to near-optimally extract SOIs from environments containing multiple signals with self-coherence at the same value of frequency separation.

1 Introduction

The need for blind adaptive signal extraction is growing in a number of signal processing applications. The ability to adapt a receiver processor to remove unknown and/or time-varying distortion and interference from a signal of interest (SOI) without using knowledge of the transmission channel or waveform to train the processor can significantly reduce cost and outage time in telephony and RF communication systems. Blind adaptive processing can also allow signal extraction to be performed in many other applications where it is impractical or impossible to provide such knowledge to the adaptive processor, e.g., in mobile radio and in regenerative satellite communication systems, where it may be too costly to provide an adaptive processor with a separate training sequence for each signal received by the transponder or receiver, and in reception of broadcast FM and in ESM/ECM systems, where the SOI and interference waveform and channel parameters are typically unknown and time-varying over the reception time.

Algorithms that have been developed in the past to perform blind adaptation include *property-restoral algorithms*, which adapt a receiver processor to restore a known set of SOI properties to the processor output signal, and *coherence-exploitation* techniques, which exploit known spatial coherence properties of the received channel (e.g., due to the discrete spatial distribution of the signals sources) to adapt the receiver processor. Examples of property restoral techniques include *Sato's algorithm* [1] and the *constant modulus algorithm (CMA)* [2], which have been used to adapt equalizers and antenna arrays to blindly correct FM, PSK, FSK and QAM signals on the basis of their low or constant modulus, and the *set-theoretic property mapping algorithms* [3], which have been used to correct signals on the basis of more general signal modulation and channel properties. Examples of coherence exploitation techniques include the *generalized sidelobe canceller (GSC)* [4] and the *signal subspace techniques* (*MUSIC* [5], *ESPRIT* [6]), which have been used to adapt antenna arrays to extract signals on the basis of their discrete distribution in the received-signal spatial spectrum.

All of these techniques suffer from shortcomings in practice. The convergence and capture characteristics of the property restoral techniques are still not well understood, a drawback that limits the application of these algorithms in automatic (unsupervised) communication systems where they must operate with a minimum of attention. While the coherence exploitation techniques are analytically more tractable, they suffer from other problems associated with measuring the spatial spectrum of the received signal. GSC and MUSIC, for instance, require accurate knowledge of the array manifold to operate, which limits their application in systems where such data is too costly or impossible to measure (e.g., if the array geometry is changing with time), while ESPRIT imposes a structural constraint on the sensor array that can be very hard to satisfy in practice, and which reduces the degrees of freedom (null-steering capability) of the overall array by 50%. All of these techniques suffer from the additional problem that that they are highly nondiscriminatory: in an unknown environment, the processors must extract *all* of the received signals and rely on downstream processing to separate the SOIs from the interferers. This drawback can be of critical importance in systems where the number of array sensors is high, or in applications where only a few signals out of many are of interest to the receiver processor.

This paper presents the new class of *Self-COherence REstoral (SCORE)* algorithms, which have the potential to overcome these limitations. A property held by most communication signals is that they are coherent with frequency-shifted and possibly conjugated versions of themselves for certain discrete values of frequency shift. This spectral self-coherence is commonly induced by periodic gating, mixing or multiplexing operations at the transmitter. For instance, self-coherence is induced at multiples of the symbol rate in PCM signals and at multiples of the pilot-tone frequency in FDM-FM signals, and conjugate self-coherence is commonly induced at twice the carrier frequency in BPSK and AM signals.

The (conjugate) self-coherence of a received signal is degraded if it is corrupted by additive interference that does not share that coherence, e.g., if a PCM SOI is corrupted at the receiver by a PCM interferer with a different symbol rate. This leads to the new class of SCORE algorithms, which adapt a receiver processor to *restore* the SOI self-coherence (and thereby reduce the interference) in the receiver output signal.

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2 The Self-Coherence Concept

A signal waveform $s(t)$ is said to be *spectrally self-coherent at frequency separation* α [8] if the correlation between $s(t)$ and $s(t)$ frequency-shifted by α is nonzero for some lag τ, i.e., if

$$\begin{aligned} \rho_{ss}^{\alpha}(\tau) &\triangleq \frac{< s(t)[s(t-\tau)e^{j2\pi\alpha t}]^* >_\infty}{\sqrt{< |s(t)|^2 >_\infty < |s(t-\tau)e^{j2\pi\alpha t}|^2 >_\infty}} \\ &= R_{ss}^{\alpha}(\tau)e^{-j\pi\alpha\tau}/R_{ss}(0) \quad \neq 0 \end{aligned} \tag{1}$$

at some value of τ, where $< \cdot >_\infty$ denotes infinite time-averaging. Similarly, a signal waveform $s(t)$ is said to be spectrally *conjugate* self-coherent at frequency separation α if

$$\begin{aligned} \rho_{ss^*}^{\alpha}(\tau) &\triangleq \frac{< s(t)[s^*(t-\tau)e^{j2\pi\alpha t}]^* >_\infty}{\sqrt{< |s(t)|^2 >_\infty < |s^*(t-\tau)e^{j2\pi\alpha t}|^2 >_\infty}} \\ &= R_{ss^*}^{\alpha}(\tau)e^{-j\pi\alpha\tau}/R_{ss}(0) \quad \neq 0 \end{aligned} \tag{2}$$

at some value of τ. The functions $\rho_{ss}^{\alpha}(\tau)$ and $\rho_{ss^*}^{\alpha}(\tau)$ are referred to here as the *self-coherence function* and the *conjugate self-coherence function*, respectively; the functions $R_{ss}^{\alpha}(\tau)$ and $R_{ss^*}^{\alpha}(\tau)$ are referred to here as the *cyclic autocorrelation function* and the *cyclic conjugate-correlation function*, respectively.

The self-coherence functions and cyclic correlation functions are developed in detail in the new *theory of spectral correlation* [7,8], where it is shown that complex *cyclostationary* and *almost-cyclostationary* waveforms exhibit self-coherence or conjugate self-coherence at discrete multiples of the time periodicities of the waveform statistics. This class of waveforms includes most communication signals; for instance, all PCM signals exhibit self-coherence at multiples of their baud-rate, and BPSK signals are in addition conjugate self-coherent at twice their carrier offset.

The function $|\rho_{ss^{(*)}}^{\alpha}(\tau)|^2$ can be interpreted as a measure of the relative strength of $s(t)$ contained within $s^{(*)}(t-\tau)e^{j2\pi\alpha t}$, where the optional conjugation $(*)$ is only applied if conjugate self-coherence is being measured. Using the Orthogonal Projection Theorem, $s^{(*)}(t-\tau)e^{j2\pi\alpha t}$ can be represented as

$$\begin{aligned} s^{(*)}(t-\tau)e^{j2\pi\alpha t} &= \rho_{ss^{(*)}}^{\alpha}(\tau)^* s(t) \\ &+ \sqrt{1-|\rho_{ss^{(*)}}^{\alpha}(\tau)|^2}\,\varepsilon_{ss^{(*)}}(t) \end{aligned} \tag{3}$$

where $s(t)$ and $\varepsilon_{ss^{(*)}}(t)$ are equal-power orthogonal waveforms ($R_{s\varepsilon} = 0$). Consequently, $s^{(*)}(t-\tau)e^{j2\pi\alpha t}$ can be thought of as a scaled and corrupted replica of $s(t)$ with a signal-to-corruption ratio $|\rho_{ss^{(*)}}^{\alpha}(\tau)|^2/[1-|\rho_{ss^{(*)}}^{\alpha}(\tau)|^2]$.

The utility of the self-coherence concept can best be seen in interference environments. Consider the environment where a scalar waveform $x(t)$ is equal to a scaled SOI $s(t)$ plus an independent interference signal $i(t)$, $x(t) = as(t) + i(t)$. If $s(t)$ is self-coherent at frequency separation α, but $i(t)$ is *not* self-coherent at α, then the cyclic autocorrelation of $x(t)$ is given by

$$R_{xx}^{\alpha}(\tau) = |a|^2 R_{ss}^{\alpha}(\tau) + R_{ii}^{\alpha}(\tau) = |a|^2 R_{ss}^{\alpha}(\tau), \tag{4}$$

i.e., the infinite time-averaged cyclic autocorrelation of $x(t)$ is unchanged by the addition of *arbitrary* interfererence, provided that the interference is not self-coherent at frequency separation α.

A useful interpretation of (4) is that the frequency-shift and (optional) conjugation operations completely decorrelate the interference component of $x(t)$ but only *partially* decorrelate the SOI component of $x(t)$. In terms of the decomposition given in (3), $x^{(*)}(t-\tau)e^{j2\pi\alpha t}$ can be expressed in terms of $x(t)$ by

$$x^{(*)}(t-\tau)e^{j2\pi\alpha t} = a_\alpha s(t) + i_\alpha(t) \tag{5}$$

where

$$\begin{aligned} a_\alpha &= a\rho_{ss^{(*)}}^{\alpha}(\tau)^* && (6) \\ i_\alpha(t) &= a\sqrt{1-|\rho_{ss^{(*)}}^{\alpha}(\tau)|^2}\,\varepsilon_{ss^{(*)}}(t) + i^{(*)}(t-\tau)e^{j2\pi\alpha t} && (7) \end{aligned}$$

and where $i_\alpha(t)$ is uncorrelated with both $s(t)$ and $i(t)$. Equation (5) motivates the development of interference cancellation techniques that use $x^{(*)}(t-\tau)e^{j2\pi\alpha t}$ as the reference signal in a conventional least-squares algorithm.

A different interpretation can be obtained by noting that the self-coherence of $x(t)$ in the above example is *reduced* when interference that is not self-coherent at α is added to the received environment. The self-coherence strength of $x(t)$ can be expressed in terms of the signal-to-interference-and-noise ratio (SINR) R_{ss}/R_{ii} as

$$|\rho_{xx^{(*)}}(\tau)| = |\rho_{ss^{(*)}}^{\alpha}(\tau)|/\left[1+\mathrm{SINR}^{-1}\right] \leq |\rho_{ss^{(*)}}^{\alpha}(\tau)| \tag{8}$$

Equation (8) motivates the development of interference cancellation techniques that extract SOIs by optimizing some direct or indirect measure of their self-coherence.

3 The SCORE Processors

The basic SCORE processors are motivated by extending the example given in Section 2 to the multiple sensor environment. Consider the environment where a multisensor antenna array is excited by a SOI $s(t)$ and by background noise and co-channel interference. If the fractional bandwidth of the receiver is small with respect to the electrical distance between the array sensors, then the received signal vector $\mathbf{x}(t)$ can be modelled by

$$\mathbf{x}(t) = \mathbf{A}s(t) + \mathbf{I}(t) \tag{9}$$

where $\mathbf{A}$, referred to here as the *SOI aperture vector*, models the polarization and direction-of-arrival (DOA) dependent antenna gains, cross-sensor phase mismatches and near-field multipath (scattering and mutual coupling) effects of the array, and where interference field $\mathbf{I}(t)$ models the remaining signals and background noise received by the array.

Given this model, $s(t)$ can be extracted from the received data by forming a processor output signal $y(t) = \mathbf{w}^\dagger \mathbf{x}(t)$ using an appropriately chosen processor vector $\mathbf{w}$. This is optimally accomplished by setting $\mathbf{w}$ equal to a *maximum-SINR* linear combiner

$$\mathbf{w}_{\max} \propto \mathbf{R}_{\mathbf{II}}^{-1}\mathbf{A} \propto \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{A}R_{ss} \tag{10}$$

where $\mathbf{R}_{\mathbf{II}}$ and $\mathbf{R}_{\mathbf{xx}}$ are the *limit* (infinite time-average) autocorrelation matrices (ACMs) of the interference and received signal vectors. These weights can also be interpreted as the optimal solution for the *least-squares cost function*

$$F_{LS}(\mathbf{w}) \triangleq < |y(t) - gs(t)|^2 >_\infty \tag{11}$$

where g is an arbitrary scalar gain constant. Conventional (non-blind) methods for computing $\mathbf{w}_{\max}$ require knowledge of the interference ACM $\mathbf{R}_{\mathbf{II}}$ and/or SOI aperture vector $\mathbf{A}$ to implement (10), or knowledge of the SOI waveform $s(t)$ to minimize (11). The goal here is to find $\mathbf{w}_{\max}$ *without* using this knowledge, i.e., to adapt $\mathbf{w}$ to approximate (10) using only knowledge of the self-coherence properties of the SOI.

This can be accomplished using the interpretation of self-coherence given in (5)-(7). We define a *reference signal* $r(t)$ by

$$r(t) \triangleq [\mathbf{c}^\dagger \mathbf{x}(t-\tau)]^{(*)} e^{j2\pi\alpha t} \tag{12}$$

where vector $\mathbf{c}$ is referred to as the *control vector* and the optional conjugation $(*)$ is only taken if conjugate self-coherence is to be restored. If $\mathbf{x}(t)$ is modelled by (9) and $s(t)$ is the sole received signal component with self-coherence (or conjugate self-coherence) at frequency separation α, then (5)-(7) can be used to show that $r(t)$ decomposes into a replica of the SOI plus a corruption term that is uncorrelated with both $s(t)$ and $\mathbf{x}(t)$

$$r(t) = a_\alpha s(t) + i_\alpha(t) \tag{13}$$

where a_α and $i_\alpha(t)$ are given by (6) and (7), respectively, with $a = (\mathbf{c}^\dagger \mathbf{A})^{(*)}$ and $i(t) = c^\dagger \mathbf{I}(t)$.

Equation (13) motivates the class of *least-squares SCORE algorithms.* We define the *least-squares SCORE cost function* by

$$F_{\text{SCORE}}(\mathbf{w};\mathbf{c}) \triangleq < |y(t) - r(t)|^2 >_T \tag{14}$$

where $y(t) = \mathbf{w}^\dagger \mathbf{x}(t)$ and $r(t)$ is given by (12), and where $< \cdot >_T$ denotes time-averaging over interval $[0,T]$. Substituting (13) into (14) and letting the averaging time grow to infinity yields

$$\begin{aligned} F_{\text{SCORE}} &= < |y(t) - [a_\alpha s(t) + i_\alpha(t)]|^2 >_T \\ &\rightarrow < |y(t) - a_\alpha s(t)|^2 >_\infty + < |i_\alpha(t)|^2 >_\infty . \end{aligned} \tag{15}$$

Since $i_\alpha(t)$ is not a function of $\mathbf{w}$, it follows that (14) becomes equivalent to the true least-squares cost function (11) and the value of $\mathbf{w}$ that minimizes (14) converges to the maximum-SINR processor as $T \rightarrow \infty$.

This result can be proved more directly by solving for the processor vector $\mathbf{w}_{\text{SCORE}}$ that optimizes (14) for infinite averaging time. Minimizing (14) with respect to $\mathbf{w}$ yields the *least-squares SCORE algorithm*

$$\mathbf{w}_{\text{SCORE}} = \hat{\mathbf{R}}_{\mathbf{xx}}^{-1} \hat{\mathbf{R}}_{\mathbf{x}r} \tag{16}$$

where $\hat{\mathbf{R}}_{\mathbf{xx}}$ and $\hat{\mathbf{R}}_{\mathbf{x}r}$ are the sample ACM and cross-correlation statistics computed over $[0,T]$. If $\mathbf{I}(t)$ is not (conjugate) self-coherent at α, then as $T \rightarrow \infty$ (16) will converge to

$$\mathbf{w}_{\text{SCORE}} \rightarrow g \mathbf{R}_{\mathbf{xx}}^{-1} \mathbf{A} R_{ss}, \quad g = (\mathbf{A}^\dagger \mathbf{c})^{(*)} |\rho^\alpha_{ss(*)}(\tau)|^2, \tag{17}$$

which is the maximum-SINR weight vector given by (10). Note that $\mathbf{w}$ converges to the maximum-SINR solution for *any* value of $\mathbf{c}$, as long as $\mathbf{c}$ is not orthogonal to $\mathbf{A}$.

The least-squares SCORE processor block diagram is shown in Figure 1. The reference signal $r(t)$ is generated by beamforming, delaying, conjugating (if conjugate self-coherence is being exploited) and frequency-shifting the received data signal. The reference signal is then used as a training signal to adapt the processor vector $\mathbf{w}$ using a block or recursive least-squares algorithm. The only control parameters used in the processor are the control vector $\mathbf{c}$, the delay τ, the conjugation control and the frequency-shift α; however, only α and the conjugation control are critical to the operation of the processor. For most communication waveforms much latitude can be allowed in the choice of $\mathbf{c}$ and τ; in theory these parameters need only be chosen to yield a nonzero value of g in (17). The frequency-shift parameter α need not be related in any way to the bandwidth or sample rate of the receiver system; however, care must be taken to avoid aliasing effects if the processor is implemented in digital form and α is large.

The least-squares SCORE processor can be improved in several ways. For instance, the strength of self-coherence at α can be increased by replacing the delay operator with a more general filtering operation, i.e., by generating $r(t)$ using

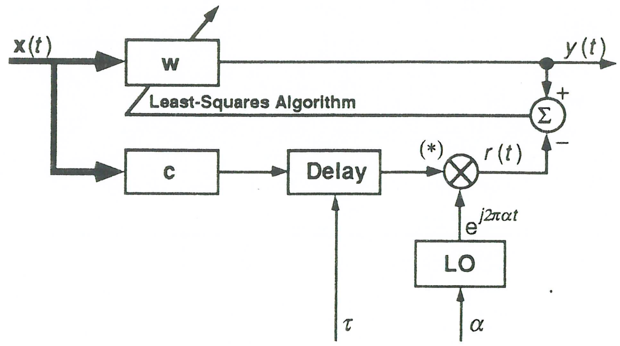

Figure 1: Least-Squares SCORE Processor

$$r(t) = [\mathbf{c}^\dagger \hat{\mathbf{x}}(t)]^{(*)} e^{j2\pi\alpha t}, \quad \hat{\mathbf{x}}(t) = h(t) \otimes \mathbf{x}(t) \tag{18}$$

where $h(t)$ is the *control filter* impulse response. The resulting optimum weight vector then converges to

$$\mathbf{w}_{\text{SCORE}} \rightarrow g \mathbf{R}_{\mathbf{xx}}^{-1} \mathbf{A} R_{ss}, \; g = (\mathbf{A}^\dagger \mathbf{c})^{(*)} |\rho^\alpha_{s\hat{s}(*)}|^2 \sqrt{\frac{R_{ss}}{R_{\hat{s}\hat{s}}}} \tag{19}$$

where $\hat{s}(t)$ is the filtered SOI. Appropriate design of the control filter can improve the performance of the SCORE processor in the presence of some signal types [9].

The critical dependence of the SCORE processor on the choice of target α can also be eased somewhat by the particular choice of least-squares algorithm used to implement (16). If an RLS algorithm with a growing rectangular window is used to solve (16), then the processor will eventually reject a received SOI if there is *any* error between the SOI self-coherence frequency and the target self-coherence frequency of the processor. In many environments, however, the SOI self-coherence frequency will not be known exactly. For instance, the SOI may be subject to Doppler shift, which has the effect of shifting the conjugate self-coherence frequency of the SOI; or the SOI may be subject to significant carrier and timing jitter, which effectively spreads the SOI self-coherence over a range of α. The SCORE processor can be made more tolerant to this error if a different choice of averaging window, e.g., an exponentially-decaying window, is used to compute $\hat{\mathbf{R}}_{\mathbf{x}r}$.

The largest improvement in SCORE processor performance can be obtained by adaptively adjusting the control vector as well as the processor vector to some appropriate setting. An algorithm for adapting $\mathbf{c}$ can be developed by motivating the least-squares SCORE algorithm from a *property-restoral* viewpoint. The same value of $\mathbf{w}_{\text{SCORE}}$ given in (16) results from maximizing the strength of the cross-correlation coefficient between $y(t)$ and $r(t)$,

$$\begin{aligned} \tilde{F}_{\text{SCORE}}(\mathbf{w};\mathbf{c}) &\triangleq |\hat{R}_{yr}|^2 / [\hat{R}_{yy} \hat{R}_{rr}] \\ &= |\mathbf{w}^\dagger \hat{\mathbf{R}}_{\mathbf{x}r}|^2 / [\mathbf{w} \hat{\mathbf{R}}_{\mathbf{xx}} \mathbf{w} \, \hat{R}_{rr}]. \qquad (20) \\ &= \frac{|\mathbf{w}^\dagger \hat{\mathbf{R}}_{\mathbf{xu}} \mathbf{c}^{(*)}|^2}{[\mathbf{w}^\dagger \hat{\mathbf{R}}_{\mathbf{xx}}][(\mathbf{c}^{(*)})^\dagger \hat{\mathbf{R}}_{\mathbf{uu}} \mathbf{c}^{(*)}]} \qquad (21) \end{aligned}$$

where $\mathbf{u}(t) \triangleq \hat{\mathbf{x}}^{(*)}(t) e^{j2\pi\alpha t}$ is defined as the *control signal.*

Equation (20) motivates the class of *cross-SCORE algorithms.* $\tilde{F}_{\text{SCORE}}$ is an indirect measurement of the self-coherence in $y(t)$ at frequency separation α; it is lowered if $\mathbf{x}(t)$ contains interference that is not self-coherent at this frequency separation. In this sense, the least-squares SCORE algorithm can be interpreted as a method for *restoring* this SOI self-coherence to the processor output signal. The cross-correlation coefficient is also degraded if interference is present in $r(t)$. Consequently, maximizing $\tilde{F}_{\text{SCORE}}$ with respect to $\mathbf{w}$ *and* $\mathbf{c}$ should restore the SOI self-coherence in both $y(t)$ *and* $r(t)$. For this reason, (21) is referred to here as the *cross-SCORE objective function.*

Jointly optimizing $\tilde{F}_{\text{SCORE}}$ with respect to $\mathbf{w}$ and $\mathbf{c}$ yields the *joint cross-SCORE eigenequation,*

$$\nu \begin{bmatrix} \hat{\mathbf{R}}_{\mathbf{xx}} & 0 \\ 0 & \hat{\mathbf{R}}_{\mathbf{uu}} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{c}^{(*)} \end{bmatrix} = \begin{bmatrix} 0 & \hat{\mathbf{R}}_{\mathbf{xu}} \\ \hat{\mathbf{R}}_{\mathbf{ux}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{c}^{(*)} \end{bmatrix} \tag{22}$$

for the maximum-eigenvalue eigenvector pair. The optimized objective function is then equal to ν^2. Equation (22) can also be decomposed into separate eigenequations in terms of $\mathbf{w}$ and $\mathbf{c}^{(*)}$,

$$\lambda \hat{\mathbf{R}}_{\mathbf{xx}} \mathbf{w} = [\hat{\mathbf{R}}_{\mathbf{xu}} \hat{\mathbf{R}}_{\mathbf{uu}}^{-1} \hat{\mathbf{R}}_{\mathbf{ux}}] \mathbf{w} \tag{23}$$

$$\lambda \hat{\mathbf{R}}_{\mathbf{uu}} \mathbf{c}^{(*)} = [\hat{\mathbf{R}}_{\mathbf{ux}} \hat{\mathbf{R}}_{\mathbf{xx}}^{-1} \hat{\mathbf{R}}_{\mathbf{xu}}] \mathbf{c}^{(*)} \tag{24}$$

where λ is related to ν by $\lambda = \nu^2$ and the optimized objective function is equal to λ. Algorithms for solving (22) or (23)-(24) are referred to here as *cross-SCORE algorithms.*

It is easily shown that the maximum-eigenvalue eigenvectors of (23) and (24) both converge to the maximum-SINR solution given in (10) if $s(t)$ is the only received signal with self-coherence (or conjugate self-coherence) at α. In this environment the Hermitian matrix on the right-hand side of (23) reduces to a rank-1 matrix as $T \to \infty$,

$$\hat{\mathbf{R}}_{\mathbf{xu}} \hat{\mathbf{R}}_{\mathbf{uu}}^{-1} \hat{\mathbf{R}}_{\mathbf{ux}} \to g \mathbf{A}\mathbf{A}^{\dagger},\; g = (\mathbf{A}^{\dagger} \mathbf{R}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}^{-1} \mathbf{A}) |R^{\alpha}_{ss^{(*)}}(\tau)|^2. \tag{25}$$

For an M-element antenna array, the eigenvectors of (23) will therefore converge to $M-1$ *noise-capture* solutions where $\mathbf{w}$ is orthogonal to $\mathbf{A}$ and λ is equal to zero and one *SOI-capture* solution where $\mathbf{w}$ is equal to $\mathbf{w}_{\text{max}}$ and λ is approximately equal to the transmitted SOI self-coherence,

$$\lambda_{\text{max}} \to (\mathbf{A}^{\dagger} \mathbf{R}_{\mathbf{xx}}^{-1} \mathbf{A})(\mathbf{A}^{\dagger} \mathbf{R}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}^{-1} \mathbf{A}) |\rho^{\alpha}_{s\hat{s}^{(*)}}|^2 \tag{26}$$

$$= \frac{|\rho^{\alpha}_{s\hat{s}^{(*)}}|^2}{(1+\gamma_x^{-2})(1+\gamma_{\hat{x}}^{-2})} \lesssim |\rho^{\alpha}_{s\hat{s}^{(*)}}|^2 \tag{27}$$

where γ_x^2 is the maximum-attainable SINR of $s(t)$ for the input data $\mathbf{x}(t)$ and $\gamma_{\hat{x}}^2$ is the maximum-attainable SINR of filtered SOI $\hat{s}(t)$ for the filtered input data $\hat{\mathbf{x}}(t)$. Similarly, $\hat{\mathbf{R}}_{\mathbf{ux}} \hat{\mathbf{R}}_{\mathbf{xx}}^{-1} \hat{\mathbf{R}}_{\mathbf{xu}}$ reduces to a rank-1 matrix in this environment as $T \to \infty$, and the eigenvectors of (24) converge to $M-1$ noise-capture solutions and one SOI-capture solution where $\mathbf{c} \propto \mathbf{R}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}^{-1} \mathbf{A} R_{\hat{s}\hat{s}}$. For this reason, the environment where only one signal is self-coherent at the target α of the SCORE processor is referred to here as the *rank-1 self-coherence environment.*

4 Performance in the Rank-1 Self-Coherence Environment

The simulator setup for the rank-1 self-coherence environment is as follows. A four-element circular array with a 10.24 MHz complex (bandpass) bandwidth, isotropic array sensors and a half-wavelength diameter is excited by white Gaussian noise, two PCM SOIs, and FM and TV interference signals. Both of the PCM signals are transmitted using Nyquist-shaped modulation pulses with 100% rolloff ($\cos^2(\frac{\pi}{2}\frac{f}{f_s})$ pulse frequency responses). The FM signal consists of a carrier frequency-modulated by a 60-552 kHz noise-loaded baseband with a 200 kHz rms frequency deviation (120-channel FDM-FM). The TV signal simulates a horizontal-synchronization pulse-train with a 15.625 kHz line rate (CCIR standard). The received data vector is converted to complex-baseband representation prior to adaptive processing. The signal parameters are given in Table 1 (*DOA* denotes direction-of-arrival and *SWNR* denotes *signal-to-white-noise-ratio*).

Table 1: Received Environment Parameters

Signal	Rate	Carrier	DOA	SWNR
16-QAM	3 Mb/s	0	−45°	15 dB
BPSK	4 Mb/s	0	60°	20 dB
FM	–	−500 kHz	30°	30 dB
TV	15.625 kHz	2 Mhz	−110°	40 dB

The two PCM SOIs are self-coherent at plus-or-minus their symbol rate, with maximum self-coherence strength of 1/6 (−8 dB) at $\tau = 0$. In addition, the BPSK SOI is conjugate self-coherent at 0 kHz, with a maximum conjugate self-coherence strength of 1 (0 dB) at $\tau = 0$. The TV signal is also self-coherent at multiples of 15.625 kHz, out to the bandwidth of the synch pulse ($\sim$ 2 MHz).

The least-squares SCORE processor is implemented using an RLS algorithm with a growing rectangular window. The weight update formula is given by

$$\mathbf{w}(n) = g(n)\, \hat{\mathbf{R}}_{\mathbf{xx}}^{-1}(n) \hat{\mathbf{R}}_{\mathbf{x}r}(n) \tag{28}$$

where $g(n)$ is a power-normalizing gain variable and $\hat{\mathbf{R}}_{(\cdot)}(n)$ denotes correlation with time-averaging over $[1, n]$,

$$\hat{\mathbf{R}}_{\mathbf{uv}}(n) \triangleq \sum_{k=0}^{n-1} \mathbf{u}(n-k) \mathbf{v}^{\dagger}(n-k). \tag{29}$$

The cross-SCORE processor is implemented using a *stochastic Rayleigh-Quotient algorithm,*

$$\mathbf{c}(n) = g_c(n)\, \hat{\mathbf{R}}_{\mathbf{uu}}^{-1}(n) \hat{\mathbf{R}}_{\mathbf{ux}}(n)\, \mathbf{w}(n-1) \tag{30}$$

$$\mathbf{w}(n) = g_w(n)\, \hat{\mathbf{R}}_{\mathbf{xx}}^{-1}(n) \hat{\mathbf{R}}_{\mathbf{xu}}(n)\, \mathbf{c}(n). \tag{31}$$

Equations (30)-(31) form a two-step procedure for finding the maximum solution to (24) using a Rayleigh-Quotient (power maximization) procedure. This algorithm converges particularly quickly in the rank-1 self-coherence environment, due to the very wide spread between the maximum and lesser eigenvalues of (24)& (23).

The performance measure used to judge the quality of the processor output signal is the *output SINR,*

$$\text{SINR} \triangleq |\mathbf{w}^{\dagger} \mathbf{A}|^2 R_{ss} / \mathbf{w}^{\dagger} \mathbf{R}_{\mathbf{II}} \mathbf{w} \tag{32}$$

where $\mathbf{A}$ and R_{ss} are the true aperture and power of the SOI and $\mathbf{R}_{\mathbf{II}}$ is the true autocorrelation matrix of the interference (noise and other signals) in the environment.

Figure 2 shows the performance obtained using the SCORE processors to capture the BPSK SOI. Two processor modes are tested: a *baud-rate restoral* mode where α is set to 4 MHz, τ is set to zero ($h(t) = \delta(t)$) and the conjugation is disabled, and a *carrier restoral* mode where α and τ are both set to zero and the conjugation is enabled. The control vector is set to $[1, 0, 0, 0]^T$ (isotropic antenna pattern) in the least-squares SCORE processor. This Figure verifies the theoretical (infinite time-average) results obtained in Section 3, and illustrates the differing convergence rates of these processors. The cross-SCORE processor converges to within 3 dB of the maximum-attainable SINR in under 100 SOI bauds in baud-rate restoral mode, and in under 20 SOI bauds in carrier restoral mode. However, the least-squares SCORE processor converges much slower: the processor performance only comes to within 5 dB of the maximum SINR after 350 SOI bauds in baud-rate restoral mode, and fails to significantly extract the SOI after even 800 SOI bauds in carrier restoral mode.

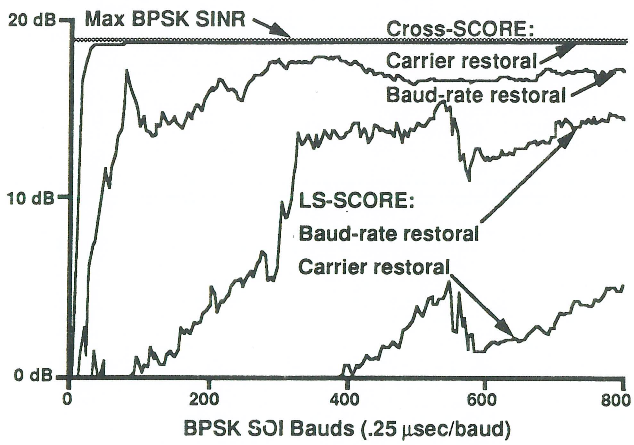

Figure 2: SCORE Performance for BPSK SOI

The relatively slow convergence of the least-squares SCORE processor is due to the large uncorrelated interference component $i_\alpha(t)$ present in the reference signal $r(t)$ when an arbitrary control vector is used on the control path. Until $i_\alpha(t)$ is averaged out by the correlation process, this corruption component will have a strong effect on the adaptation of the processor weights. This effect is greatly reduced when the control vector is also adapted to restore self-coherence; the dominant corruption component remaining in $r(t)$ after $\mathbf{c}$ is optimized is the irreducible self-interference component (7), which is small if $|\rho^{\alpha}_{s\hat{s}(*)}|$ is close to unity. This also explains why the cross-SCORE processor converges much faster in carrier restoral mode than in baud-rate restoral mode: the SOI conjugate self-coherence at $\alpha = 0$ is six times stronger than the SOI self-coherence at $\alpha = 4$ MHz.

Figure 3 shows the performance of the cross-SCORE processor when it is configured to restore the baud-rate of the BPSK SOI ($\alpha = 4$ MHz, $\tau = 0$, conjugation disabled), and when it is configured to restore the baud-rate of the 16-QAM SOI (same τ and conjugation; $\alpha = 4$ MHz). This Figure demonstrates a key feature of the SCORE processor: the ability to *sort* through interference environments and extract SOIs on the basis of their differing self-coherence properties. When the frequency-shift is set to 4 MHz, the SCORE processor captures the BPSK SOI; changing α to 3 MHz, however, causes the SCORE processor to reject the BPSK SOI and capture the 16-QAM signal. In both cases the processor converges to within 3 dB of the optimal performance within 100 (BPSK) SOI bauds.

Figure 4 shows the performance of the carrier restoring cross-SCORE processor for varying degree of error in the target α. This Figure illustrates the ability of the SCORE processor to tolerate error in the assumed SOI self-coherence, a particularly important problem when conjugate self-coherence (which is affected by Doppler shift) is being restored by the processor. In all cases where α is in error the processor eventually rejects the BPSK SOI; however, as Figure 4 shows, the time required to accomplish this is long when the error is small. Furthermore, even when the error is large and the SOI capture time is short, the SINR of the captured SOI can reach a high value before the SCORE processor begins to reject the SOI. In many applications, this SINR will be high enough to allow a more robust (but less discriminatory) algorithm such as a CMA to take over adaptation of the array. The capture time may also be long enough to allow the error in α to be estimated and removed over the SOI transmission time.

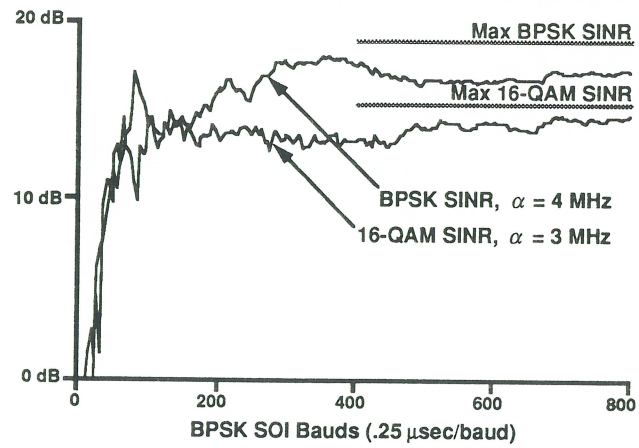

Figure 3: Signal Sorting Via Self-Coherence Frequency

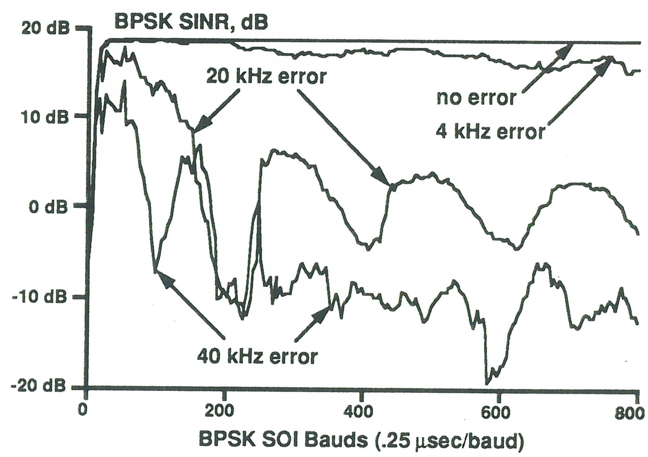

Figure 4: Tolerance to Target Self-Coherence Error

5 Performance in the Rank-L Self-Coherence Environment

The performance of the SCORE processors in the *rank-L self-coherence environment* where L signals with (conjugate) self-coherence at the target α are received by the array is also of importance. Two environments are of particular interest: the *multipath* environment where L correlated signals (reflections) with (conjugate) self-coherence at α are impinging on the array, and the *multiple-SOI* environment where L *un*correlated signals with (conjugate) self-coherence at the same α are impinging on the array.

A comprehensive examination of the SCORE processor performance in this environment cannot be reported here due to lack of space. However, it can be shown [9,10] that the cross-SCORE eigenequations (23) and (24) will have L linearly-independent solutions with $\lambda > 0$ in these environments, correponding to capture of the L received signals that are (conjugate) self-coherent at α and optimal rejection of background noise and any other received signals that are *not* (conjugate) self-coherent at α. Moreover, if self-coherence is being restored by the processor (conjugation is disabled) and the L signals are uncorrelated with each other (multiple-SOI environment) and have distinct self-coherence strengths, then each of these solutions will correspond to near-optimal capture of each *individual* SOI. At the least, therefore, the SCORE processor will be able to *screen out* interference in environments containing multipath interference or uncorrelated SOIs with the same self-coherence properties; in many of these environments, the SCORE processor will also be able to *separate* SOIs on the basis of self-coherence strength as well as frequency.

Figure 5 demonstrates this last property by replacing the 3 Mb/s 16-QAM SOI with a 4 Mb/s BPSK signal with 50% Nyquist rolloff in the environment simulated in Section 4. The two BPSK signals are uncorrelated and have distinct self-coherence strengths at $\alpha = 4$ MHz, $\tau = 0$ (1/6 for the SOI with 100% Nyquist rolloff vs. 1/14 for the SOI with 50% Nyquist rolloff); consequently, the two highest-eigenvalue solutions to (23) should capture each of the SOIs. This is shown in Figure 5: the eigenvector corresponding to the largest eigenvalue of (23) captures the BPSK signal with 100% Nyquist rolloff, while the eigenvector corresponding to the next-largest eigenvalue of (23) captures the BPSK signal with 50% Nyquist rolloff. In both cases, the SINR of the captured signals converge to within 3 dB of their maximum attainable value.

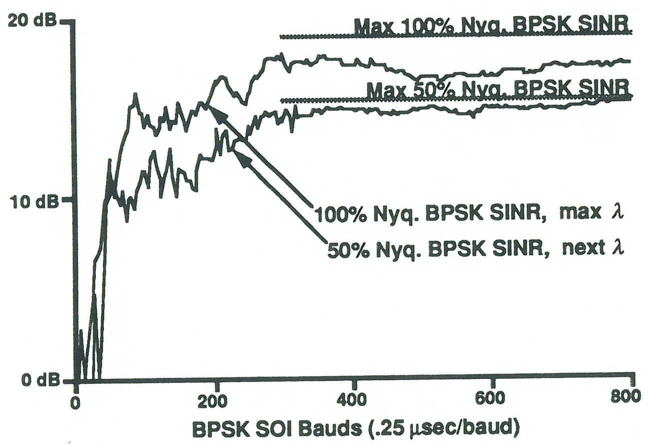

Figure 5: Signal Sorting Via Eigenequation Solution

6 Conclusions

A new class of algorithms for blind adaptation of antenna arrays, the *self-coherence restoral (SCORE)* algorithms, have been introduced. Two new processor architectures, the *least-squares SCORE processor* and the *cross-SCORE processor* have been developed, analyzed and simulated in the *rank-1* and *rank-L* self-coherence environments where 1 to L signals with self-coherence or conjugate self-coherence at a known value of frequency separation and arbitrary interference without (conjugate) self-coherence at that value of frequency-shift are received by an antenna array. It has been shown analytically and via computer simulations that the SCORE processors will capture a SOI with maximum SINR in the rank-1 self-coherence environment, given only knowledge of the self-coherence properties of the SOI, e.g., the SOI symbol-rate or carrier frequency. It has also been shown via computer simulation that the cross-SCORE processor can capture SOIs even if their self-coherence properties are only approximately known (e.g., to within 4 kHz Doppler shift), and that the cross-SCORE processor can capture SOIs with near-optimum SINR in the rank-L self-coherence environment. These properties have been used to demonstrate the ability of the SCORE processor to *sort* through environments to extract and separate multiple PCM SOIs on the basis of their differing symbol-rate or (if their symbol rates are equal) differing self-coherence strength.

These results show that the SCORE approach provides a promising alternative to the existing blind adaptation techniques. The SCORE processors have unambiguous and analytically tractable convergence and capture properties, giving them an advantage over property restoral techniques in automatic processing applications. The SCORE algorithms also do not require knowledge of (or impose constraints on) the sensor array geometry or individual sensor characteristics to operate, and have simple and flexible RLS-based implementations, giving them cost and complexity advantages over the spatial coherence-exploiting techniques. The highly discriminatory capture properties of the SCORE approach make it ideal for directed-search applications where a few SOIs with well-known modulation properties must be extracted from a dense interference environment.

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